

The University of the State of New York  
REGENTS HIGH SCHOOL EXAMINATION**GEOMETRY**

Wednesday, August 14, 2019 — 12:30 to 3:30 p.m., only

Student Name: Mr. SibulSchool Name: John Dewey HS

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**

**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

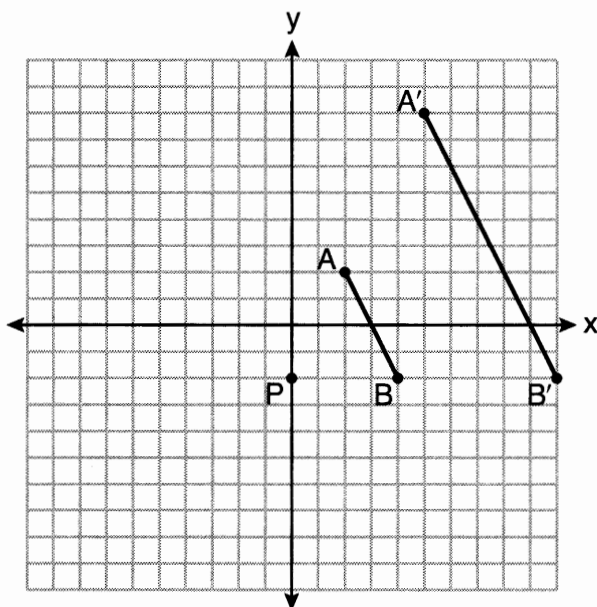
**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

- 1 On the set of axes below,  $\overline{AB}$  is dilated by a scale factor of  $\frac{5}{2}$  centered at point  $P$ .



Which statement is always true?

- (1)  $\overline{PA} \cong \overline{AA'}$  (3)  $AB = A'B'$   
 (2)  $\overline{AB} \parallel \overline{A'B'}$  (4)  $\frac{5}{2}(A'B') = AB$

- 2 The coordinates of the vertices of parallelogram  $CDEH$  are  $C(-5,5)$ ,  $D(2,5)$ ,  $E(-1,-1)$ , and  $H(-8,-1)$ . What are the coordinates of  $P$ , the point of intersection of diagonals  $\overline{CE}$  and  $\overline{DH}$ ?

- (1)  $(-2,3)$  (3)  $(-3,2)$   
 (2)  $(-2,2)$  (4)  $(-3,-2)$

$$\frac{-5 + -1}{2} = -3 \quad \frac{5 + -1}{2} = 2$$

Use this space for computations.

3 The coordinates of the endpoints of  $\overline{QS}$  are  $Q(-9,8)$  and  $S(9,-4)$ . Point  $R$  is on  $\overline{QS}$  such that  $QR:RS$  is in the ratio of 1:2. What are the coordinates of point  $R$ ?

- (1) (0,2)
- (2) (3,0)

- (3) (-3,4)
- (4) (-6,6)

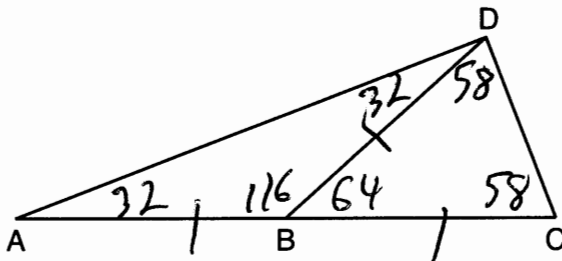
$$-9 + \frac{1}{3}(9 - (-9)) = -3$$

$$8 + \frac{1}{3}(-4 - 8) = 4$$

4 If the altitudes of a triangle meet at one of the triangle's vertices, then the triangle is

- (1) a right triangle
- (2) an acute triangle
- (3) an obtuse triangle
- (4) an equilateral triangle

5 In the diagram below of  $\triangle ACD$ ,  $\overline{DB}$  is a median to  $\overline{AC}$ , and  $\overline{AB} \cong \overline{DB}$ .



If  $m\angle DAB = 32^\circ$ , what is  $m\angle BDC$ ?

- (1)  $32^\circ$
- (2)  $52^\circ$
- (3)  $58^\circ$
- (4)  $64^\circ$

Use this space for computations.

