The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY (Common Core)

Wednesday, August 12, 2015 — 8:30 to 11:30 a.m., only

Student Name: *Mr. Sibol*

School Name: *JMAP*

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. [48]

1 A parallelogram must be a rectangle when its

(1) diagonals are perpendicular

(2) diagonals are congruent

(3) opposite sides are parallel

(4) opposite sides are congruent

2 If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?

(1) reflection over the x-axis

(2) translation to the left 5 and down 4

(3) dilation centered at the origin with scale factor 2

(4) rotation of 270° counterclockwise about the origin

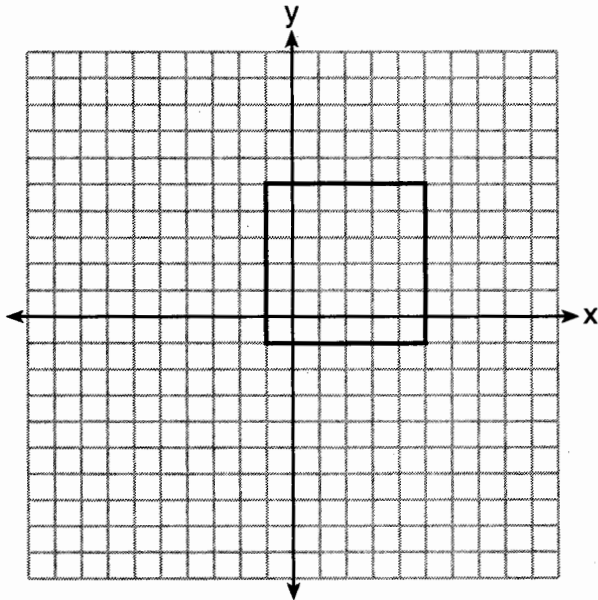3 If the rectangle below is continuously rotated about side w, which solid figure is formed?
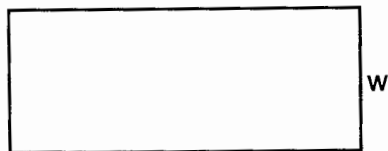
(1) pyramid   (3) cone

(2) rectangular prism   (4) cylinder

4 Which expression is always equivalent to sin x when $0° < x < 90°$?

(1) $\cos (90° - x)$ (3) $\cos (2x)$
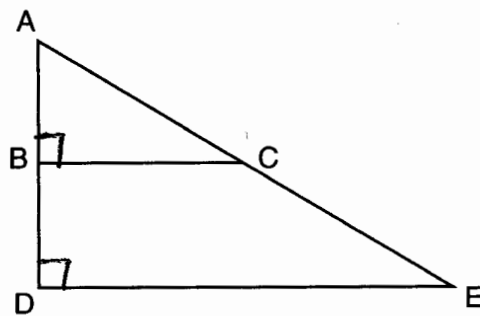
(2) $\cos (45° - x)$ (4) $\cos x$

5 In the diagram below, a square is graphed in the coordinate plane.

A reflection over which line does *not* carry the square onto itself?

(1) $x = 5$ (3) $y = x$

(2) $y = 2$ (4) $x + y = 4$

6 The image of $\triangle ABC$ after a dilation of scale factor k centered at point A is $\triangle ADE$, as shown in the diagram below.
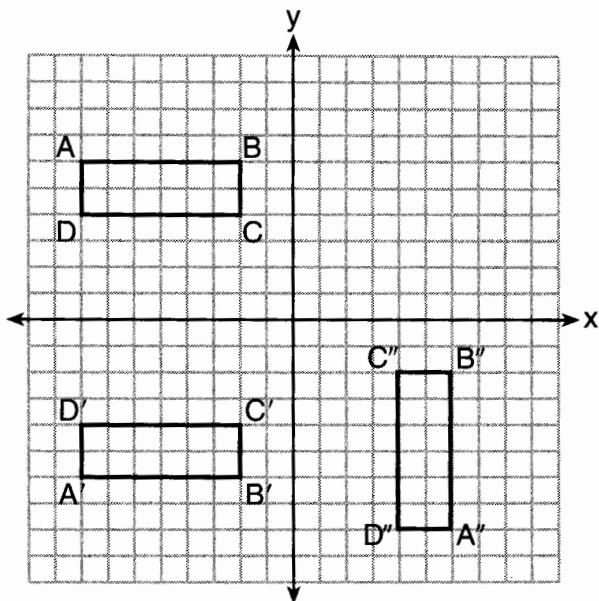

Which statement is always true?

