The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, June 20, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP, Inc.

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

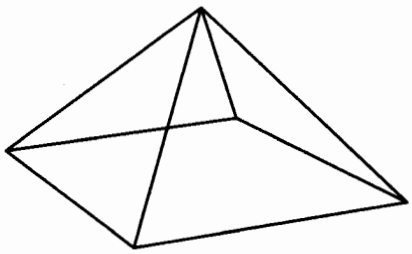
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

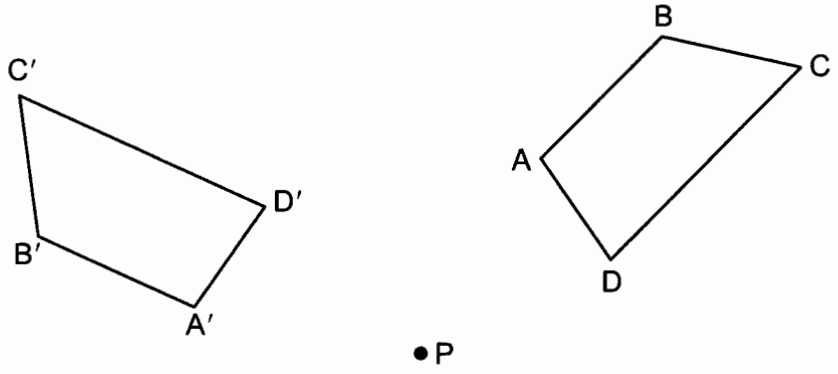
1 A square pyramid is intersected by a plane passing through the vertex and perpendicular to the base.



Which two-dimensional shape describes this cross section?

(1) square
(2) triangle
(3) pentagon
(4) rectangle

2 Trapezoid $ABCD$ is drawn such that $\overline{AB} \parallel \overline{DC}$.
Trapezoid $A'B'C'D'$ is the image of trapezoid $ABCD$ after a rotation of 110° counterclockwise about point P.



Which statement is always true?

(1) $\angle A \cong \angle D'$
(2) $\overline{AC} \cong \overline{B'D'}$
(3) $\overline{A'B'} \parallel \overline{D'C'}$
(4) $\overline{B'A'} \cong \overline{C'D'}$

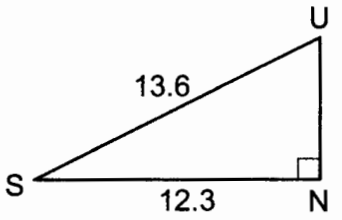
Use this space for computations.

3 What is the volume of a right circular cone that has a height of 7.2 centimeters and a radius of 2.5 centimeters, to the *nearest tenth of a cubic centimeter*?

(1) 37.7
(2) 47.1
(3) 113.1
(4) 141.4

$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(2.5)^2(7.2)$
≈ 47.1

4 In the diagram below of right triangle SUN, where $\angle N$ is a right angle, $SU = 13.6$ and $SN = 12.3$.

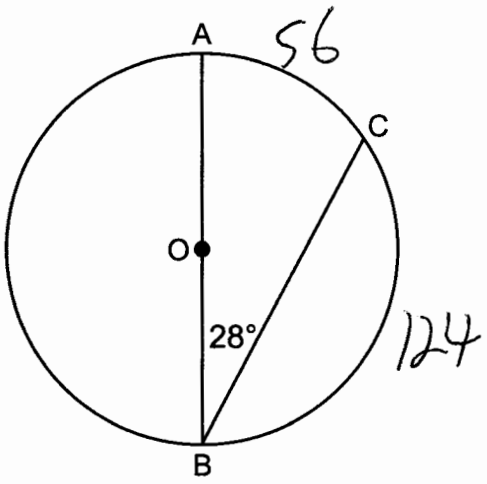


What is $m\angle S$, to the *nearest degree*?

(1) 25°
(2) 42°
(3) 48°
(4) 65°

$\cos S = \frac{12.3}{13.6}$
$S \approx 25$

5 In the diagram below of circle O, diameter $\overline{AOB}$ and chord $\overline{CB}$ are drawn, and $m\angle B = 28°$.

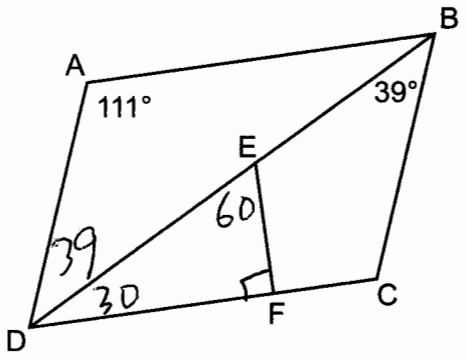


What is $m\overset{\frown}{BC}$?

(1) 56°
(2) 124°
(3) 152°
(4) 166°

Use this space for computations.