6 What are the coordinates of the center and the length of the radius of the circle whose equation is  $x^2 + y^2 = 8x - 6y + 39$ ?

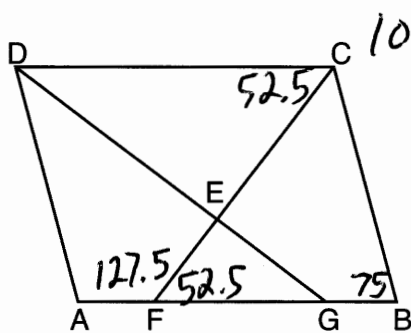
- (1) center  $(-4, 3)$  and radius 64
- (2) center  $(4, -3)$  and radius 64
- (3) center  $(-4, 3)$  and radius 8
- (4) center  $(4, -3)$  and radius 8

$$x^2 - 8x + y^2 + 6y = 39$$

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 39 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 64$$

7 In the diagram below of parallelogram  $ABCD$ ,  $\overline{AFGB}$ ,  $\overline{CF}$  bisects  $\angle DCB$ ,  $\overline{DG}$  bisects  $\angle ADC$ , and  $\overline{CF}$  and  $\overline{DG}$  intersect at  $E$ .



If  $m\angle B = 75^\circ$ , then the measure of  $\angle EFA$  is

- (1)  $142.5^\circ$
- (2)  $127.5^\circ$
- (3)  $52.5^\circ$
- (4)  $37.5^\circ$

8 What is an equation of a line that is perpendicular to the line whose equation is  $2y + 3x = 1$ ?

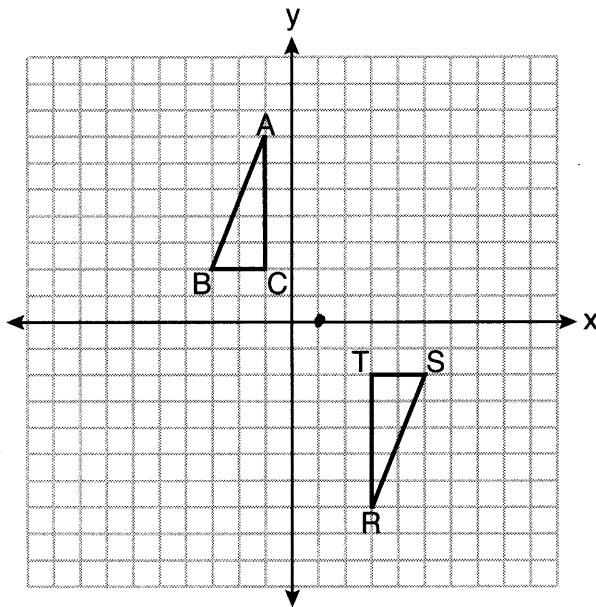
- (1)  $y = \frac{2}{3}x + \frac{5}{2}$
- (2)  $y = \frac{3}{2}x + 2$
- (3)  $y = -\frac{2}{3}x + 1$
- (4)  $y = -\frac{3}{2}x + \frac{1}{2}$

$$M = \frac{-A}{B} = \frac{-3}{2}$$

$$M_{\perp} = \frac{2}{3}$$

Use this space for computations.

9 Triangles  $ABC$  and  $RST$  are graphed on the set of axes below.



Which sequence of rigid motions will prove  $\triangle ABC \cong \triangle RST$ ?

- (1) a line reflection over  $y = x$
- (2) a rotation of  $180^\circ$  centered at  $(1, 0)$
- (3) a line reflection over the  $x$ -axis followed by a translation of 6 units right
- (4) a line reflection over the  $x$ -axis followed by a line reflection over  $y = 1$

10 If the line represented by  $y = -\frac{1}{4}x - 2$  is dilated by a scale factor of 4 centered at the origin, which statement about the image is true?

- (1) The slope is  $-\frac{1}{4}$  and the  $y$ -intercept is  $-8$ .
- (2) The slope is  $-\frac{1}{4}$  and the  $y$ -intercept is  $-2$ .
- (3) The slope is  $-1$  and the  $y$ -intercept is  $-8$ .
- (4) The slope is  $-1$  and the  $y$ -intercept is  $-2$ .