(1) $2AB = AD$ (3) $AC = CE$

(2) $\overline{AD} \perp \overline{DE}$ (4) $\overline{BC} \parallel \overline{DE}$

7 A sequence of transformations maps rectangle $ABCD$ onto rectangle $A''B''C''D''$, as shown in the diagram below.

Which sequence of transformations maps $ABCD$ onto $A'B'C'D'$ and then maps $A'B'C'D'$ onto $A''B''C''D''$?

(1) a reflection followed by a rotation

(2) a reflection followed by a translation

(3) a translation followed by a rotation

(4) a translation followed by a reflection

8 In the diagram of parallelogram $FRED$ shown below, $\overline{ED}$ is extended to A, and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.
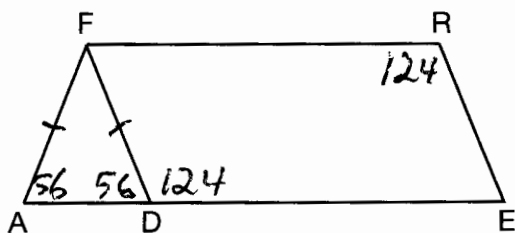
If $m\angle R = 124°$, what is $m\angle AFD$?

(1) 124°   (3) 68°

(2) 112°   (4) 56°

9 If $x^2 + 4x + y^2 - 6y - 12 = 0$ is the equation of a circle, the length of the radius is

(1) 25

(3) 5

(2) 16

(4) 4

Handwritten work:
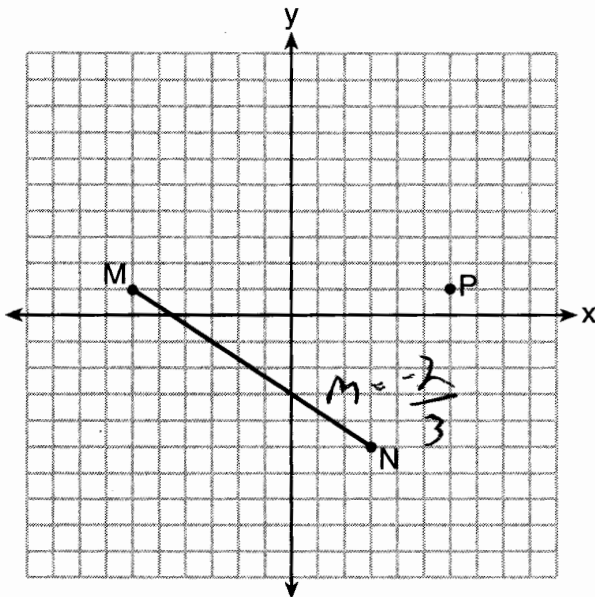$$x^2 + 4x + 4 + y^2 - 6y + 9 - 25 = 0$$
$$(x+2)^2 + (y-3)^2 = 25$$

10 Given $\overline{MN}$ shown below, with $M(-6,1)$ and $N(3,-5)$, what is an equation of the line that passes through point $P(6,1)$ and is parallel to $\overline{MN}$?

Handwritten work:
$$1 = \left(-\tfrac{2}{3}\right)6 + b$$
$$1 = -4 + b$$
$$5 = b$$

$M = -\dfrac{2}{3}$ (written on graph)

(1) $y = -\dfrac{2}{3}x + 5$

(3) $y = \dfrac{3}{2}x + 7$

(2) $y = -\dfrac{2}{3}x - 3$

(4) $y = \dfrac{3}{2}x - 8$

11 Linda is designing a circular piece of stained glass with a diameter of 7 inches. She is going to sketch a square inside the circular region.