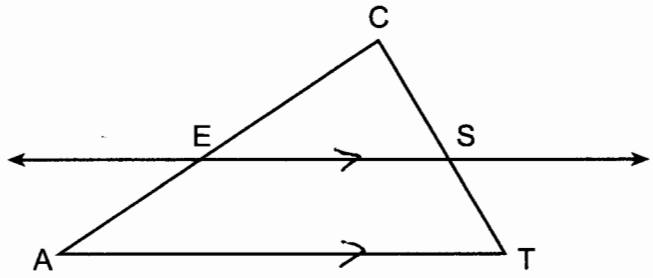
6 In the diagram below of parallelogram $ABCD$, diagonal $\overline{BED}$ and $\overline{EF}$ are drawn, $\overline{EF} \perp \overline{DFC}$, $m\angle DAB = 111°$, and $m\angle DBC = 39°$.



What is $m\angle DEF$?

(1) 30°
(2) 51°
(3) 60°
(4) 120°

7 In the diagram below of $\triangle ACT$, $\overleftrightarrow{ES}$ is drawn parallel to $\overline{AT}$ such that E is on $\overline{CA}$ and S is on $\overline{CT}$.

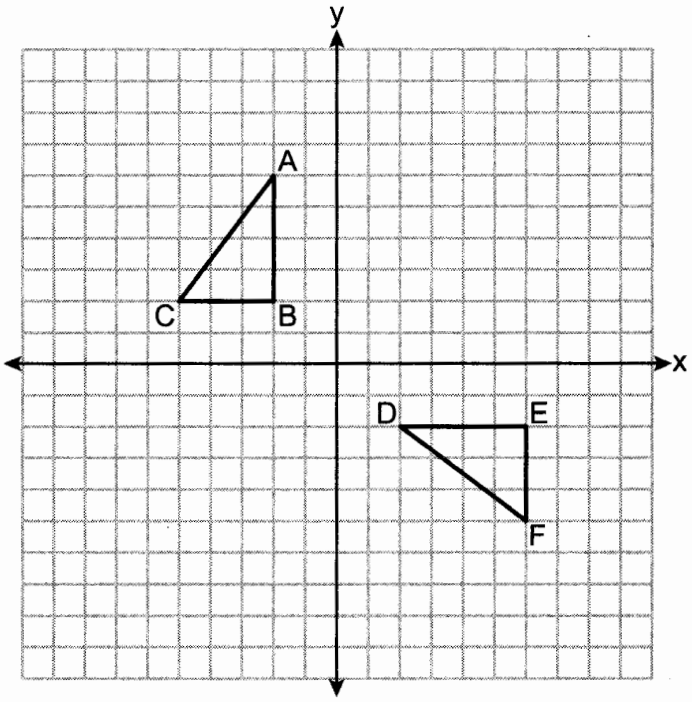


Which statement is always true?

(1) $\frac{CE}{CA} = \frac{CS}{ST}$
(2) $\frac{CE}{ES} = \frac{EA}{AT}$
(3) $\frac{CE}{EA} = \frac{CS}{ST}$
(4) $\frac{CE}{ST} = \frac{EA}{CS}$

Use this space for computations.

8 On the set of axes below, congruent triangles ABC and DEF are drawn.



Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

(1) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 8 units to the right.

(2) A counterclockwise rotation of 90 degrees about the origin, followed by a reflection over the y-axis.

(3) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 4 units down.

(4) A clockwise rotation of 90 degrees about the origin, followed by a reflection over the x-axis.

9 An equation of circle M is $x^2 + y^2 + 6x - 2y + 1 = 0$. What are the coordinates of the center and the length of the radius of circle M?

(1) center $(3,-1)$ and radius 9

(2) center $(3,-1)$ and radius 3

(3) center $(-3,1)$ and radius 9

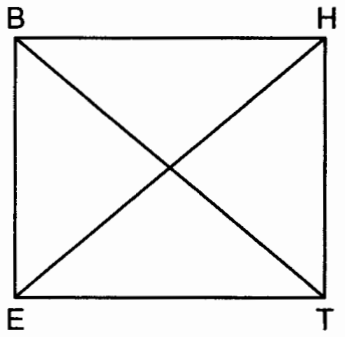
(4) center $(-3,1)$ and radius 3

$x^2+6x+9+y^2-2y+1 = -1+9+1$

$(x+3)^2+(y-1)^2 = 9$

Use this space for computations.

10 Parallelogram *BETH*, with diagonals $\overline{BT}$ and $\overline{HE}$, is drawn below.



Which additional statement is sufficient to prove that *BETH* is a rectangle?

(1) $\overline{BT} \perp \overline{HE}$

(2) $\overline{BE} \parallel \overline{HT}$

(3) $\overline{BT} \cong \overline{HE}$

(4) $\overline{BE} \cong \overline{ET}$

11 A gardener wants to buy enough mulch to cover a rectangular garden that is 3 feet by 10 feet. One bag contains 2 cubic feet of mulch and costs \$3.66. How much will the minimum number of bags cost to cover the garden with mulch 3 inches deep?

