

GEOMETRY

Friday, June 21, 2019 — 9:15 a.m. to 12:15 p.m., only

Student Name: _____

Mr. S. Bol

School Name: _____

JMRP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

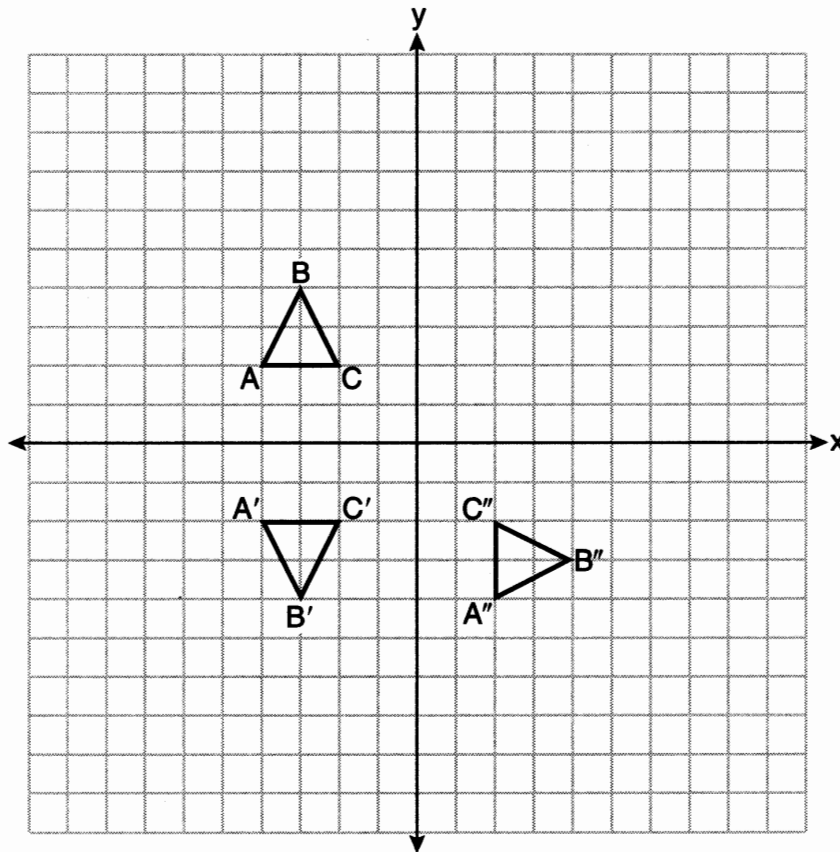
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

- 1 On the set of axes below, triangle ABC is graphed. Triangles $A'B'C'$ and $A''B''C''$, the images of triangle ABC , are graphed after a sequence of rigid motions.



Identify which sequence of rigid motions maps $\triangle ABC$ onto $\triangle A'B'C'$ and then maps $\triangle A'B'C'$ onto $\triangle A''B''C''$.

- (1) a rotation followed by another rotation
- (2) a translation followed by a reflection
- (3) a reflection followed by a translation
- (4) a reflection followed by a rotation

Use this space for computations.

2 The table below shows the population and land area, in square miles, of four counties in New York State at the turn of the century.

| County | 2000 Census Population | 2000 Land Area (mi ²) |
|----------|------------------------|-----------------------------------|
| Broome | 200,536 | 706.82 |
| Dutchess | 280,150 | 801.59 |
| Niagara | 219,846 | 522.95 |
| Saratoga | 200,635 | 811.84 |

Pop
Area
284
349
420
247

Which county had the greatest population density?

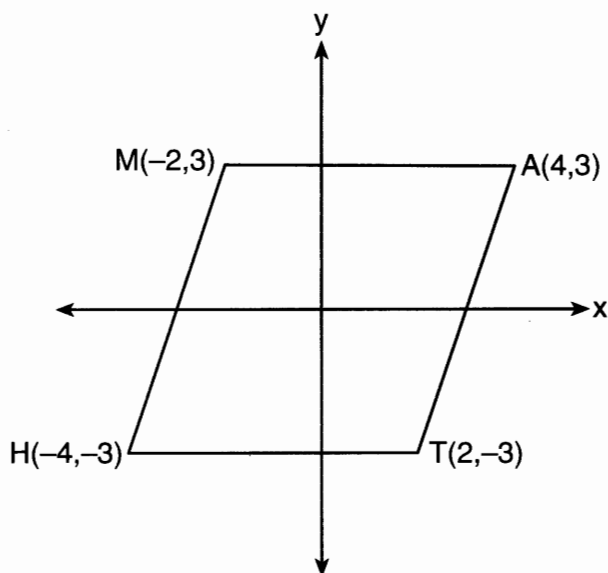
- (1) Broome (3) Niagara
(2) Dutchess (4) Saratoga

3 If a rectangle is continuously rotated around one of its sides, what is the three-dimensional figure formed?

- (1) rectangular prism (3) sphere
(2) cylinder (4) cone

Use this space for computations.