To the *nearest tenth of an inch*, the largest possible length of a side of the square is
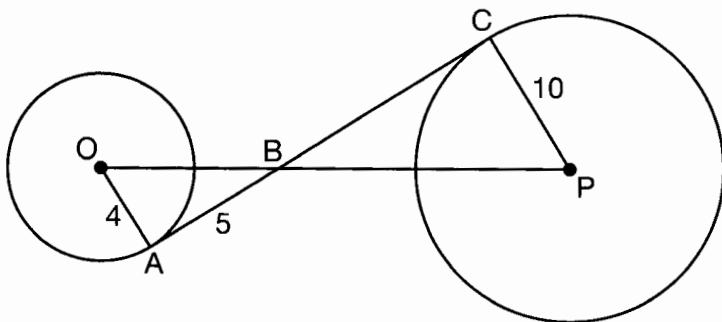
(1) 3.5

(3) 5.0

(2) 4.9

(4) 6.9

Handwritten work:
$$s^2 + s^2 = 7^2$$
$$2s^2 = 49$$
$$s = \sqrt{24.5} \approx 4.9$$

12 In the diagram shown below, $\overline{AC}$ is tangent to circle O at A and to circle P at C, $\overline{OP}$ intersects $\widehat{AC}$ at B, $OA = 4$, $AB = 5$, and $PC = 10$.
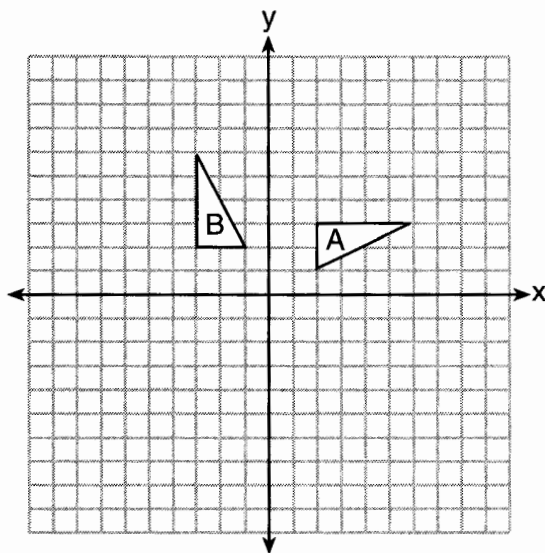

What is the length of $\overline{BC}$?

(1) 6.4

(3) 12.5

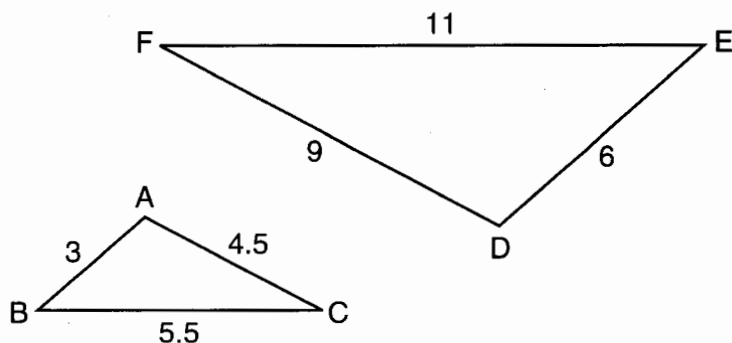(2) 8

(4) 16

$$5 \cdot \frac{10}{4} = \frac{50}{4} = 12.5$$

13 In the diagram below, which single transformation was used to map triangle A onto triangle B?

(1) line reflection

(3) dilation

(2) rotation

(4) translation

14 In the diagram below, △DEF is the image of △ABC after a clockwise
rotation of 180° and a dilation where AB = 3, BC = 5.5, AC = 4.5,
DE = 6, FD = 9, and EF = 11.

Which relationship must always be true?

(1) $\dfrac{m\angle A}{m\angle D} = \dfrac{1}{2}$

(3) $\dfrac{m\angle A}{m\angle C} = \dfrac{m\angle F}{m\angle D}$

(2) $\dfrac{m\angle C}{m\angle F} = \dfrac{2}{1}$

(4) $\dfrac{m\angle B}{m\angle E} = \dfrac{m\angle C}{m\angle F}$

15 In the diagram below, quadrilateral ABCD is inscribed in circle P.
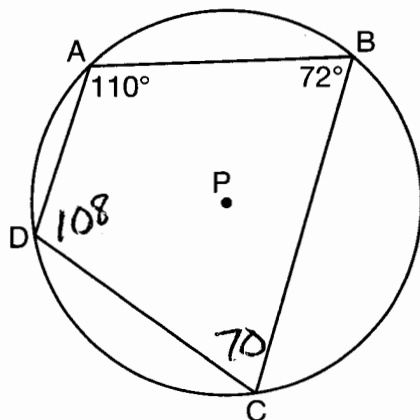
What is m∠ADC?