(1) \$3.66

(2) \$10.98

(3) \$14.64

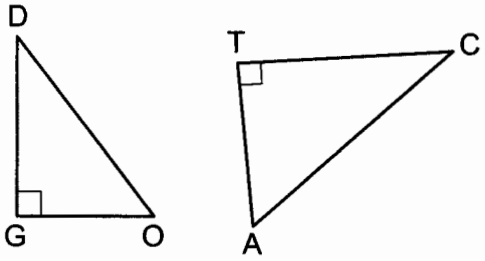
(4) \$29.28

$V = 3 \times 10 \times \frac{1}{4} = 7.5$

$\frac{7.5}{2} = 4$ bags

$4 \times 3.66 = 14.64$

12 In the diagram below, $\triangle DOG \sim \triangle CAT$, where $\angle G$ and $\angle T$ are right angles.



Which expression is always equivalent to sin *D*?

(1) cos *A*

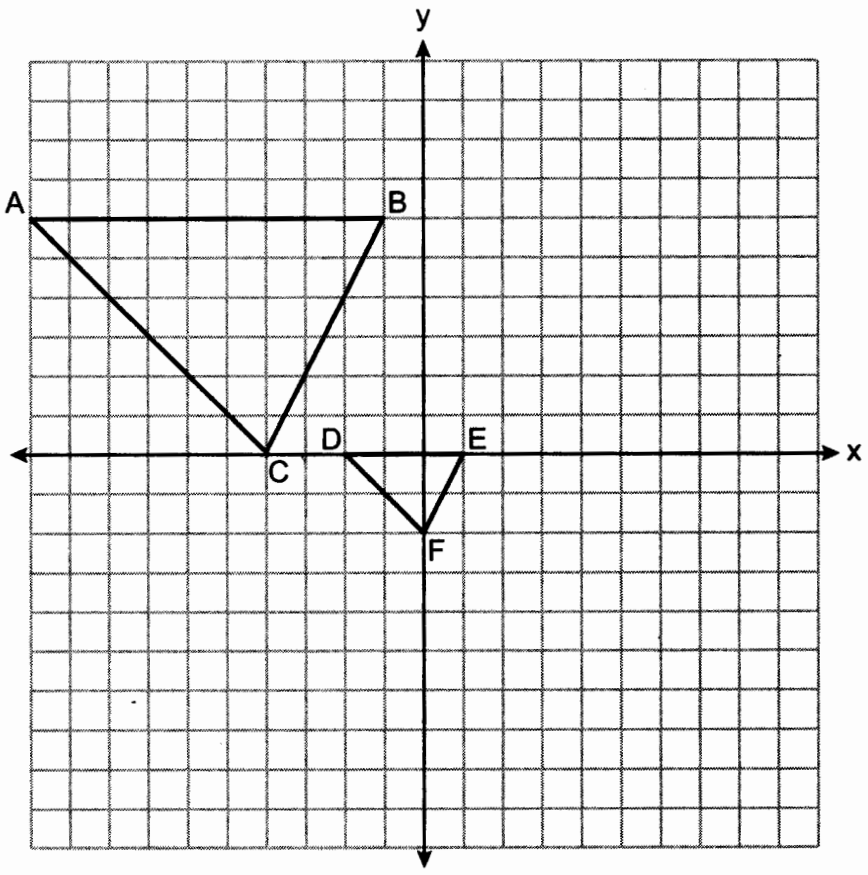
(2) sin *A*

(3) tan *A*

(4) cos *C*

Use this space for computations.

13 On the set of axes below, $\triangle DEF$ is the image of $\triangle ABC$ after a dilation of scale factor $\frac{1}{3}$.



The center of dilation is at

(1) (0,0)
(2) (2,−3)
(3) (0,−2)
(4) (−4,0)

$$x_0 = \frac{kx_1 - x_2}{k-1}$$

$$x_0 = \frac{\frac{1}{3}(-4) - 0}{\frac{1}{3} - 1}$$

$$x_0 = \frac{-4/3}{-2/3}$$

$$x_0 = 2$$

$$y_0 = \frac{ky_1 - y_2}{k-1}$$

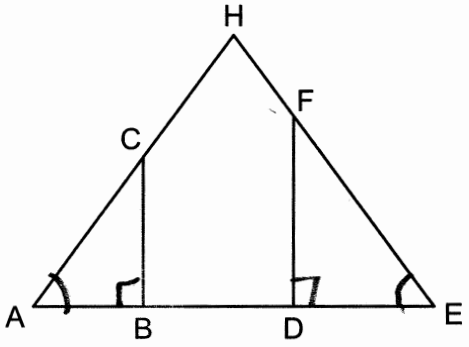
$$y_0 = \frac{\frac{1}{3}(0) - -2}{\frac{1}{3} - 1}$$

$$y_0 = \frac{2}{-2/3}$$

$$y_0 = -3$$

Use this space for computations.

14 In the diagram below of isosceles triangle AHE with the vertex angle at H, $\overline{CB} \perp \overline{AE}$ and $\overline{FD} \perp \overline{AE}$.



Which statement is always true?