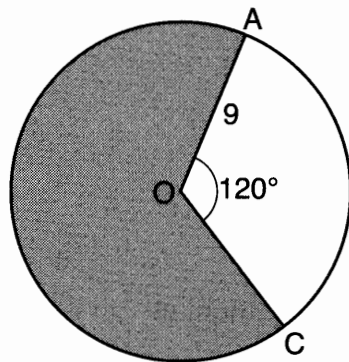
A dilation centered at the origin preserves parallelism  
 $(0, -2) \rightarrow (0, -8)$

Use this space for computations.

11 Square *MATH* has a side length of 7 inches. Which three-dimensional object will be formed by continuously rotating square *MATH* around side  $\overline{AT}$ ?

- (1) a right cone with a base diameter of 7 inches
- (2) a right cylinder with a diameter of 7 inches
- (3) a right cone with a base radius of 7 inches
- (4) a right cylinder with a radius of 7 inches

12 Circle *O* with a radius of 9 is drawn below. The measure of central angle *AOC* is  $120^\circ$ .



$$\frac{(360-120)}{360} (\pi) (9)^2$$
$$54\pi$$

What is the area of the shaded sector of circle *O*?

- (1)  $6\pi$
- (2)  $12\pi$
- (3)  $27\pi$
- (4)  $54\pi$

13 In quadrilateral *QRST*, diagonals  $\overline{QS}$  and  $\overline{RT}$  intersect at *M*. Which statement would always prove quadrilateral *QRST* is a parallelogram?

- (1)  $\angle TQR$  and  $\angle QRS$  are supplementary.
- (2)  $\overline{QM} \cong \overline{SM}$  and  $\overline{QT} \cong \overline{RS}$
- (3)  $\overline{QR} \cong \overline{TS}$  and  $\overline{QT} \cong \overline{RS}$
- (4)  $\overline{QR} \cong \overline{TS}$  and  $\overline{QT} \parallel \overline{RS}$

Use this space for computations.

14 A standard-size golf ball has a diameter of 1.680 inches. The material used to make the golf ball weighs 0.6523 ounce per cubic inch. What is the weight, to the nearest hundredth of an ounce, of one golf ball?

- (1) 1.10  
(2) 1.62

- (3) 2.48  
(4) 3.81

$$\frac{4}{3} \pi \cdot \left(\frac{1.68}{2}\right)^3 \cdot 0.6523 \approx 1.62$$

15 Chelsea is sitting 8 feet from the foot of a tree. From where she is sitting, the angle of elevation of her line of sight to the top of the tree is  $36^\circ$ . If her line of sight starts 1.5 feet above ground, how tall is the tree, to the nearest foot?

- (1) 8  
(2) 7

- (3) 6  
(4) 4

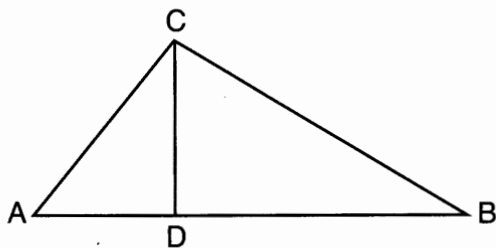
$$\tan 36 = \frac{x}{8}$$

$$x \approx 5.8$$

$$\underline{+1.5}$$

16 In the diagram below of right triangle  $ABC$ , altitude  $\overline{CD}$  intersects hypotenuse  $\overline{AB}$  at  $D$ .

7.3



Which equation is always true?

(1)  $\frac{AD}{AC} = \frac{CD}{BC}$

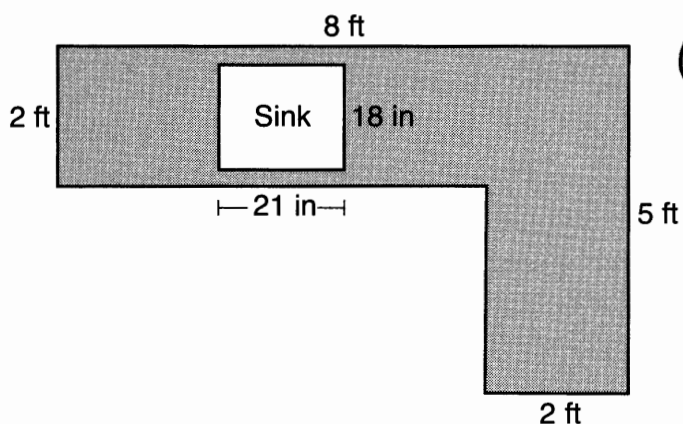
(3)  $\frac{AC}{CD} = \frac{BC}{CD}$

(2)  $\frac{AD}{CD} = \frac{BD}{CD}$

(4)  $\frac{AD}{AC} = \frac{AC}{BD}$

Use this space for computations.