(1) 70°

(3) 108°

(2) 72°

(4) 110°

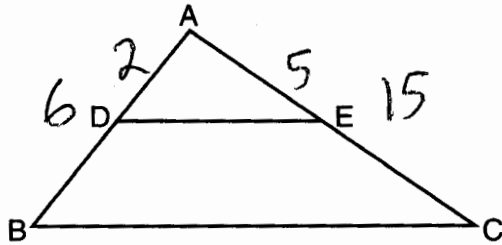**16** A hemispherical tank is filled with water and has a diameter of 10 feet. If water weighs 62.4 pounds per cubic foot, what is the total weight of the water in a full tank, to the *nearest pound*?

(1) 16,336

(3) 130,690

(2) 32,673

(4) 261,381

$V = \dfrac{\frac{4}{3}\pi\left(\frac{10}{2}\right)^3}{2} \approx 261.8$

261.8
$\times 62.4$
$16,336$

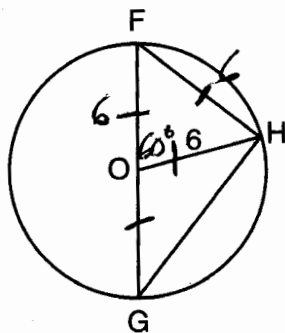**17** In the diagram below, $\triangle ABC \sim \triangle ADE$.

Which measurements are justified by this similarity?

(1) $AD = 3$, $AB = 6$, $AE = 4$, and $AC = 12$

(2) $AD = 5$, $AB = 8$, $AE = 7$, and $AC = 10$

(3) $AD = 3$, $AB = 9$, $AE = 5$, and $AC = 10$

(4) $AD = 2$, $AB = 6$, $AE = 5$, and $AC = 15$

$\dfrac{2}{6} = \dfrac{5}{15}$

18 Triangle FGH is inscribed in circle O, the length of radius $\overline{OH}$ is 6, and $\overline{FH} \cong \overline{OG}$.
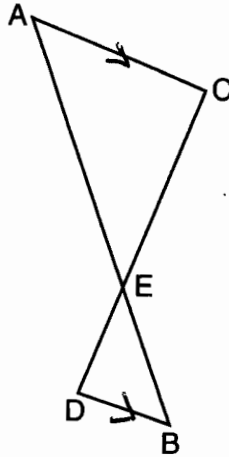

What is the area of the sector formed by angle FOH?

(1) 2π

(3) 6π

(2) $\dfrac{3}{2}\pi$

(4) 24π

$\dfrac{60}{360} \cdot 6^2\pi$

6π

[8]

19 As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at E, and $\overline{AC} \parallel \overline{BD}$.

Given $\triangle AEC \sim \triangle BED$, which equation is true?

(1) $\dfrac{CE}{DE} = \dfrac{EB}{EA}$

(3) $\dfrac{EC}{AE} = \dfrac{BE}{ED}$

(2) $\dfrac{AE}{BE} = \dfrac{AC}{BD}$

(4) $\dfrac{ED}{EC} = \dfrac{AC}{BD}$

20 A triangle is dilated by a scale factor of 3 with the center of dilation at the origin. Which statement is true?

(1) The area of the image is nine times the area of the original triangle.

(2) The perimeter of the image is nine times the perimeter of the original triangle.

(3) The slope of any side of the image is three times the slope of the corresponding side of the original triangle.

(4) The measure of each angle in the image is three times the measure of the corresponding angle of the original triangle.

$3^2 = 9$

21 The Great Pyramid of Giza was constructed as a regular pyramid with a square base. It was built with an approximate volume of 2,592,276 cubic meters and a height of 146.5 meters. What was the length of one side of its base, to the *nearest meter*?

$2592276 = \dfrac{1}{3} s^2 \cdot 146.5$

(1) 73

(3) 133

(2) 77

(4) 230

$230 \approx s$

22 A quadrilateral has vertices with coordinates $(-3,1)$, $(0,3)$, $(5,2)$, and $(-1,-2)$. Which type of quadrilateral is this?