(1) $\frac{AH}{AC} = \frac{EH}{EF}$

(2) $\frac{AC}{EF} = \frac{AB}{ED}$

(3) $\frac{AB}{ED} = \frac{CB}{FE}$

(4) $\frac{AD}{AB} = \frac{BE}{DE}$

15 Rectangle $ABCD$ has two vertices at coordinates $A(-1,-3)$ and $B(6,5)$. The slope of $\overline{BC}$ is

(1) $-\frac{7}{8}$

(2) $\frac{7}{8}$

(3) $-\frac{8}{7}$

(4) $\frac{8}{7}$

$\frac{-3-5}{-1-6} = \frac{-8}{-7} = \frac{8}{7}$

16 In right triangle ABC, $m\angle A = 90°$, $m\angle B = 18°$, and $AC = 8$.

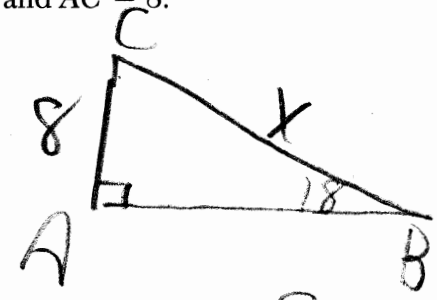
To the *nearest tenth*, the length of $\overline{BC}$ is

(1) 2.5

(2) 8.4

(3) 24.6

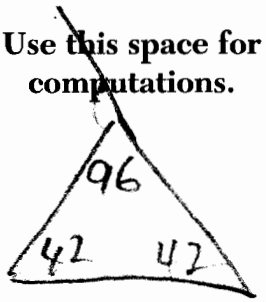
(4) 25.9



$\sin 18 = \frac{8}{x}$

$x \approx 25.9$

Use this space for computations.

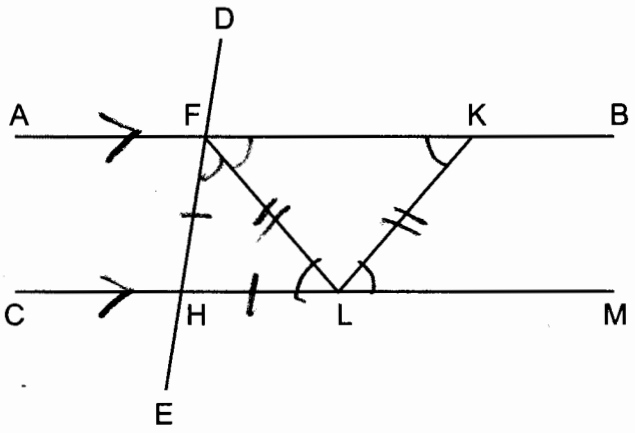


180 - 96

17 The measure of one of the base angles of an isosceles triangle is 42°. The measure of an exterior angle at the vertex of the triangle is

(1) 42°
(2) 84°
(3) 96°
(4) 138°

18 In the diagram below, $\overline{AFKB} \parallel \overline{CHLM}$, $\overline{FH} \cong \overline{LH}$, $\overline{FL} \cong \overline{KL}$, and $\overline{LF}$ bisects $\angle HFK$.



Which statement is always true?

(1) $2(m\angle HLF) = m\angle CHE$
(2) $2(m\angle FLK) = m\angle LKB$
(3) $m\angle AFD = m\angle BKL$
(4) $m\angle DFK = m\angle KLF$

19 The line whose equation is $6x + 3y = 3$ is dilated by a scale factor of 2 centered at the point (0,0). An equation of its image is

(1) $y = -2x + 1$
(2) $y = -2x + 2$
(3) $y = -4x + 1$
(4) $y = -4x + 2$

Use this space for computations.

20 Which figure will *not* carry onto itself after a 120-degree rotation about its center?

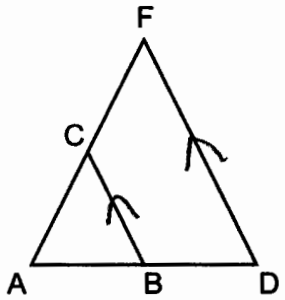
(1) equilateral triangle

(2) regular hexagon

(3) regular octagon

(4) regular nonagon

21 Triangle ADF is drawn and $\overline{BC} \parallel \overline{DF}$.



Which statement must be true?

(1) $\frac{AB}{BC} = \frac{BD}{DF}$

(2) $BC = \frac{1}{2}DF$

(3) $AB:AD = AC:CF$

(4) $\angle ACB \cong \angle AFD$

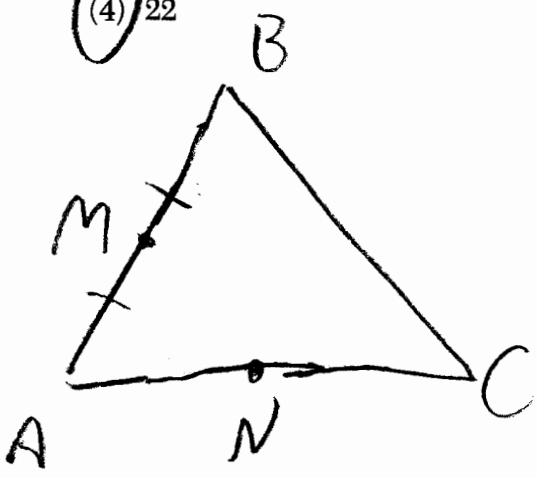
22 In $\triangle ABC$, M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{AC}$. If $MN = x + 13$ and $BC = 5x - 1$, what is the length of $\overline{MN}$?