- 17 A countertop for a kitchen is modeled with the dimensions shown below. An 18-inch by 21-inch rectangle will be removed for the installation of the sink.

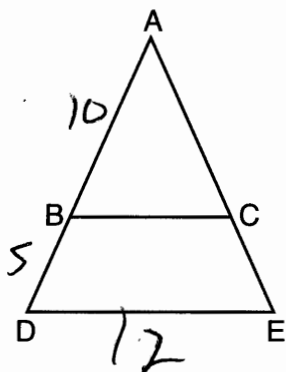


$$(8 \times 2) + (3 \times 2) - \left( \frac{18}{12} \times \frac{21}{12} \right) \approx 19$$

What is the area of the top of the installed countertop, to the nearest square foot?

- (1) 26                      (3) 22  
 (2) 23                      (4) 19

- 18 In the diagram below,  $\overline{BC}$  connects points  $B$  and  $C$  on the congruent sides of isosceles triangle  $ADE$ , such that  $\triangle ABC$  is isosceles with vertex angle  $A$ .



$$\frac{10}{x} = \frac{15}{12}$$

$$x = 8$$

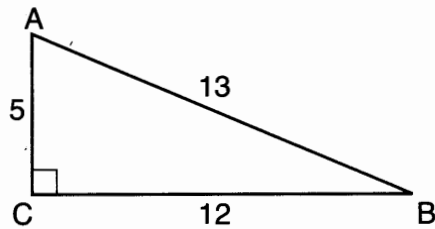
If  $AB = 10$ ,  $BD = 5$ , and  $DE = 12$ , what is the length of  $\overline{BC}$ ?

- (1) 6                      (3) 8  
 (2) 7                      (4) 9



Use this space for  
computations.

19 In  $\triangle ABC$  below, angle  $C$  is a right angle.



Which statement must be true?

- (1)  $\sin A = \cos B$                       (3)  $\sin B = \tan A$   
(2)  $\sin A = \tan B$                       (4)  $\sin B = \cos B$

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20 In right triangle  $RST$ , altitude  $\overline{TV}$  is drawn to hypotenuse  $\overline{RS}$ .

If  $RV = 12$  and  $RT = 18$ , what is the length of  $\overline{SV}$ ?

- (1)  $6\sqrt{5}$                                       (3)  $6\sqrt{6}$   
(2) 15    (4) 27

$$\begin{aligned} 18^2 &= 12(x+12) \\ 324 &= 12(x+12) \\ 27 &= x+12 \\ 15 &= x \end{aligned}$$

21 What is the volume, in cubic centimeters, of a right square pyramid with base edges that are 64 cm long and a slant height of 40 cm?

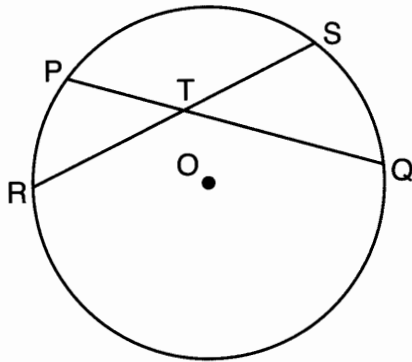
- (1) 8192.0                                      (3) 32,768.0  
(2)  $13,653.\bar{3}$                                       (4)  $54,613.\bar{3}$