(1) rhombus
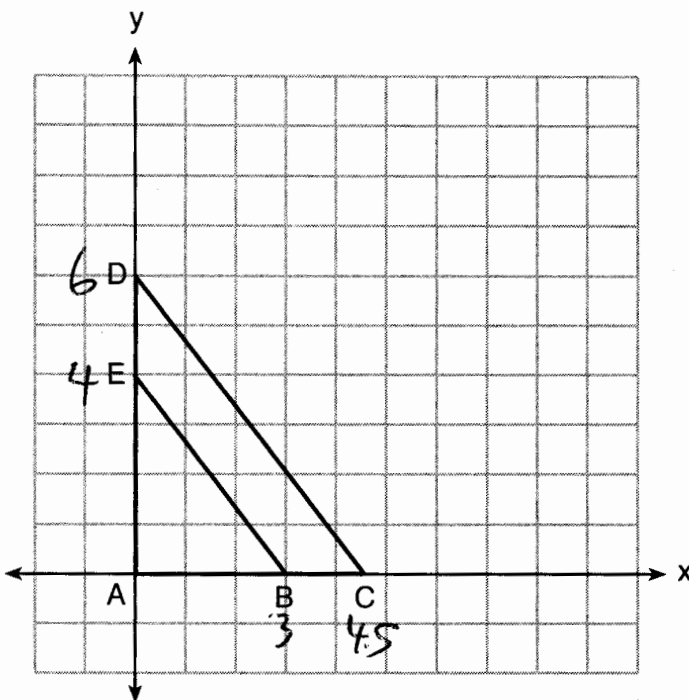
(2) rectangle

(3) square

(4) trapezoid

Handwritten computations:
$$\frac{-2-1}{-1--3} ; \frac{-3}{2} \qquad \frac{3-2}{0-5} ; \frac{1}{-5}$$
$$\frac{3-1}{0--3} ; \frac{2}{3} \qquad \frac{2-2}{5--1} ; \frac{4}{6} ; \frac{2}{3}$$

23 In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.

The ratio of the lengths of $\overline{BE}$ to $\overline{CD}$ is

(1) $\frac{2}{3}$

(2) $\frac{3}{2}$

(3) $\frac{3}{4}$

(4) $\frac{4}{3}$

Handwritten computations:
$$\frac{4}{6} = \frac{3}{4.5} ; \frac{2}{3}$$

24 Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3,8)$. The line's image is

(1) $y = 3x - 8$

(2) $y = 3x - 4$

(3) $y = 3x - 2$

(4) $y = 3x - 1$

Handwritten note: Since the line contains the center of dilation, the y-intercept is unchanged

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the *nearest thousandth*.
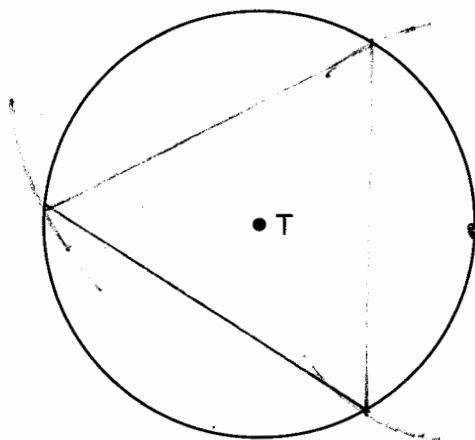
State which type of wood the cube is made of, using the density table below.

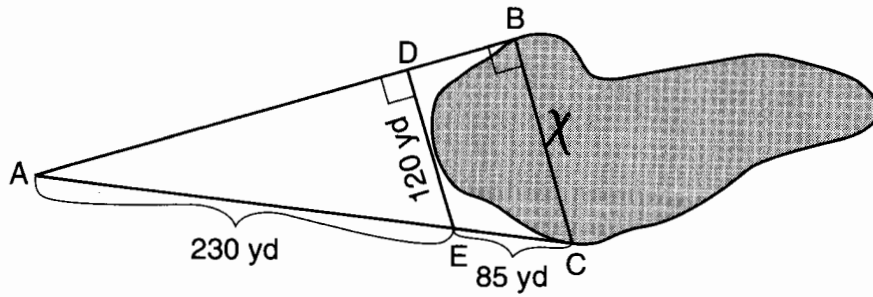| Type of Wood	Density (g/cm^3)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711

$$\frac{137.8}{6^3} \approx .638$$

26 Construct an equilateral triangle inscribed in circle T shown below.

[Leave all construction marks.]