(1) 3.5

(2) 9

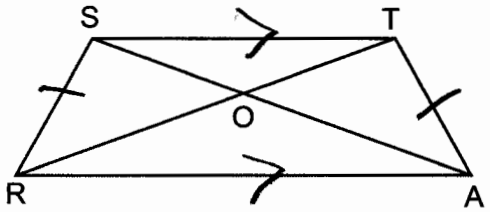
(3) 16.5

(4) 22



Use this space for computations.

23 In the diagram below of isosceles trapezoid $STAR$, diagonals $\overline{AS}$ and $\overline{RT}$ intersect at O and $\overline{ST} \parallel \overline{RA}$, with nonparallel sides $\overline{SR}$ and $\overline{TA}$.



Which pair of triangles are *not* always similar?

(1) $\triangle STO$ and $\triangle ARO$

(2) $\triangle SOR$ and $\triangle TOA$

(3) $\triangle SRA$ and $\triangle ATS$

(4) $\triangle SRT$ and $\triangle TAS$

24 The endpoints of $\overline{AB}$ are $A(0,4)$ and $B(-4,6)$. Which equation of a line represents the perpendicular bisector of $\overline{AB}$?

(1) $y = -\frac{1}{2}x + 4$

(2) $y = -2x + 1$

(3) $y = 2x + 8$

(4) $y = 2x + 9$

$(-2, 5)$

$\frac{6-4}{-4-0} = \frac{2}{-4} = -\frac{1}{2}$ $\quad m_{\perp} = 2$

$y - 5 = 2(x + 2)$

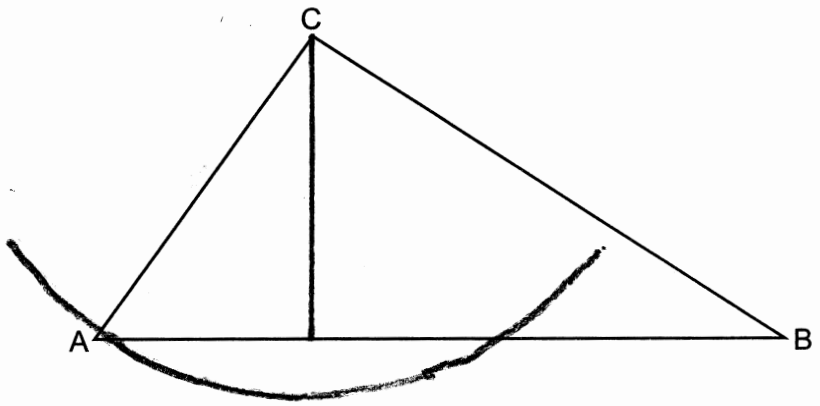
$y = 2x + 4 + 5$

$y = 2x + 9$

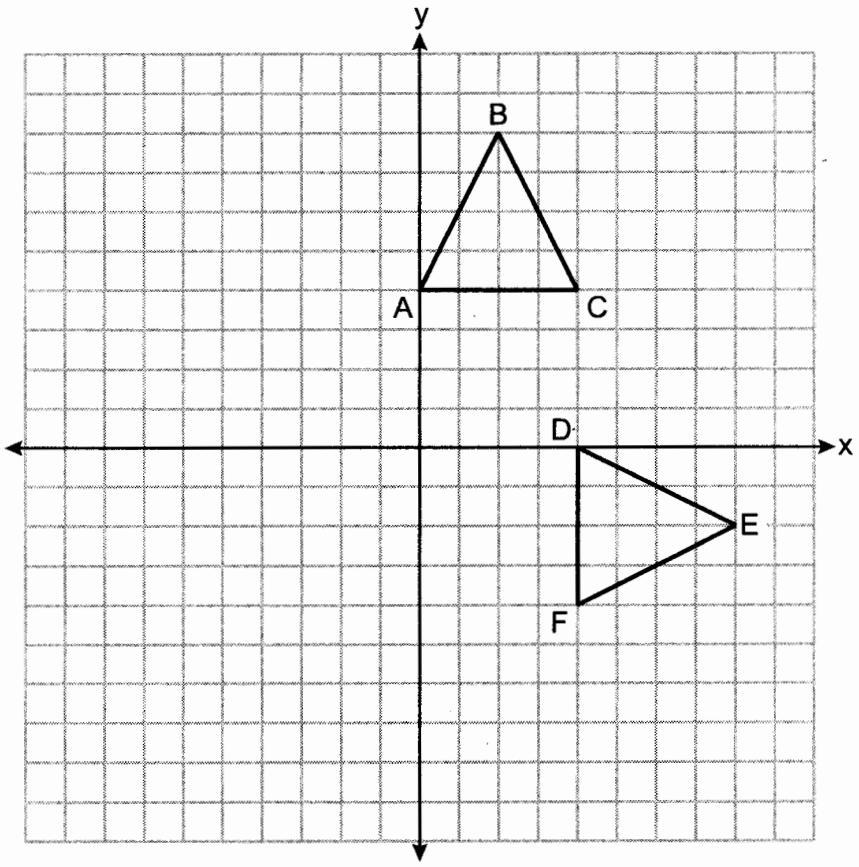
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 In $\triangle ABC$ below, use a compass and straightedge to construct the altitude from C to $\overline{AB}$. [Leave all construction marks.]