4 Which transformation carries the parallelogram below onto itself?



- (1) a reflection over $y = x$
- (2) a reflection over $y = -x$
- (3) a rotation of 90° counterclockwise about the origin
- (4) a rotation of 180° counterclockwise about the origin

5 After a dilation centered at the origin, the image of \overline{CD} is $\overline{C'D'}$. If the coordinates of the endpoints of these segments are $C(6,-4)$, $D(2,-8)$, $C'(9,-6)$, and $D'(3,-12)$, the scale factor of the dilation is

- (1) $\frac{3}{2}$
- (2) $\frac{2}{3}$
- (3) 3
- (4) $\frac{1}{3}$

$$\frac{9}{6} = \frac{-6}{-4} = \frac{3}{2} = \frac{-12}{-8}$$

Use this space for computations.

6 A tent is in the shape of a right pyramid with a square floor. The square floor has side lengths of 8 feet. If the height of the tent at its center is 6 feet, what is the volume of the tent, in cubic feet?

- (1) 48
(2) 128
(3) 192
(4) 384

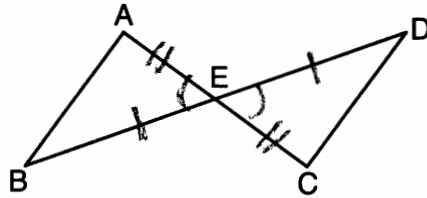
$$V = \frac{1}{3} (8)^2 (6) = 128$$

$$m = -\frac{A}{B} = \frac{3}{4}$$

7 The line $-3x + 4y = 8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image?

- (1) $y = \frac{4}{3}x + 8$
(2) $y = \frac{3}{4}x + 8$
(3) $y = -\frac{3}{4}x - 8$
(4) $y = -\frac{4}{3}x - 8$

8 In the diagram below, \overline{AC} and \overline{BD} intersect at E .



Which information is always sufficient to prove $\triangle ABE \cong \triangle CDE$?

- (1) $\overline{AB} \parallel \overline{CD}$ AAA
(2) $\overline{AB} \cong \overline{CD}$ and $\overline{BE} \cong \overline{DE}$ ASS
(3) E is the midpoint of \overline{AC} . SA
(4) \overline{BD} and \overline{AC} bisect each other. SAS

9 The expression $\sin 57^\circ$ is equal to

- (1) $\tan 33^\circ$
(2) $\cos 33^\circ$
(3) $\tan 57^\circ$
(4) $\cos 57^\circ$

$$90 - 57 = 33$$

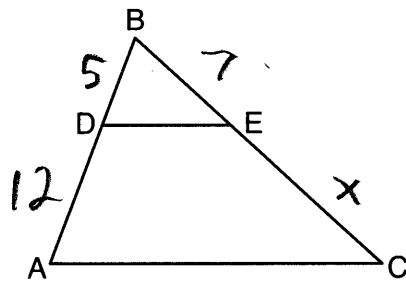
Use this space for computations.

10 What is the volume of a hemisphere that has a diameter of 12.6 cm, to the nearest tenth of a cubic centimeter?

- (1) 523.7 (3) 4189.6
(2) 1047.4 (4) 8379.2

$$\frac{1}{2} \cdot \frac{4}{3} \pi \left(\frac{12.6}{2}\right)^3 \approx 513.7$$

11 In the diagram below of $\triangle ABC$, D is a point on \overline{BA} , E is a point on \overline{BC} , and \overline{DE} is drawn.



$$\begin{aligned} 5x &= 12 \cdot 7 \\ 5x &= 84 \\ x &= 16.8 + 7 = 23.8 \end{aligned}$$

If $BD = 5$, $DA = 12$, and $BE = 7$, what is the length of \overline{BC} so that $\overline{AC} \parallel \overline{DE}$?