27 To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram.
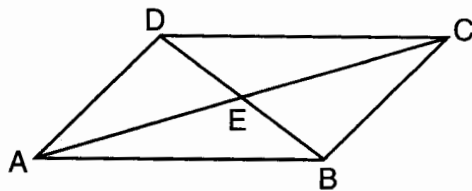

Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard*.

$$\frac{120}{230} = \frac{x}{315}$$

$$x \approx 164$$

28 In parallelogram *ABCD* shown below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E*.

Prove: ∠*ACD* ≅ ∠*CAB*

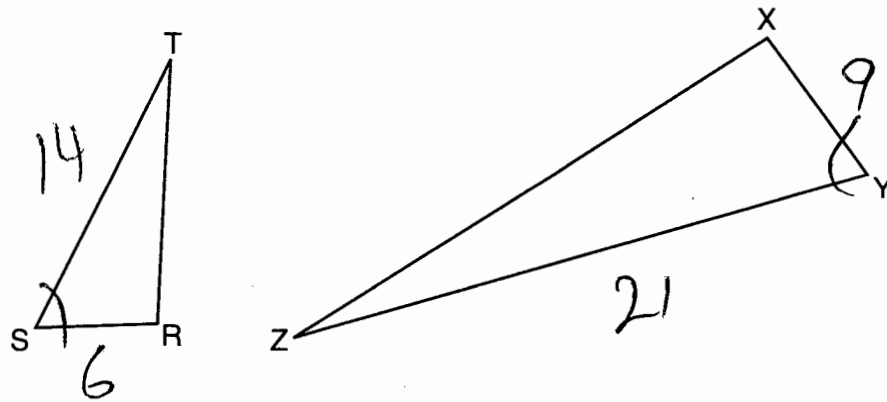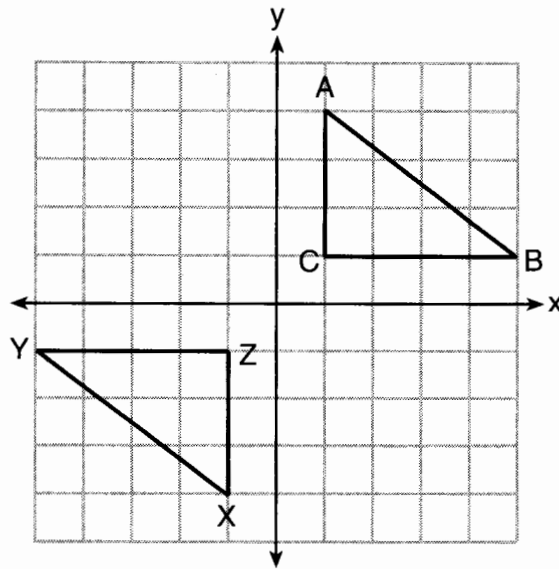| Statement	Reason
① Parallelogram *ABCD*, diagonals $\overline{AC}$ & $\overline{BD}$ intersect at *E*.	① Given
② $\overline{DC} \parallel \overline{AB}$, $\overline{DA} \parallel \overline{CB}$	② Opposite sides of a parallelogram are parallel
③ ∠*ACD* ≅ ∠*CAB*	③ Alternate interior angles formed by parallel lines & a transversal are congruent.

29 Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.

(student handwritten diagram of triangle T, S, R with sides 14 and 6, and triangle X, Y, Z with sides 9 and 21)

$$\frac{6}{14} ; \frac{9}{21}$$

$$126 = 126$$

$$SAS$$

30 In the diagram below, △*ABC* and △*XYZ* are graphed.

Use the properties of rigid motions to explain why △*ABC* ≅ △*XYZ*.

The transformation is a rotation, & a rotation is a rigid motion

31 The endpoints of $\overline{DEF}$ are $D(1,4)$ and $F(16,14)$. Determine and state the coordinates of point E, if $DE{:}EF = \dfrac{2{:}3}{5}$.

$$\frac{2}{5} \cdot (16-1) = 6$$

$$\frac{2}{5} \cdot (14-4) = 4$$

$$(7, 8)$$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7°. A short time later, at point D, the angle of elevation was 16°.
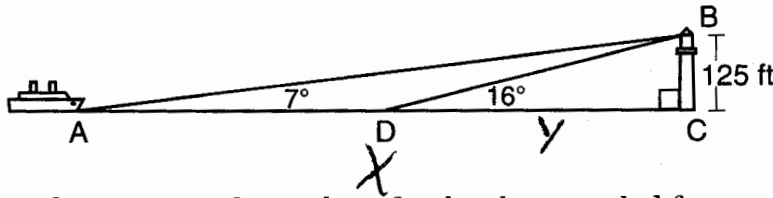
To the *nearest foot*, determine and state how far the ship traveled from point A to point D.