26 Triangles *ABC* and *DEF* are graphed on the set of axes below.



Describe a sequence of transformations that maps $\triangle ABC$ onto $\triangle DEF$.

$T_{4,-4}$, followed by a 90° clockwise rotation about point D.

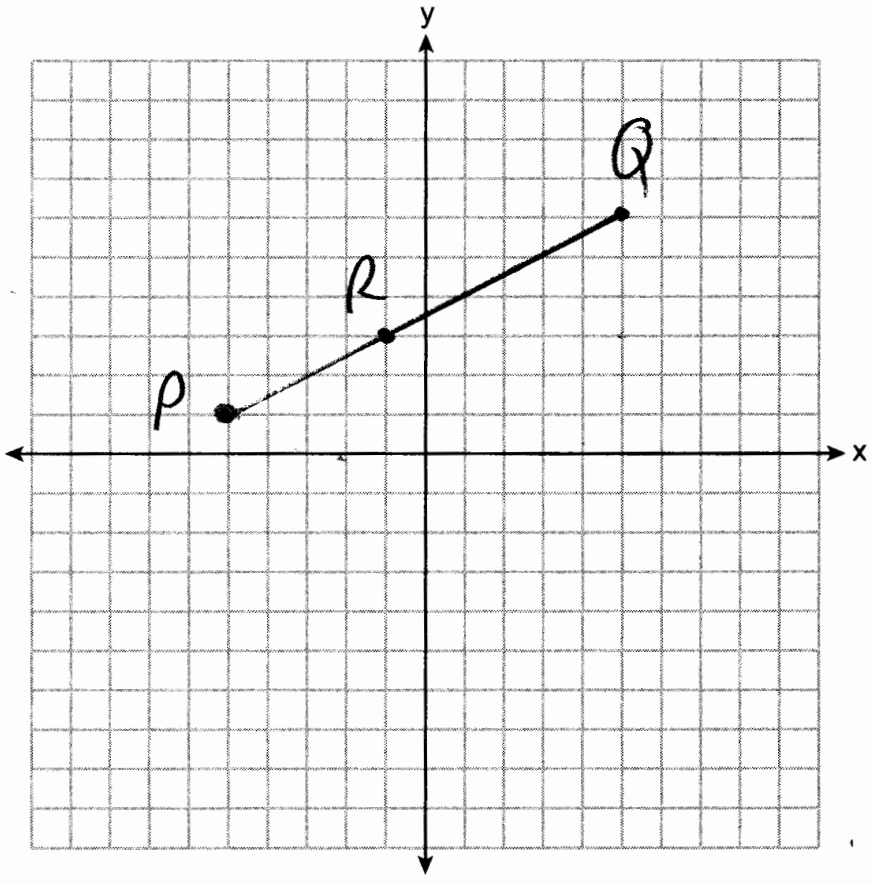
27 Line segment PQ has endpoints $P(-5,1)$ and $Q(5,6)$, and point R is on $\overline{PQ}$. Determine and state the coordinates of R, such that $PR:RQ = 2:3$.

[The use of the set of axes below is optional.]

$-5+\frac{2}{5}(5--5) = -5+\frac{2}{5}(10) = -1$

$1+\frac{2}{5}(6-1) = 1+\frac{2}{5}(5) = 3$

$(-1,3)$



28 A circle has a radius of 6.4 inches. Determine and state, to the *nearest square inch*, the area of a sector whose arc measures 80°.

$\frac{80}{360}\pi(6.4)^2 \approx 29$

29 A large snowman is made of three spherical snowballs with radii of 1 foot, 2 feet, and 3 feet, respectively. Determine and state the amount of snow, in cubic feet, that is used to make the snowman.

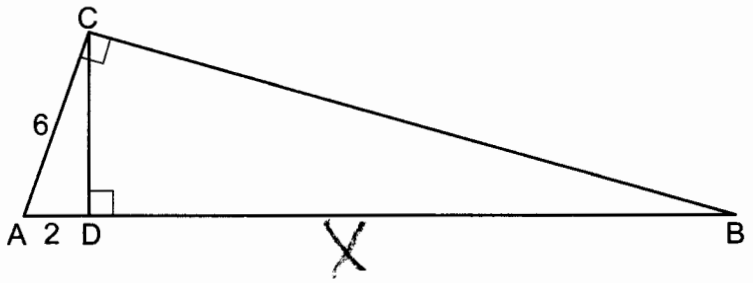
[Leave your answer in terms of π.]