$$h = \sqrt{40^2 - \left(\frac{64}{2}\right)^2} = 24$$

$$V = \frac{1}{3} (64)^2 (24) = 32768$$

22 In the diagram below, chords  $\overline{PQ}$  and  $\overline{RS}$  of circle  $O$  intersect at  $T$ .

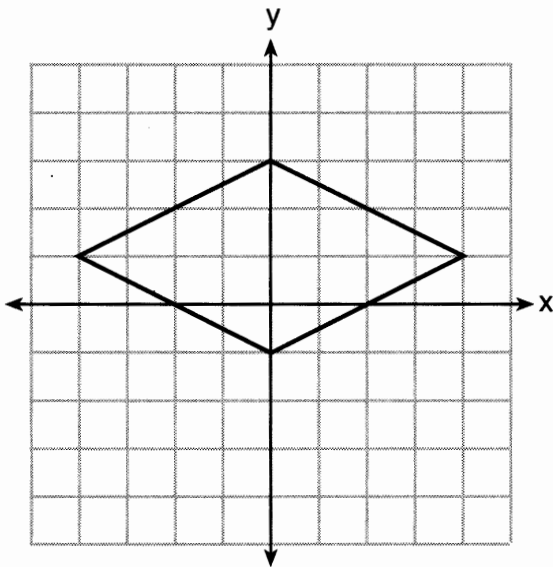
Use this space for computations.



Which relationship must always be true?

- (1)  $RT = TQ$
- (2)  $RT = TS$
- (3)  $RT + TS = PT + TQ$
- (4)  $RT \times TS = PT \times TQ$

23 A rhombus is graphed on the set of axes below.



Which transformation would carry the rhombus onto itself?

- (1)  $180^\circ$  rotation counterclockwise about the origin
- (2) reflection over the line  $y = \frac{1}{2}x + 1$
- (3) reflection over the line  $y = 0$
- (4) reflection over the line  $x = 0$

Use this space for  
computations.

24 A 15-foot ladder leans against a wall and makes an angle of  $65^\circ$  with the ground. What is the horizontal distance from the wall to the base of the ladder, to the *nearest tenth of a foot*?

(1) 6.3

(3) 12.9

(2) 7.0

(4) 13.6

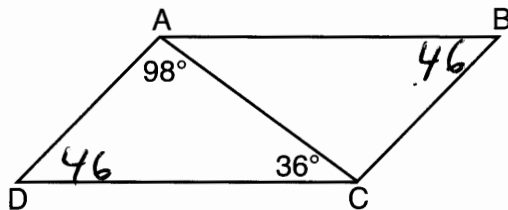
$$\cos 65 = \frac{x}{15}$$
$$x \approx 6.3$$

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## Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

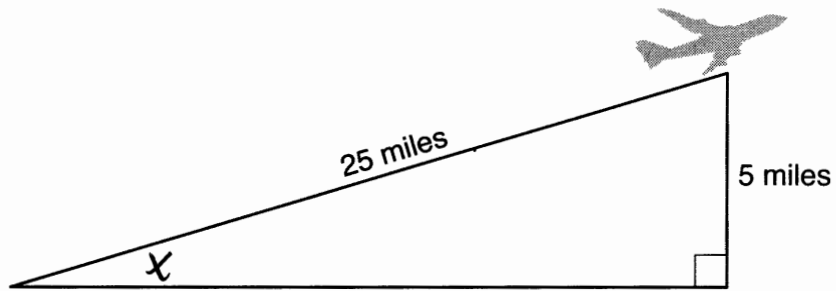
25 In parallelogram  $ABCD$  shown below,  $m\angle DAC = 98^\circ$  and  $m\angle ACD = 36^\circ$ .



What is the measure of angle  $B$ ? Explain why.

$\angle D = 46^\circ$  because the angles of a  $\triangle$  equal  $180^\circ$ .  $\angle B = 46^\circ$  because opposite angles of a  $\square$  are congruent.

- 26 An airplane took off at a constant angle of elevation. After the plane traveled for 25 miles, it reached an altitude of 5 miles, as modeled below.

