The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, August 17, 2022 — 12:30 to 3:30 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

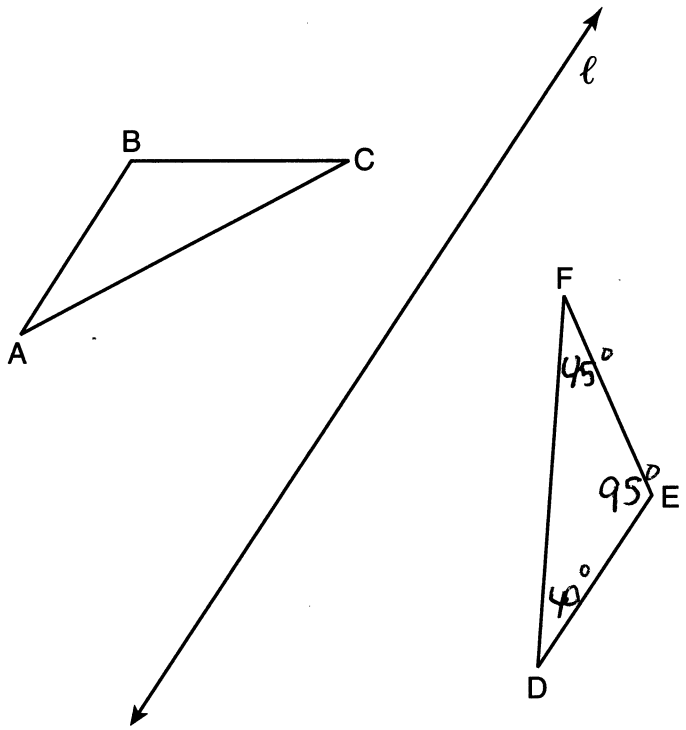
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, $\triangle ABC$ is reflected over line ℓ to create $\triangle DEF$.

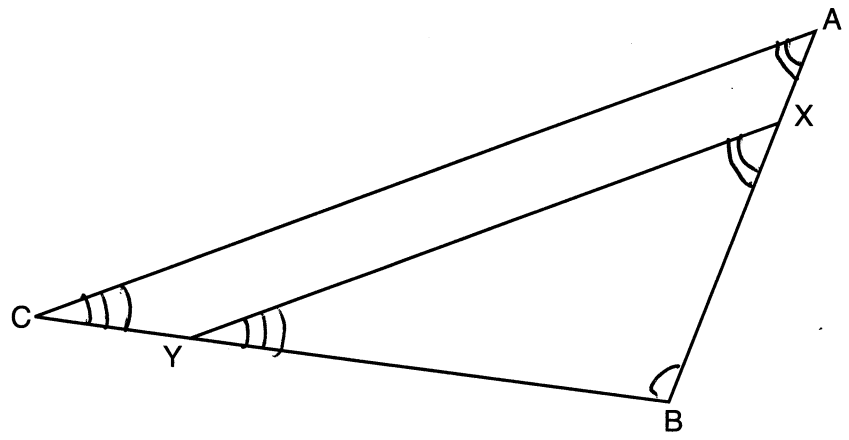


If $m\angle A = 40°$ and $m\angle B = 95°$, what is $m\angle F$?

(1) 40°
(2) 45°
(3) 85°
(4) 95°

Use this space for computations.

2 The diagram below shows triangle ABC with point X on side $\overline{AB}$ and point Y on side $\overline{CB}$.



Which information is sufficient to prove that $\triangle BXY \sim \triangle BAC$?

(1) $\angle B$ is a right angle.
(2) $\overline{XY}$ is parallel to $\overline{AC}$.
(3) $\triangle ABC$ is isosceles.
(4) $\overline{AX} \cong \overline{CY}$

3 Quadrilateral *MATH* is congruent to quadrilateral *WXYZ*. Which statement is always true?

(1) $MA = XY$
(2) $m\angle H = m\angle W$
(3) Quadrilateral *WXYZ* can be mapped onto quadrilateral *MATH* using a sequence of rigid motions.
(4) Quadrilateral *MATH* and quadrilateral *WXYZ* are the same shape, but not necessarily the same size.

4 A quadrilateral has diagonals that are perpendicular but *not* congruent. This quadrilateral could be

(1) a square
(2) a rhombus
(3) a rectangle
(4) an isosceles trapezoid

Use this space for computations.