$\frac{4}{3}\pi(1)^3 + \frac{4}{3}\pi(2)^3 + \frac{4}{3}\pi(3)^3$

$\frac{4}{3}\pi + \frac{32\pi}{3} + \frac{108\pi}{3}$

48π

30 In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$, $AD = 2$ and $AC = 6$.



Determine and state the length of $\overline{AB}$.

$6^2 = 2(x+2)$

$36 = 2x + 4$

$32 = 2x$

$16 = x$

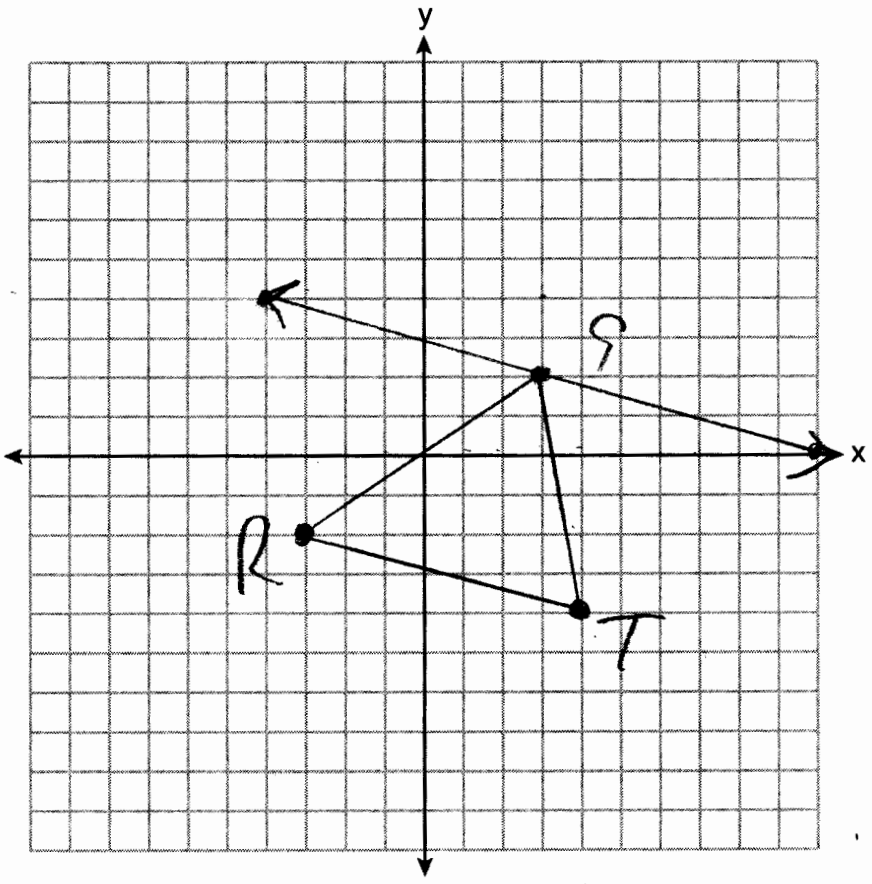
$\overline{AB} = 16+2 = 18$

31 Triangle RST has vertices with coordinates $R(-3,-2)$, $S(3,2)$ and $T(4,-4)$. Determine and state an equation of the line parallel to $\overline{RT}$ that passes through point S.

[The use of the set of axes below is optional.]

$$\frac{-2--4}{-3-4} = \frac{2}{-7}$$

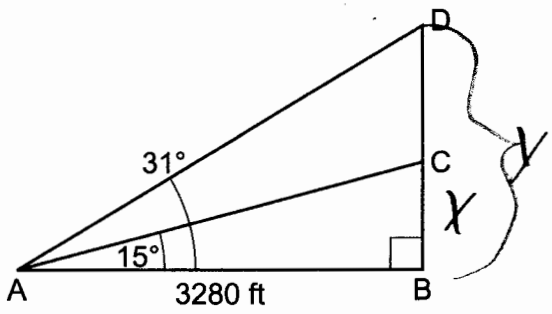
$$y-2 = -\frac{2}{7}(x-3)$$



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area *A*, 3280 feet away from launch pad *B*. After launch, the rocket was sighted at *C* with an angle of elevation of 15°. The rocket was later sighted at *D* with an angle of elevation of 31°.



Determine and state, to the *nearest foot*, the distance the rocket traveled between the two sightings, *C* and *D*.

$\tan 15 = \frac{X}{3280}$ $\tan 31 = \frac{Y}{3280}$

$X \approx 878.9$ $Y \approx 1970.8$

$1970.8 - 878.9 = 1091.9 \approx 1092$

33 A small can of soup is a right circular cylinder with a base diameter of 7 cm and a height of 9 cm. A large container is also a right circular cylinder with a base diameter of 9 cm and a height of 13 cm.

Determine and state the volume of the small can and the volume of the large container to the *nearest cubic centimeter*.

$\pi(3.5)^2(9) \approx 346$

$\pi(4.5)^2(13) \approx 827$

What is the minimum number of small cans that must be opened to fill the large container? Justify your answer.