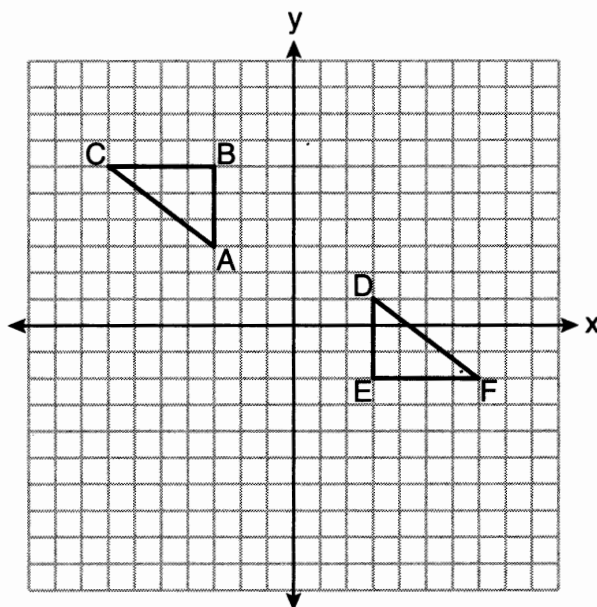


To the *nearest tenth of a degree*, what was the angle of elevation?

$$\sin x = \frac{5}{25}$$

$$x \approx 11.5$$

27 On the set of axes below,  $\triangle ABC \cong \triangle DEF$ .

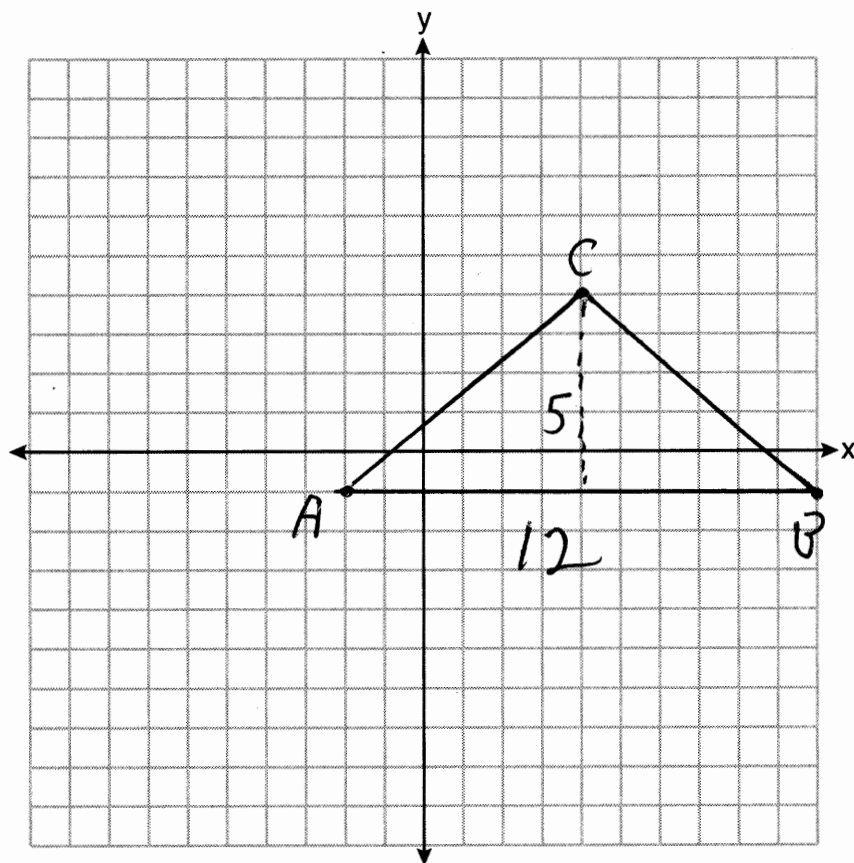


Describe a sequence of rigid motions that maps  $\triangle ABC$  onto  $\triangle DEF$ .

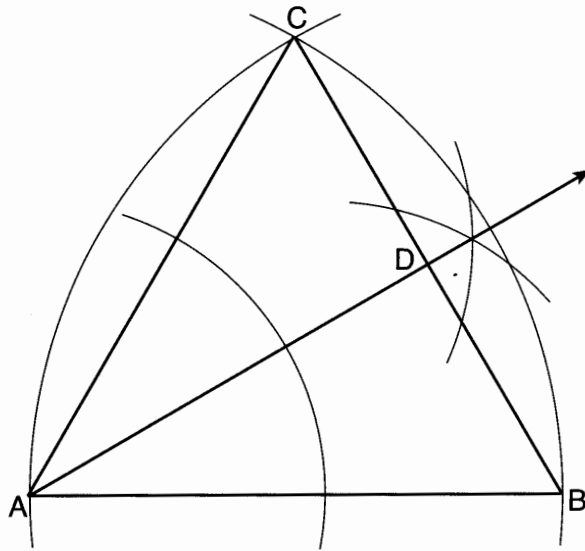
$r_{y=2} \circ r_{y\text{-axis}}$

28 The vertices of  $\triangle ABC$  have coordinates  $A(-2, -1)$ ,  $B(10, -1)$ , and  $C(4, 4)$ . Determine and state the area of  $\triangle ABC$ . [The use of the set of axes below is optional.]