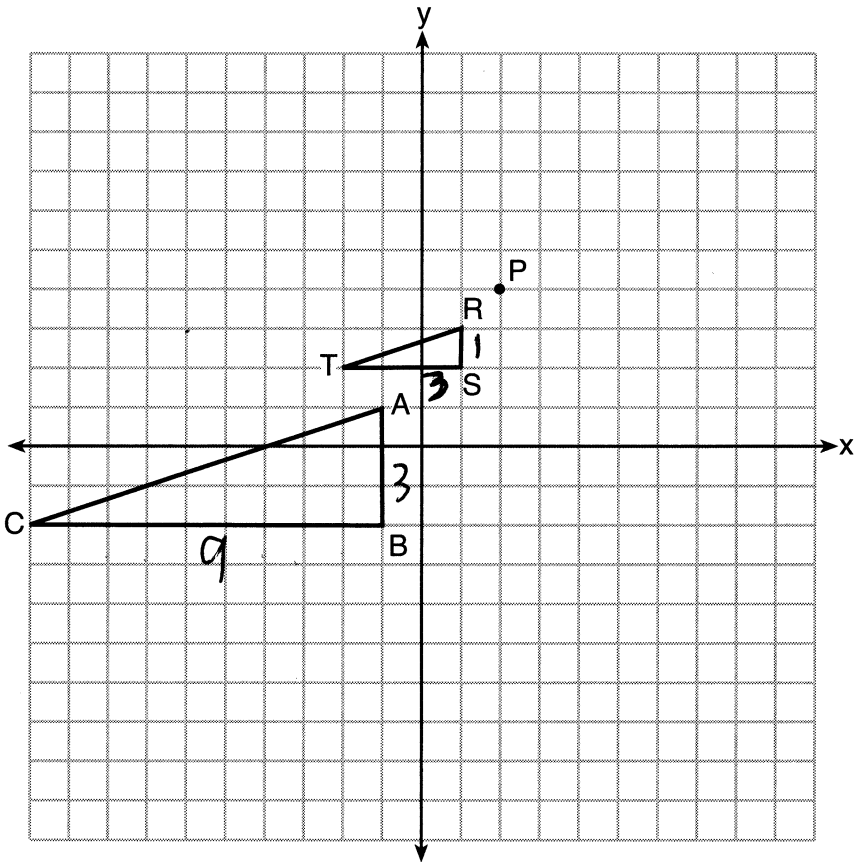
5 Which regular polygon has a minimum rotation of 36° about its center that carries the polygon onto itself?

(1) pentagon
(3) nonagon
(2) octagon
(4) decagon

$\frac{360}{n} = 36$

$n = 10$

6 On the set of axes below, $\triangle RST$ is the image of $\triangle ABC$ after a dilation centered at point P.

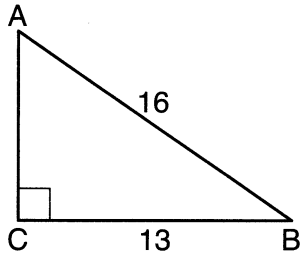


The scale factor of the dilation that maps $\triangle ABC$ onto $\triangle RST$ is

(1) $\frac{1}{3}$
(3) 3
(2) 2
(4) $\frac{2}{3}$

Use this space for computations.

7 In the diagram of $\triangle ABC$ below, $m\angle C = 90°$, $CB = 13$, and $AB = 16$.



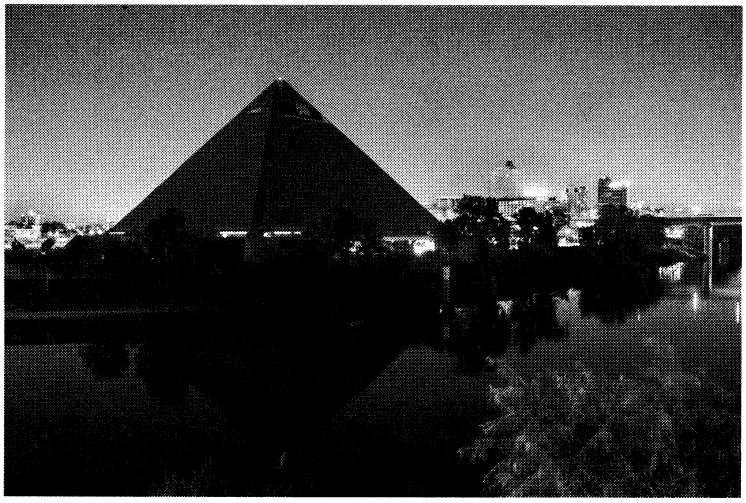
$\sin A = \frac{13}{16}$

$A \approx 54$

What is the measure of $\angle A$, to the *nearest degree*?

(1) 36°
(2) 39°
(3) 51°
(4) 54°

8 The Pyramid of Memphis, in Tennessee, stands 107 yards tall and has a square base whose side is 197 yards long.



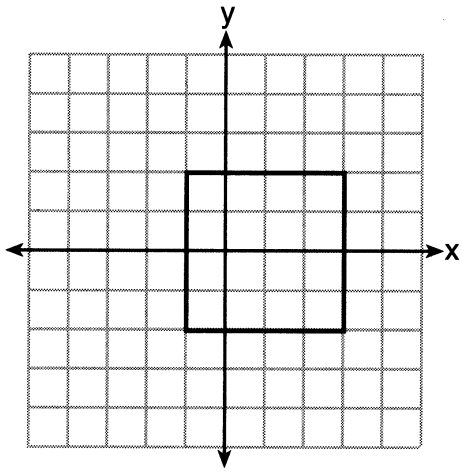
What is the volume of the Pyramid of Memphis, to the *nearest cubic yard*?

(1) 751,818
(2) 1,384,188
(3) 2,076,212
(4) 4,152,563

$\frac{1}{3}(197)^2(107)$

Use this space for computations.

9 A square is graphed on the set of axes below, with vertices at $(-1,2)$, $(-1,-2)$, $(3,-2)$, and $(3,2)$.



Which transformation would *not* carry the square onto itself?