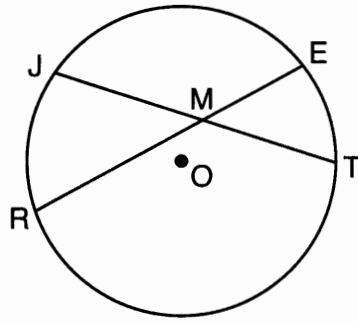
- (1) 23.8 (3) 15.6
(2) 16.8 (4) 8.6

12 A quadrilateral must be a parallelogram if

- (1) one pair of sides is parallel and one pair of angles is congruent
(2) one pair of sides is congruent and one pair of angles is congruent
(3) one pair of sides is both parallel and congruent
(4) the diagonals are congruent

13 In the diagram below of circle O , chords \overline{JT} and \overline{ER} intersect at M .

Use this space for computations.



If $EM = 8$ and $RM = 15$, the lengths of \overline{JM} and \overline{TM} could be

- (1) 12 and 9.5
- (2) 14 and 8.5

- (3) 16 and 7.5
- (4) 18 and 6.5

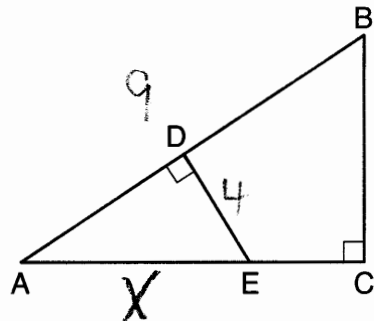
$$8 \cdot 15 = 16 \cdot 7.5$$

14 Triangles $\triangle JOE$ and $\triangle SAM$ are drawn such that $\angle E \cong \angle M$ and $\overline{EJ} \cong \overline{MS}$. Which mapping would *not* always lead to $\triangle JOE \cong \triangle SAM$?

- (1) $\angle J$ maps onto $\angle S$
- (2) $\angle O$ maps onto $\angle A$

- (3) \overline{EO} maps onto \overline{MA}
- (4) \overline{JO} maps onto \overline{SA} SSA

15 In $\triangle ABC$ shown below, $\angle ACB$ is a right angle, E is a point on \overline{AC} , and \overline{ED} is drawn perpendicular to hypotenuse \overline{AB} .



If $AB = 9$, $BC = 6$, and $DE = 4$, what is the length of \overline{AE} ?

- (1) 5
- (2) 6

- (3) 7
- (4) 8

$$\frac{4}{x} = \frac{6}{9}$$

$$6x = 36$$

$$x = 6$$

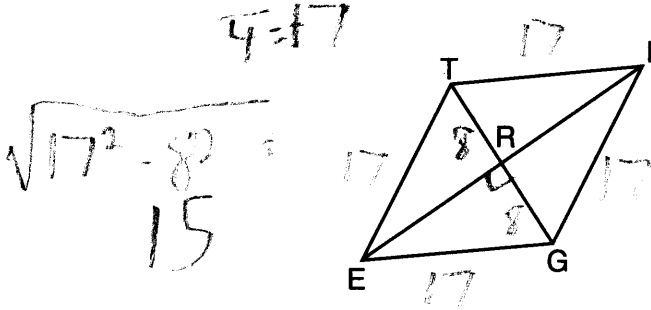
$$M = \frac{-(-2)}{3} = \frac{2}{3}$$

Use this space for computations.

16 Which equation represents a line parallel to the line whose equation is $-2x + 3y = -4$ and passes through the point $(1,3)$?

- (1) $y - 3 = -\frac{3}{2}(x - 1)$ (3) $y + 3 = -\frac{3}{2}(x + 1)$
 (2) $y - 3 = \frac{2}{3}(x - 1)$ (4) $y + 3 = \frac{2}{3}(x + 1)$

17 In rhombus $TIGE$, diagonals \overline{TG} and \overline{IE} intersect at R . The perimeter of $TIGE$ is 68, and $TG = 16$.

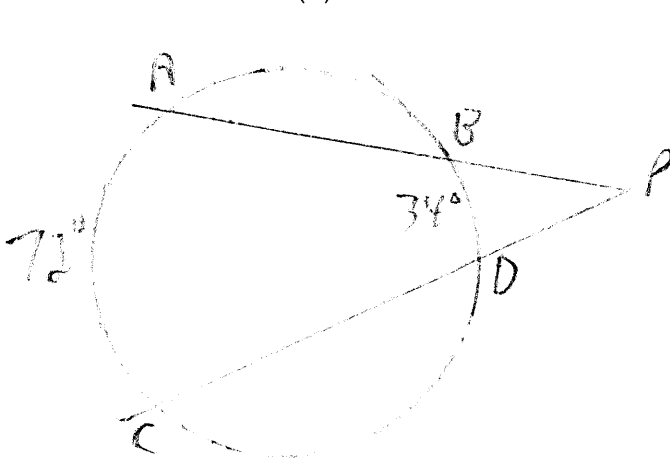


What is the length of diagonal \overline{IE} ?