$$\tan 7 = \frac{125}{X}$$

$$X \approx 1018$$

$$\tan 16 = \frac{125}{Y}$$

$$Y \approx 436$$

$$582$$

33 Triangle *ABC* has vertices with *A*(*x*,3), *B*(−3,−1), and C(−1,−4).

Determine and state a value of *x* that would make triangle *ABC* a right triangle. Justify why △*ABC* is a right triangle.

[The use of the set of axes below is optional.]

$$-1 = \frac{2}{3}(-3) + b$$

$$-1 = -2 + b$$

$$1 = b$$

$$3 = \frac{2}{3}x + 1$$

$$2 = \frac{2}{3}x$$

$$3 = x$$

or

$$-4 = \frac{2}{3}(-1) + b$$

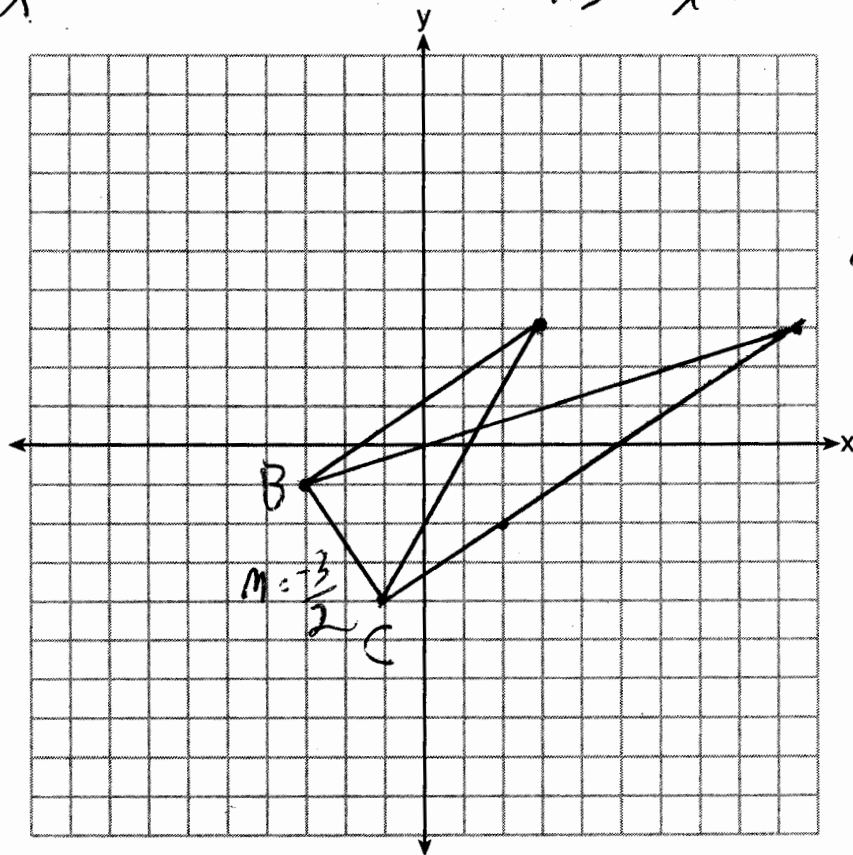$$\frac{-12}{3} = \frac{-2}{3} + b$$

$$\frac{-10}{3} = b$$

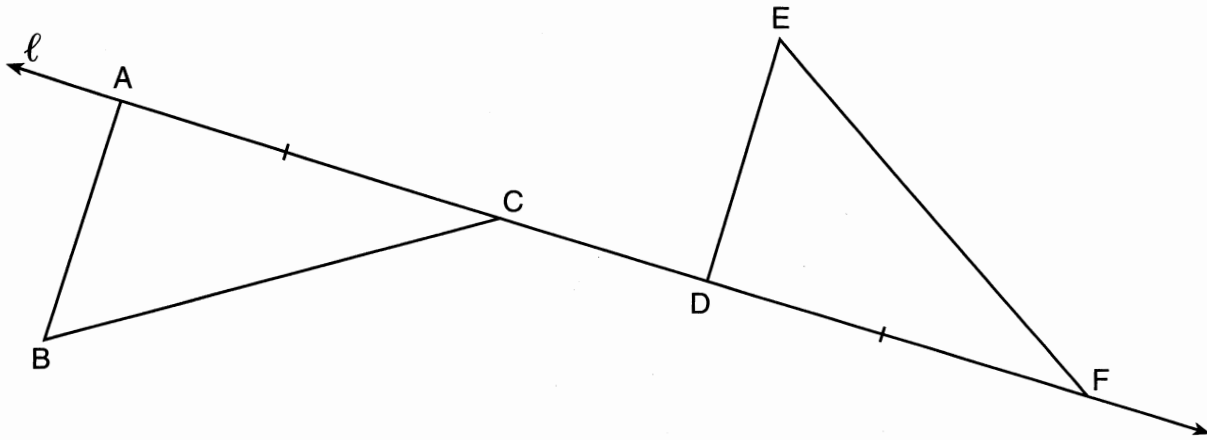$$3 = \frac{2}{3}x - \frac{10}{3}$$

$$9 = 2x - 10$$

$$19 = 2x$$

$$9.5 = x$$

The slopes of ⊥ lines are opposite reciprocals

$M = \frac{-3}{2}$

34 In the diagram below, $\overline{AC} \cong \overline{DF}$ and points A, C, D, and F are collinear on line ℓ.

Let $\triangle D'E'F'$ be the image of $\triangle DEF$ after a translation along ℓ, such that point D is mapped onto point A. Determine and state the location of F'. Explain your answer.

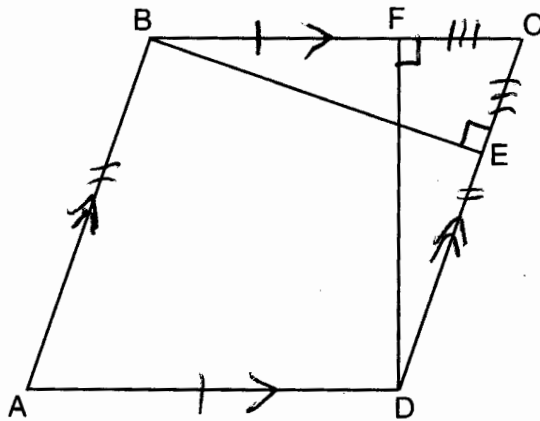Translations preserve distance. If point D is mapped onto point A, point F would map onto point C.

Let $\triangle D''E''F''$ be the image of $\triangle D'E'F'$ after a reflection across line ℓ. Suppose that E'' is located at B. Is $\triangle DEF$ congruent to $\triangle ABC$? Explain your answer.