(1) reflection over the y-axis

(2) reflection over the x-axis

(3) rotation of 180 degrees around point (1,0)

(4) reflection over the line $y = x - 1$

10 If scalene triangle XYZ is similar to triangle QRS and $m\angle X = 90°$, which equation is always true?

(1) $\sin Y = \sin S$

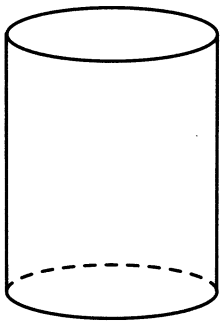
(2) $\cos R = \cos Z$

(3) $\cos Y = \sin Q$

(4) $\sin R = \cos Z$

Use this space for computations.

11 A plane intersects a cylinder perpendicular to its bases.



This cross section can be described as a

(1) rectangle
(2) parabola
(3) triangle
(4) circle

12 An equation of line p is $y = \frac{1}{3}x + 4$. An equation of line q is $y = \frac{2}{3}x + 8$. Which statement about lines p and q is true?

(1) A dilation of $\frac{1}{2}$ centered at the origin will map line q onto line p.
(2) A dilation of 2 centered at the origin will map line p onto line q.
(3) Line q is not the image of line p after a dilation because the lines are not parallel.
(4) Line q is not the image of line p after a dilation because the lines do not pass through the origin.

13 The coordinates of the endpoints of $\overline{SC}$ are $S(-7,3)$ and $C(2,-6)$. If point M is on $\overline{SC}$, what are the coordinates of M such that $SM:MC$ is 1:2?

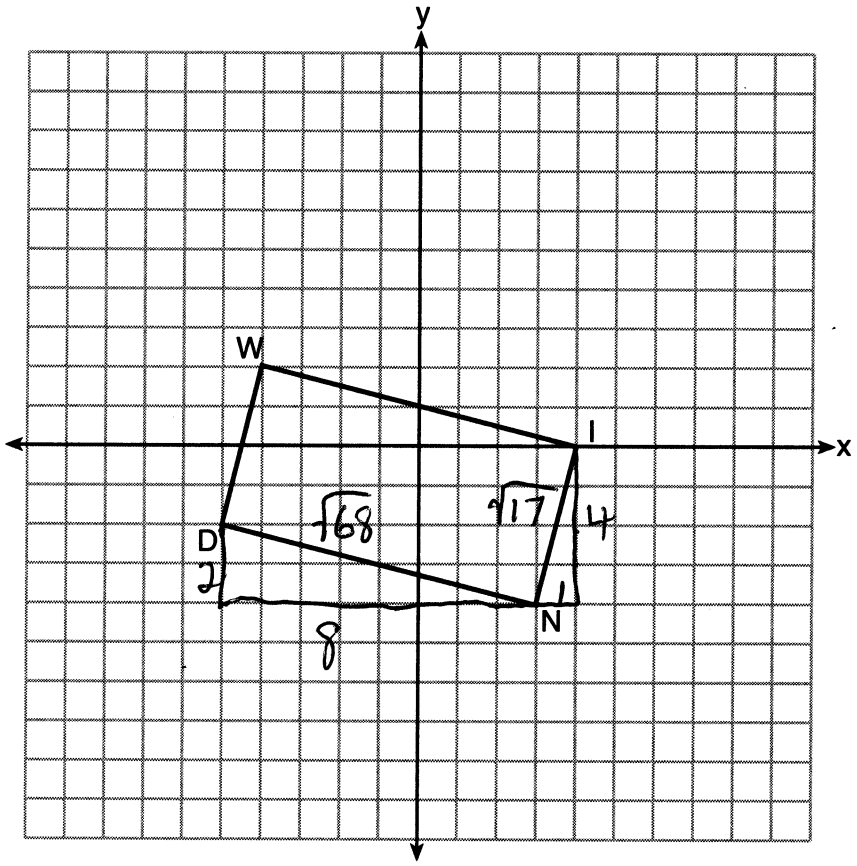
(1) $(-4,0)$
(2) $(0,-4)$
(3) $(-1,-3)$
(4) $(-\frac{5}{2},-\frac{3}{2})$

$-7 + \frac{1}{3}(2 - -7) = -4$

$3 + \frac{1}{3}(-6-3) = 0$

Use this space for computations.

14 On the set of axes below, rectangle *WIND* has vertices with coordinates $W(-4,2)$, $I(4,0)$, $N(3,-4)$, and $D(-5,-2)$.



What is the area of rectangle *WIND*?

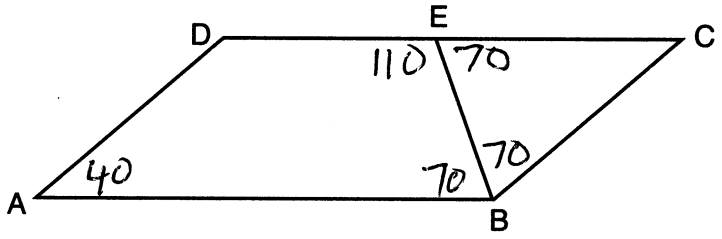
(1) 17
(2) 31
(3) 32
(4) 34

$\sqrt{68} \cdot \sqrt{17}$

$\sqrt{4} \cdot \sqrt{17}\sqrt{17}$

$2 \cdot 17$

15 In parallelogram *ABCD* shown below, $\overline{EB}$ bisects $\angle ABC$.



If $m\angle A = 40°$, then $m\angle BED$ is

(1) 40°
(2) 70°
(3) 110°
(4) 140°

Assume isosceles

Use this space for computations.