- (1) 15 (3) 34
 (2) 30 (4) 52

18 In circle O two secants, \overline{ABP} and \overline{CDP} , are drawn to external point P . If $m\widehat{AC} = 72^\circ$, and $m\widehat{BD} = 34^\circ$, what is the measure of $\angle P$?

- (1) 19° (3) 53°
 (2) 38° (4) 106°



$$\frac{72 - 34}{2} = 19$$

Use this space for computations.

19 What are the coordinates of point C on the directed segment from $A(-8,4)$ to $B(10,-2)$ that partitions the segment such that $AC:CB$ is $2:1$?

- (1) $(1,1)$ (3) $(2,-2)$
 (2) $(-2,2)$ (4) $(4,0)$

$$-8 + \frac{2}{3}(10 - -8) = 4$$

$$4 + \frac{2}{3}(-2 - 4) = 0$$

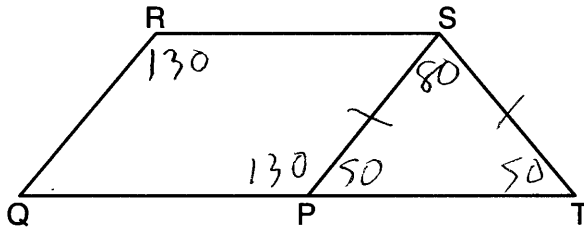
20 The equation of a circle is $x^2 + 8x + y^2 - 12y = 144$. What are the coordinates of the center and the length of the radius of the circle?

- (1) center $(4,-6)$ and radius 12
 (2) center $(-4,6)$ and radius 12
 (3) center $(4,-6)$ and radius 14
 (4) center $(-4,6)$ and radius 14

$$x^2 + 8x + 16 + y^2 - 12y + 36 = 144 + 16 + 36$$

$$(x+4)^2 + (y-6)^2 = 196$$

21 In parallelogram $PQRS$, \overline{QP} is extended to point T and \overline{ST} is drawn.



If $\overline{ST} \cong \overline{SP}$ and $m\angle R = 130^\circ$, what is $m\angle PST$?

- (1) 130° (3) 65°
 (2) 80° (4) 50°

Use this space for computations.

22 A 12-foot ladder leans against a building and reaches a window 10 feet above ground. What is the measure of the angle, to the nearest degree, that the ladder forms with the ground?

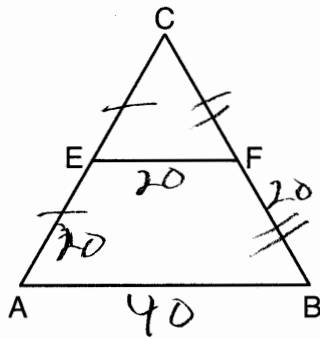
- (1) 34
 (2) 40
 (3) 50
 (4) 56



$$\sin X = \frac{10}{12}$$

$$X \approx 56$$

23 In the diagram of equilateral triangle ABC shown below, E and F are the midpoints of \overline{AC} and \overline{BC} , respectively.



$$2(2x + 8) = 7x - 2$$

$$4x + 16 = 7x - 2$$

$$18 = 3x$$

$$6 = x$$

If $EF = 2x + 8$ and $AB = 7x - 2$, what is the perimeter of trapezoid $ABFE$?

- (1) 36
 (2) 60
 (3) 100
 (4) 120

$$7(6) - 2 = 40$$

24 Which information is *not* sufficient to prove that a parallelogram is a square?

- (1) The diagonals are both congruent and perpendicular.
 (2) The diagonals are congruent and one pair of adjacent sides are congruent.
 (3) The diagonals are perpendicular and one pair of adjacent sides are congruent.
 (4) The diagonals are perpendicular and one pair of adjacent sides are perpendicular.