Yes. $\overline{AC} \cong \overline{DF}$ & points are collinear on line ℓ. A reflection preserves distance

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 In the diagram of parallelogram *ABCD* below, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, and $\overline{CE} \cong \overline{CF}$.

Prove *ABCD* is a rhombus.

Statement	Reason
① Parallelogram ABCD, BE⊥CED, DF⊥BFC, CE≅CF	① Given
② ∠BEC ≅ ∠DFC	② Perpendicular lines form right angles, which are congruent
③ ∠FCD ≅ ∠BCE	③ Reflexive property
④ △BEC ≅ △DFC	④ ASA
⑤ BC ≅ CD	⑤ CPCTC
⑥ ABCD is a rhombus	⑥ A parallelogram with consecutive congruent sides is a rhombus

36 Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches. To the *nearest cubic inch*, what will be the total volume of 100 candles?

$V = \frac{1}{3}\pi\left(\frac{3}{2}\right)^2 \cdot 8 = 18.85$

$$\begin{array}{r} \times 100 \\ \hline 1885 \end{array}$$
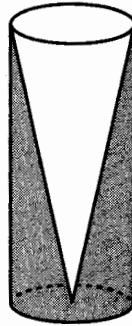
Walter goes to a hobby store to buy the wax for his candles. The wax costs $0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?

$1885 \, in^3 \cdot \dfrac{0.52 \, oz}{in^3} , \dfrac{\$0.10}{oz} = \$98.02$

If Walter spent a total of $37.83 for the molds and charges $1.95 for each candle, what is Walter's profit after selling 100 candles?

$1.95(100) - (37.83 + 98.02)$

$\$59.15$