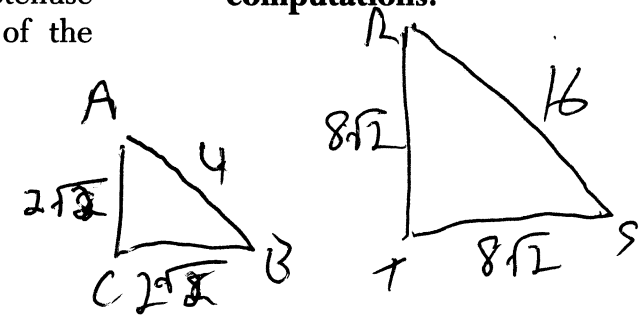
16 In right triangles ABC and RST, hypotenuse $AB = 4$ and hypotenuse $RS = 16$. If $\triangle ABC \sim \triangle RST$, then 1:16 is the ratio of the corresponding

(1) legs 1:4

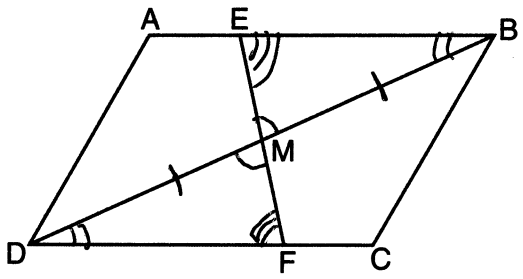
(2) areas

(3) ~~volumes~~

(4) perimeters 1:4



17 Parallelogram $ABCD$ with diagonal $\overline{DB}$ is drawn below. Line segment EF is drawn such that it bisects $\overline{DB}$ at M.



Which triangle congruence method would prove that $\triangle EMB \cong \triangle FMD$?

(1) ASA, only

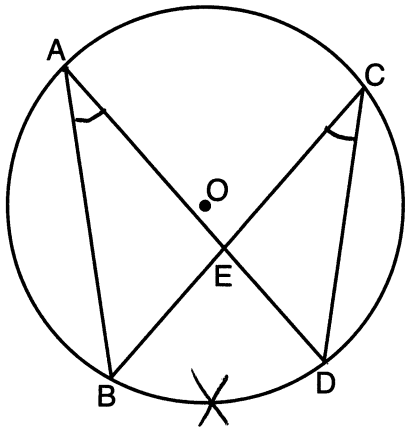
(2) AAS, only

(3) both ASA and AAS

(4) neither ASA nor AAS

Use this space for computations.

18 In the diagram below of circle O, chords $\overline{AD}$ and $\overline{BC}$ intersect at E, and chords $\overline{AB}$ and $\overline{CD}$ are drawn.



Which statement must always be true?

(1) $\overline{AB} \cong \overline{CD}$

(2) $\overline{AD} \cong \overline{BC}$

(3) $\angle B \cong \angle C$

(4) $\angle A \cong \angle C$

19 What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + y^2 - 12y - 20.25 = 0$?

(1) center (0,6) and radius 7.5

(2) center (0,−6) and radius 7.5

(3) center (0,12) and radius 4.5

(4) center (0,−12) and radius 4.5

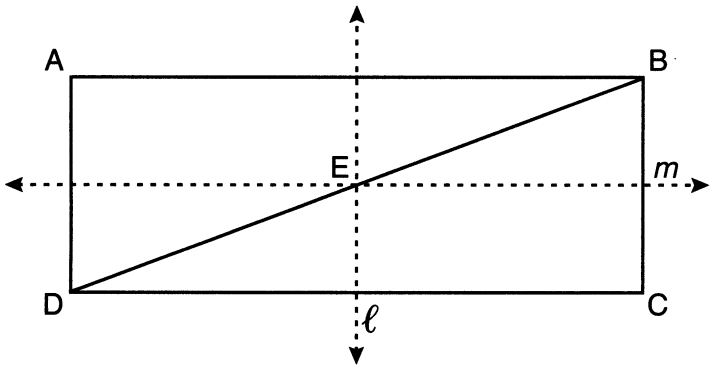
$x^2 + y^2 - 12x + 36 = 20.25 + 36$

$x^2 + (y-6)^2 = 56.25$

$\sqrt{56.25} = 7.5$

Use this space for computations.

20 In the diagram below, $ABCD$ is a rectangle, and diagonal $\overline{BD}$ is drawn. Line ℓ, a vertical line of symmetry, and line m, a horizontal line of symmetry, intersect at point E.



Which sequence of transformations will map $\triangle ABD$ onto $\triangle CDB$?