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

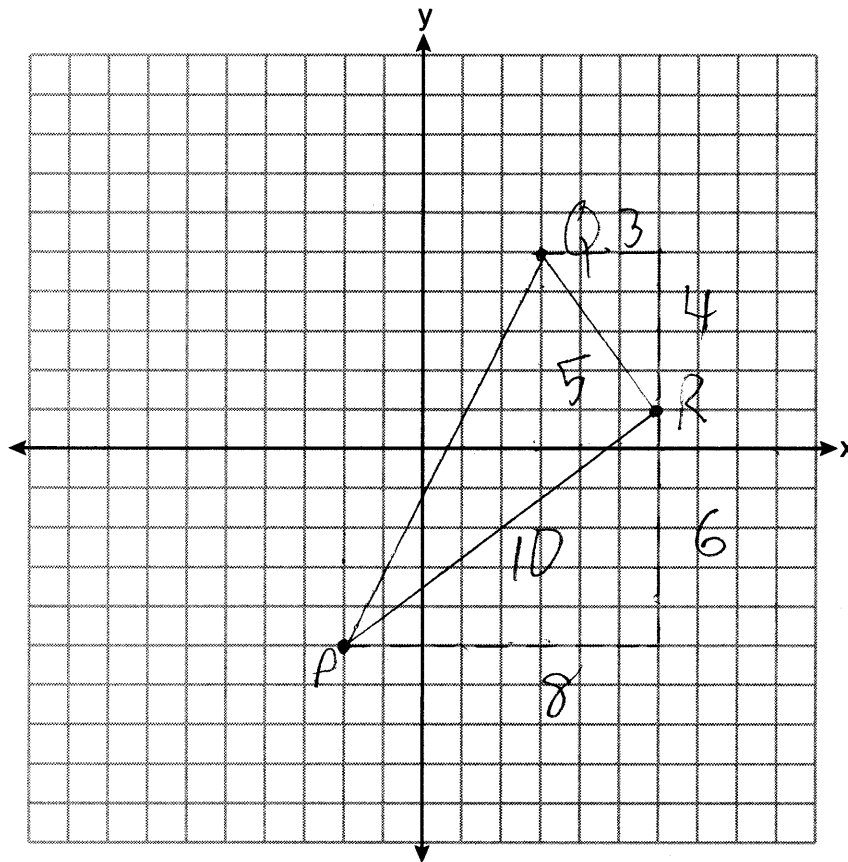
25 Triangle $A'B'C'$ is the image of triangle ABC after a dilation with a scale factor of $\frac{1}{2}$ and centered at point A . Is triangle ABC congruent to triangle $A'B'C'$? Explain your answer.

No, a dilation does not preserve distance.

26 Determine and state the area of triangle PQR , whose vertices have coordinates $P(-2,-5)$, $Q(3,5)$, and $R(6,1)$.

[The use of the set of axes below is optional.]

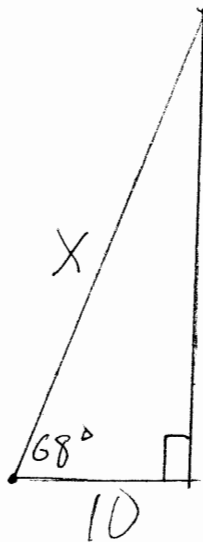
$$\frac{1}{2}(5)(10) = 25$$



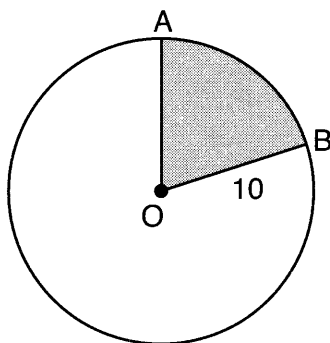
- 27 A support wire reaches from the top of a pole to a clamp on the ground. The pole is perpendicular to the level ground and the clamp is 10 feet from the base of the pole. The support wire makes a 68° angle with the ground. Find the length of the support wire to the *nearest foot*.

$$\cos 68 = \frac{10}{x}$$

$$x \approx 27$$



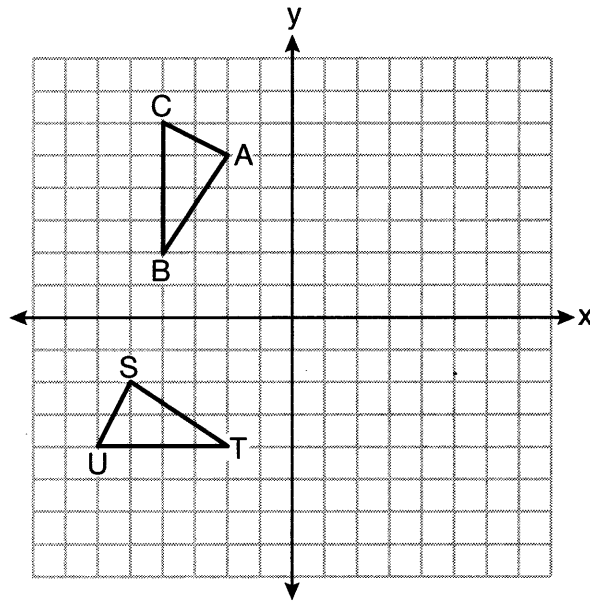
28 In the diagram below, circle O has a radius of 10.