$$A = \frac{1}{2} (12)(5) = 30$$



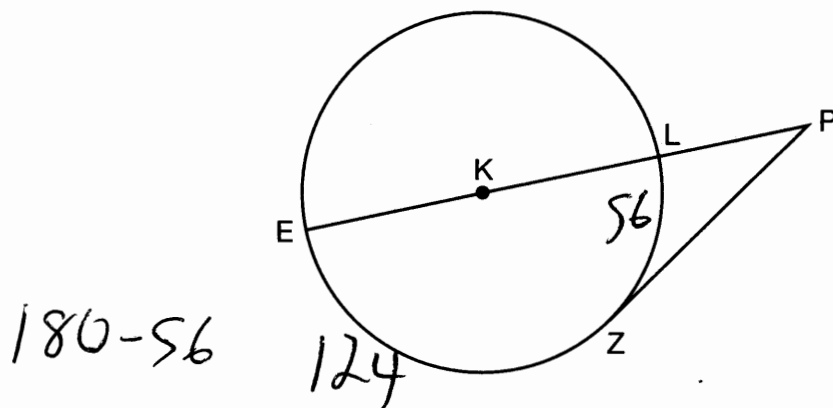
29 Using the construction below, state the degree measure of  $\angle CAD$ . Explain why.



$\triangle CAD$  is an equilateral triangle, so  $\angle CAB = 60^\circ$ . Since  $\overrightarrow{AD}$  is an angle bisector,  $\angle CAD = 30^\circ$ .



30 In the diagram below of circle  $K$ , secant  $\overline{PLKE}$  and tangent  $\overline{PZ}$  are drawn from external point  $P$ .



If  $m\widehat{LZ} = 56^\circ$ , determine and state the degree measure of angle  $P$ .

$$\frac{124 - 56}{2} = 34$$

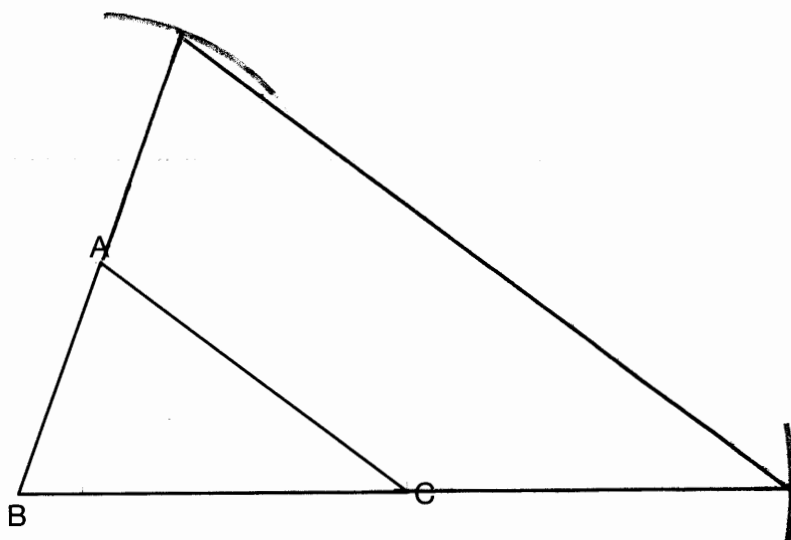
**31** A large water basin is in the shape of a right cylinder. The inside of the basin has a diameter of  $8\frac{1}{4}$  feet and a height of 3 feet. Determine and state, to the *nearest cubic foot*, the number of cubic feet of water that it will take to fill the basin to a level of  $\frac{1}{2}$  foot from the top.