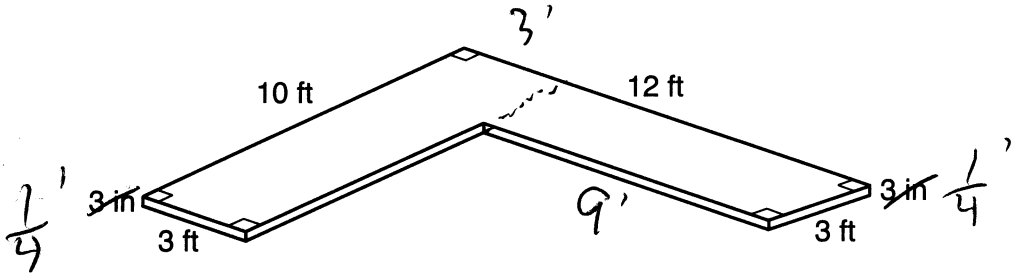
(1) a reflection over line ℓ followed by a 180° rotation about point E

(2) a reflection over line ℓ followed by a reflection over line m

(3) a 180° rotation about point B

(4) a reflection over $\overline{DB}$

21 The diagram below models a countertop designed for a kitchen. The countertop is made of solid oak and is 3 inches thick.



If oak weighs approximately 44 pounds per cubic foot, the approximate weight, in pounds, of the countertop is

(1) 630

(2) 730

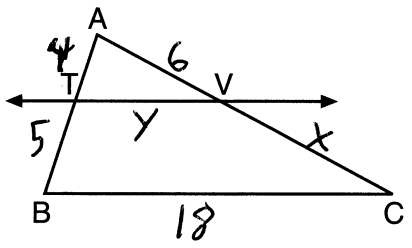
(3) 750

(4) 870

$44\left(\left(10 \times 3 \times \frac{1}{4}\right) + \left(9 \times 3 \times \frac{1}{4}\right)\right) = 627$

Use this space for computations.

22 In the diagram below of $\triangle ABC$, $\overleftrightarrow{TV}$ intersects $\overline{AB}$ and $\overline{AC}$ at points T and V respectively, and $m\angle ATV = m\angle ABC$.



$\frac{4}{5} = \frac{6}{x}$

$x = 7.5$

$\frac{4}{9} = \frac{y}{18}$

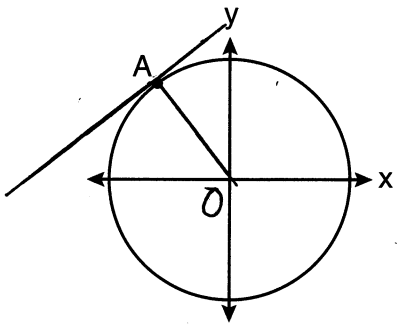
$y = 8$

If $AT = 4$, $BC = 18$, $TB = 5$, and $AV = 6$, what is the perimeter of quadrilateral $TBCV$?

(1) 38.5 $= 5 + 18 + 7.5 + 8$ (3) 40.5

(2) 39.5 (4) 44.9

23 A circle centered at the origin passes through $A(-3,4)$.



slope of $\overline{OA}$ is $-\frac{4}{3}$

$m_{\perp} = \frac{3}{4}$

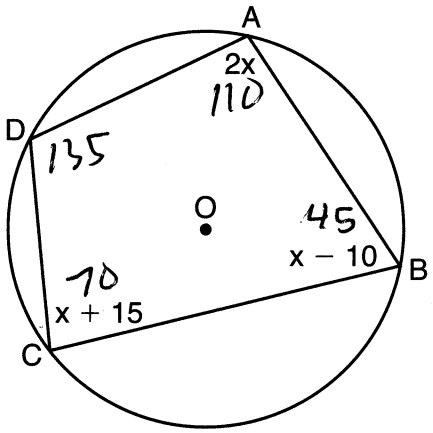
What is the equation of the line tangent to the circle at A?

(1) $y-4=\frac{4}{3}(x+3)$ (3) $y+4=\frac{4}{3}(x-3)$

(2) $y-4=\frac{3}{4}(x+3)$ (4) $y+4=\frac{3}{4}(x-3)$

Use this space for computations.

24 In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, $m\angle A = (2x)°$, $m\angle B = (x-10)°$, and $m\angle C = (x+15)°$.



$2x + x + 15 = 180$

$3x = 165$

$x = 55$

$180 - 45 = 135$

What is $m\angle D$?

(1) 55°

(2) 70°

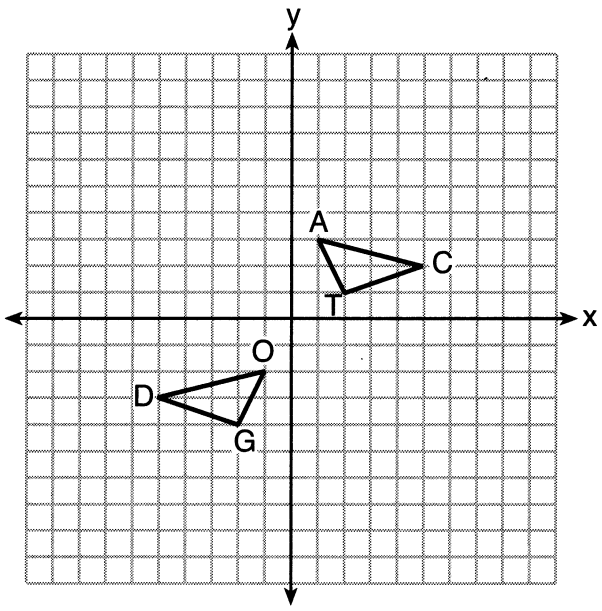
(3) 110°

(4) 135°

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

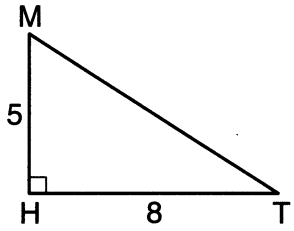
25 On the set of axes below, $\triangle DOG \cong \triangle CAT$.