$\frac{827}{346} \approx 2.4$

3 cans

34 Parallelogram *MATH* has vertices $M(-7,-2)$, $A(0,4)$, $T(9,2)$, and $H(2,-4)$.

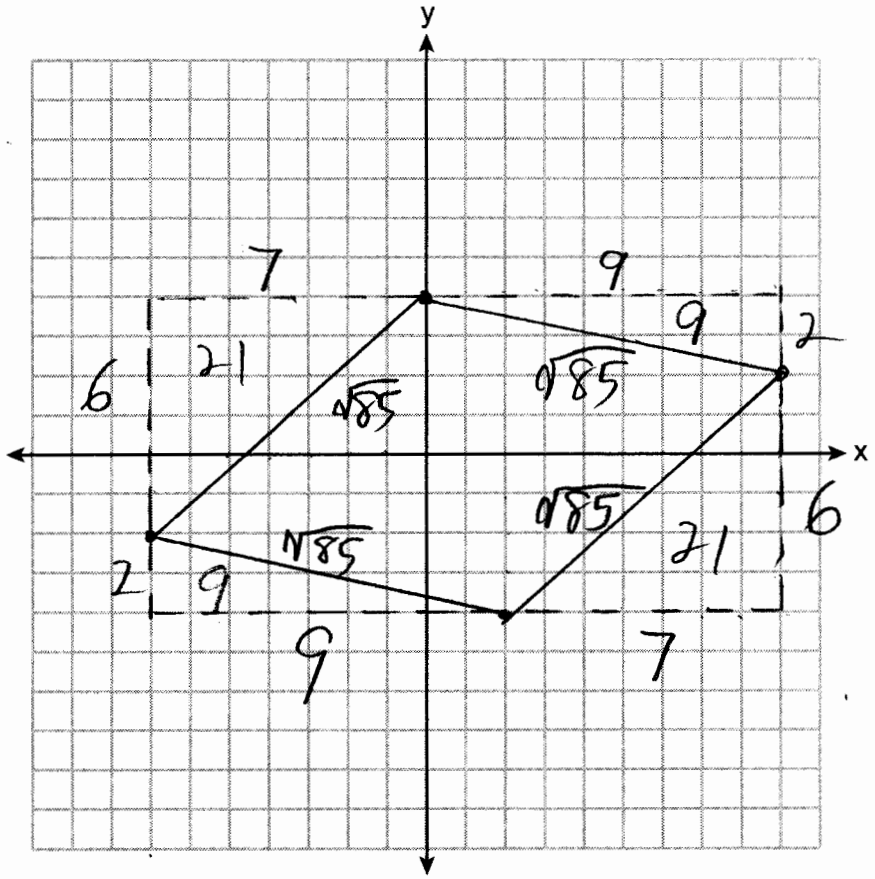
Prove that parallelogram *MATH* is a rhombus.

[The use of the set of axes below is optional.]

A rhombus has four congruent sides. Since each side measures $\sqrt{85}$, all four sides of MATH are congruent ∴ MATH is a rhombus.

Determine and state the area of *MATH*.

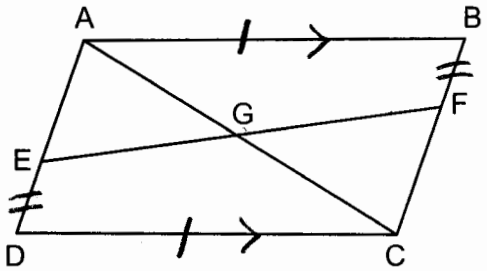
$16 \cdot 8 - (21 + 9 + 21 + 9) = 68$



Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Quadrilateral $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, diagonal $\overline{AC}$ intersects $\overline{EF}$ at G, and $\overline{DE} \cong \overline{BF}$



Prove: G is the midpoint of $\overline{EF}$

STATEMENT	REASON
(1) Quad $ABCD$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$ diagonal $\overline{AC}$ intersects $\overline{EF}$ at G, $\overline{DE} \cong \overline{BF}$	(1) Given
(2) $ABCD$ is a parallelogram	(2) A quadrilateral with a pair of opposite sides that are $\parallel$ & $\cong$ is a parallelogram
(3) $\overline{AD} \cong \overline{CB}$	(3) Opposite sides of a parallelogram are $\cong$
(4) $\overline{AE} \cong \overline{CF}$	(4) Subtraction postulate
(5) $\overline{AD} \parallel \overline{CB}$	(5) Opposite sides of a parallelogram are $\parallel$
(6) $\angle EAG \cong \angle FCG$	(6) If parallel lines are cut by a transversal the alternate interior angles are $\cong$
(7) $\angle AGE \cong \angle CGF$	(7) Vertical angles
(8) $\triangle AEG \cong \triangle CFG$	(8) AAS
(9) $\overline{EG} \cong \overline{FG}$	(9) CPCTC
(10) G is the midpoint of $\overline{EF}$	(10) Since G divides $\overline{EF}$ into two equal parts, G is the midpoint.

Work space for question 35 is continued on the next page.