$$\left(\frac{2.5}{3}\right)(\pi) \left(\frac{8.25}{2}\right)^2 (3) \approx 134$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

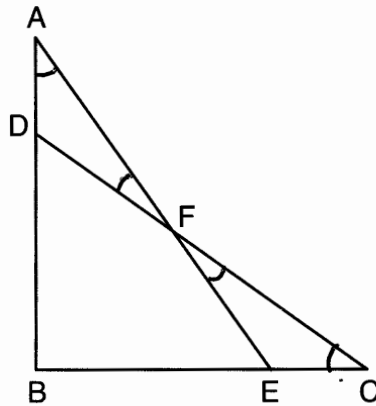
- 32 Triangle  $ABC$  is shown below. Using a compass and straightedge, construct the dilation of  $\triangle ABC$  centered at  $B$  with a scale factor of 2.  
[Leave all construction marks.]



Is the image of  $\triangle ABC$  similar to the original triangle? Explain why.

Yes, because a dilation preserves angle measure.

33 In the diagram below,  $\triangle ABE \cong \triangle CBD$ .



Prove:  $\triangle AFD \cong \triangle CFE$

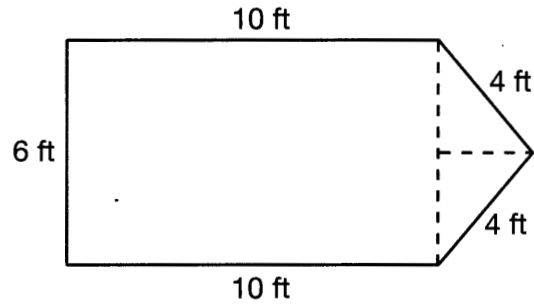
Statement	Reason
① $\triangle ABE \cong \triangle CBD$	① Given
② $\angle A \cong \angle C$	② CPCTC
③ $\angle AFD \cong \angle CFE$	③ Vertical angles are congruent
④ $\overline{AB} \cong \overline{CB} + \overline{DB} \cong \overline{EB}$	④ CPCTC
⑤ $\overline{AD} \cong \overline{CE}$	⑤ Segment subtraction
⑥ $\triangle AFD \cong \triangle CFE$	⑥ AAS

- 34 A cargo trailer, pictured below, can be modeled by a rectangular prism and a triangular prism. Inside the trailer, the rectangular prism measures 6 feet wide and 10 feet long. The walls that form the triangular prism each measure 4 feet wide inside the trailer. The diagram below is of the floor, showing the inside measurements of the trailer.

Cargo Trailer



Cargo Trailer Floor



If the inside height of the trailer is 6.5 feet, what is the total volume of the inside of the trailer, to the *nearest cubic foot*?

$$\left( (10 \times 6) + \frac{1}{2}(7-6)(7-4)(7-4) \right) (6.5) \approx 442$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. For the question in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 The coordinates of the vertices of  $\triangle ABC$  are  $A(1,2)$ ,  $B(-5,3)$ , and  $C(-6,-3)$ .

Prove that  $\triangle ABC$  is isosceles.

[The use of the set of axes on the next page is optional.]

$$AB = \sqrt{(-5-1)^2 + (3-2)^2} = \sqrt{37}$$

$$BC = \sqrt{(-5--6)^2 + (3--3)^2} = \sqrt{37}$$

Since  $\overline{AB} = \overline{BC}$ ,  $\triangle ABC$  is isosceles

State the coordinates of point  $D$  such that quadrilateral  $ABCD$  is a square.

$$(0, -4)$$

Question 35 is continued on the next page.

Question 35 continued

Prove that your quadrilateral  $ABCD$  is a square.  
[The use of the set of axes below is optional.]

$$AD = \sqrt{(1-0)^2 + (2-4)^2} = \sqrt{37}$$

$$CD = \sqrt{(-6-0)^2 + (-3--4)^2} = \sqrt{37}$$

$$M_{\overline{AB}} = \frac{3-2}{-5-1} = -\frac{1}{6} \quad M_{\overline{CB}} = \frac{3--3}{-5--6} = 6$$

$ABCD$  is a square because all four sides are congruent, consecutive sides are  $\perp$  since slopes are opposite reciprocals, & so  $\angle B$  is a right angle.