Describe a sequence of transformations that maps $\triangle DOG$ onto $\triangle CAT$.

Reflect over y-axis
Translate 5 up

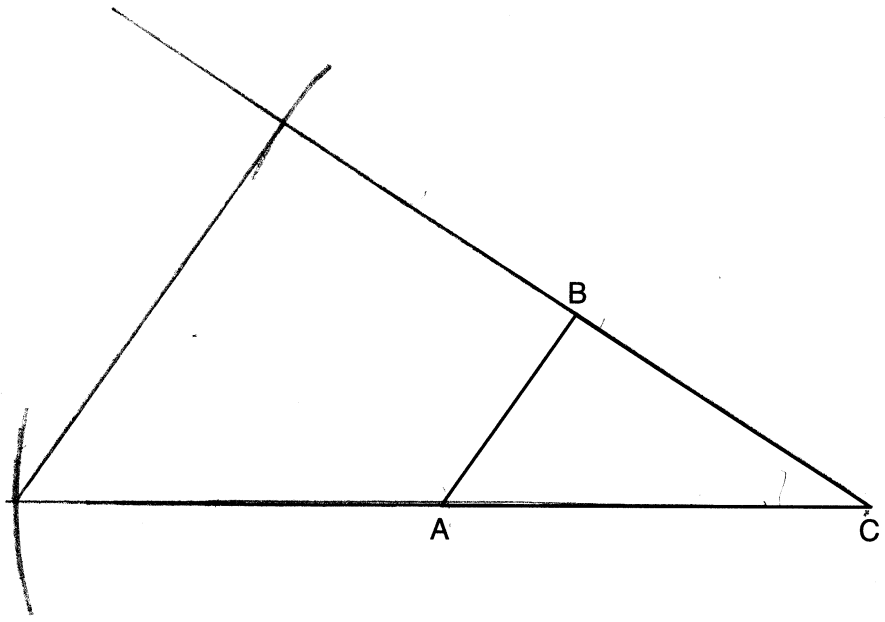
26 In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$.



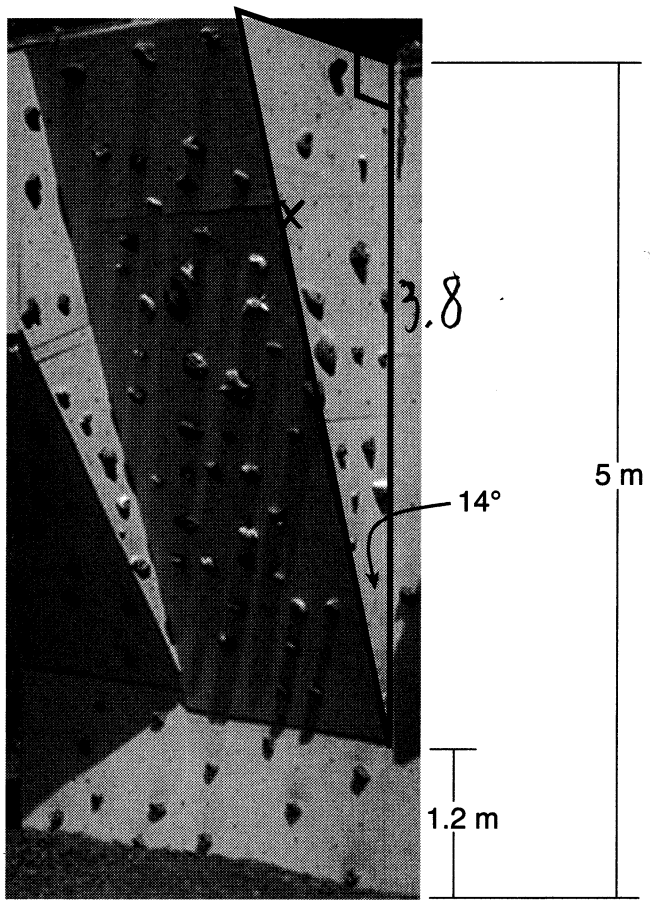
Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

$\frac{1}{3}\pi(8^2)5 \approx 335.1$

27 Using a compass and straightedge, dilate triangle ABC by a scale factor of 2 centered at C. [Leave all construction marks.]



28 A rock-climbing wall at a local park has a right triangular section that slants toward the climber, as shown in the picture below. The height of the wall is 5 meters and the slanted section begins 1.2 meters up the wall at an angle of 14 degrees.

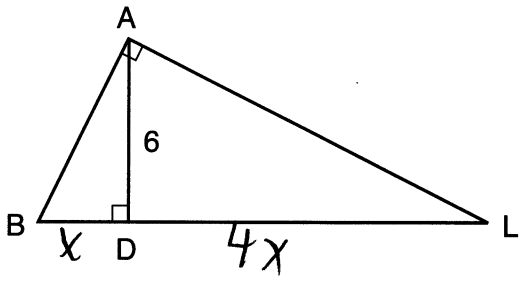


Determine and state, to the *nearest hundredth*, the number of meters in the length of the section of the wall that is slanted (hypotenuse).

$\cos 14^\circ = \frac{3.8}{X}$

$X \approx 3.92$

29 In the diagram below of right triangle BAL, altitude $\overline{AD}$ is drawn to hypotenuse $\overline{BDL}$. The length of $\overline{AD}$ is 6.



If the length of $\overline{DL}$ is four times the length of $\overline{BD}$, determine and state the length of $\overline{BD}$.

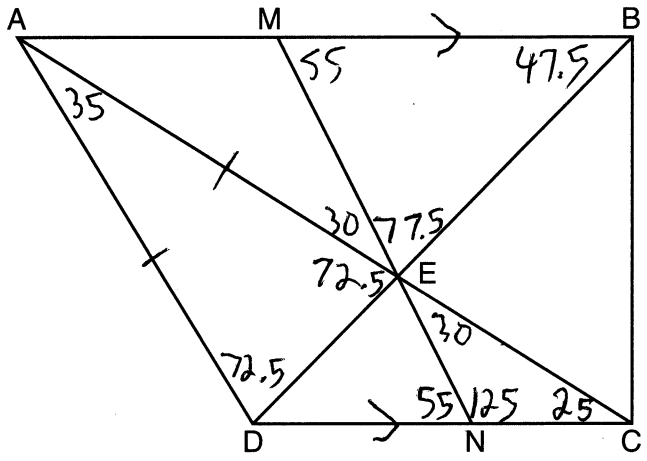
$4x \cdot x = 6^2$

$4x^2 = 36$

$x^2 = 9$

$x = 3$

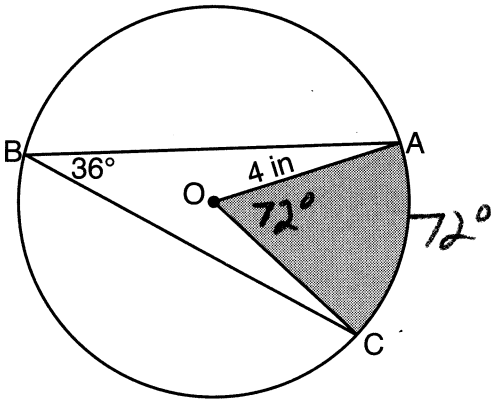
30 Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$.



If $m\angle DAE = 35°$, $m\angle DCE = 25°$, and $m\angle NEC = 30°$, determine and state $m\angle ABD$.

47.5

31 In the diagram below of circle O, the measure of inscribed angle ABC is 36° and the length of $\overline{OA}$ is 4 inches.



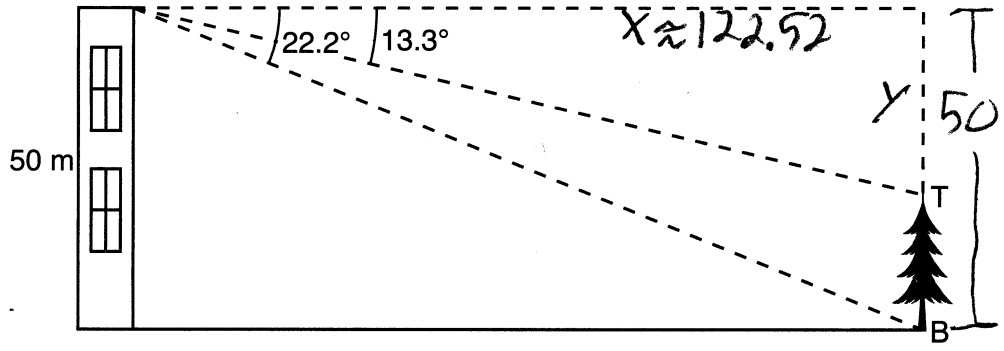
Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.

$\left(\frac{72}{360}\right)\pi(4^2) \approx 10.1$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°.



Determine and state, to the *nearest meter*, the height of the tree.

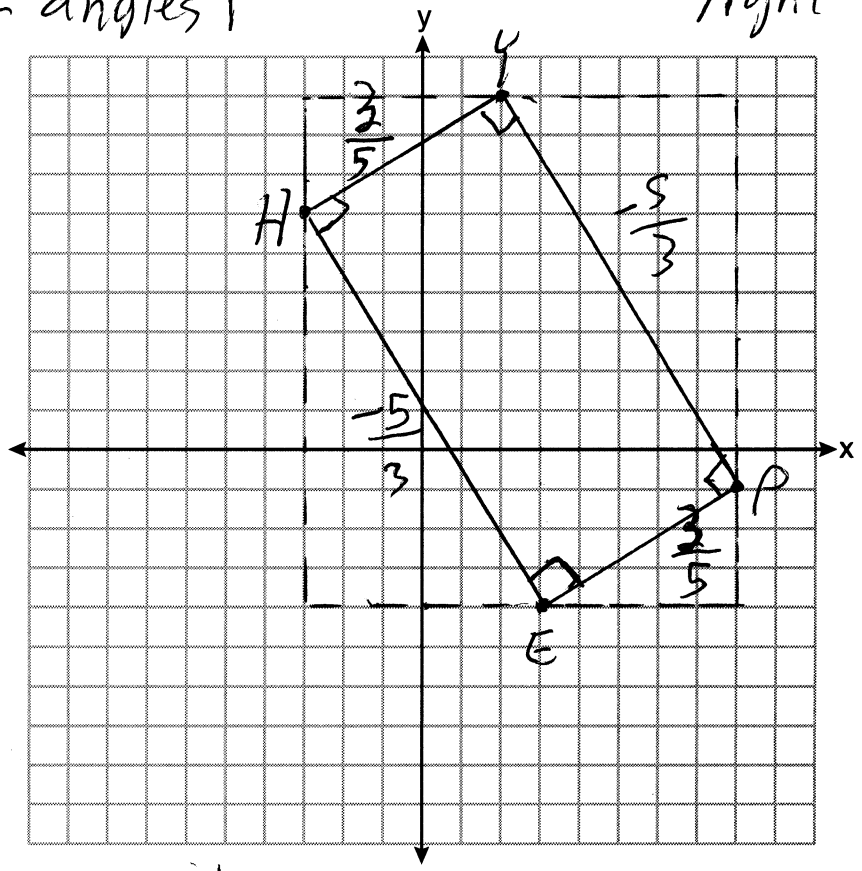
$\tan 22.2 = \frac{50}{X}$ $X \approx 122.52$

$\tan 13.3 = \frac{Y}{122.52}$ $Y \approx 29$

$50 - 29 = 21$

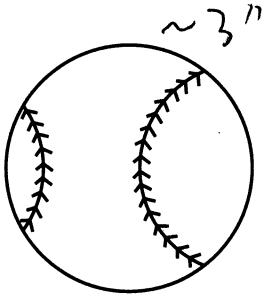
33 The coordinates of the vertices of quadrilateral $HYPE$ are $H(-3,6)$, $Y(2,9)$, $P(8,-1)$, and $E(3,-4)$. Prove $HYPE$ is a rectangle. [The use of the set of axes below is optional.]

① Quad $HYPE$ at $H(-3,6)$, $Y(2,9)$, $P(8,-1)$ $E(3,-4)$	① Given
② Slope of $\overline{HY}$ and $\overline{PE}$ is $\frac{3}{5}$; slope of $\overline{YP}$ and $\overline{EH}$ is $-\frac{5}{3}$	② Slope formula graphically
③ $\overline{HY} \perp \overline{YP}$; $\overline{PE} \perp \overline{EH}$; $\overline{YP} \perp \overline{PE}$; $\overline{EH} \perp \overline{HY}$	③ The slopes of perpendicular lines are opposite reciprocals
④ $\angle H, \angle Y, \angle P, \angle E$ are right angles	④ Perpendicular lines form right angles
⑤ $HYPE$ is a rectangle	⑤ A rectangle has four right $\angle$s.



34 A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.

24" X 12" X 18"



Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs.

8 X 4 X 6 = 192

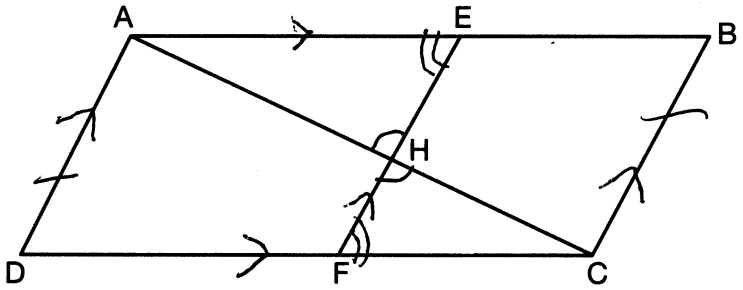
The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

$192 \cdot \left(\frac{4}{3}\pi\right)\left(\frac{2.94}{2}\right)^3 (.025) \approx 64$ lbs

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Quadrilateral $ABCD$, $\overline{AC}$ and $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, and $\overline{AD} \cong \overline{BC}$



Prove: $(EH)(CH) = (FH)(AH)$

1) Quad $ABCD$, $\overline{AC}$ & $\overline{EF}$ intersect at H, $\overline{EF} \parallel \overline{AD}$, $\overline{EF} \parallel \overline{BC}$, $\overline{AD} \cong \overline{BC}$	1) Given
2) $\angle EHA \cong \angle FHC$	2) Vertical angles are congruent
3) $\overline{AD} \parallel \overline{BC}$	3) Transitive property of parallel lines
4) $ABCD$ is a parallelogram	4) Quadrilateral with a pair of sides both parallel & congruent
5) $\overline{AB} \parallel \overline{CD}$	5) Opposite sides of a parallelogram
6) $\angle AEH \cong \angle CFH$	6) Alternate interior angles formed by parallel lines & transversal
7) $\triangle AEH \sim \triangle CFH$	7) AA
8) $\frac{EH}{FH} = \frac{AH}{CH}$	8) Corresponding sides of similar triangles are proportional
9) $(EH)(CH) = (FH)(AH)$	9) Product of means equals product of extremes