If $m\widehat{AB} = 72^\circ$, find the area of shaded sector AOB , in terms of π .

$$\frac{72}{360} (\pi) (10^2) = 20\pi$$

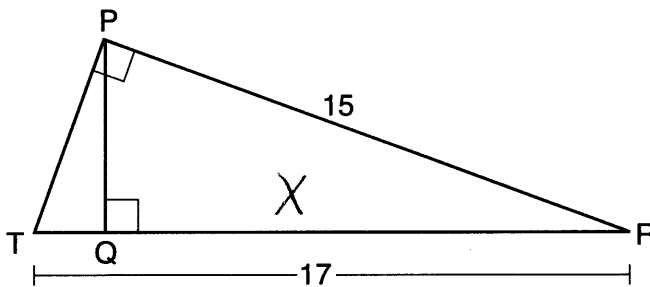
29 On the set of axes below, $\triangle ABC \cong \triangle STU$.



Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.

Rotation 90° about the origin

- 30 In right triangle PRT , $m\angle P = 90^\circ$, altitude \overline{PQ} is drawn to hypotenuse \overline{RT} , $RT = 17$, and $PR = 15$.



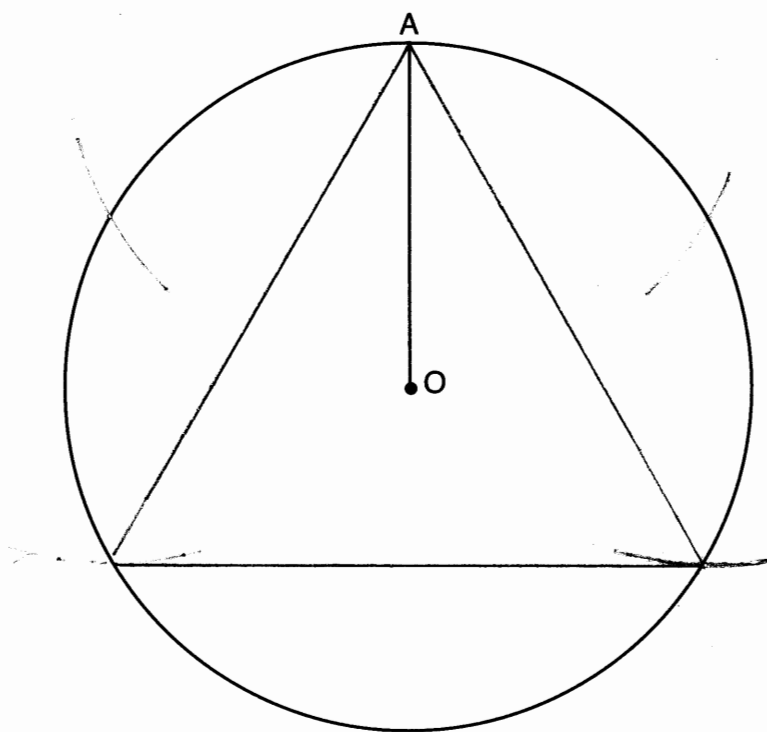
Determine and state, to the *nearest tenth*, the length of \overline{RQ} .

$$17x = 15^2$$

$$17x = 225$$

$$x \approx 13.2$$

31 Given circle O with radius \overline{OA} , use a compass and straightedge to construct an equilateral triangle inscribed in circle O . [Leave all construction marks.]



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Riley plotted $A(-1,6)$, $B(3,8)$, $C(6,-1)$, and $D(1,0)$ to form a quadrilateral.

Prove that Riley's quadrilateral $ABCD$ is a trapezoid.

[The use of the set of axes on the next page is optional.]

$$m_{\overline{AD}} = \frac{0-6}{1-(-1)} = -3$$

$$m_{\overline{BC}} = \frac{-1-8}{6-3} = -3$$

$\overline{AD} \parallel \overline{BC}$ because their slope is equal.

$ABCD$ is a trapezoid because it has a pair of parallel sides.

Question 32 is continued on the next page.

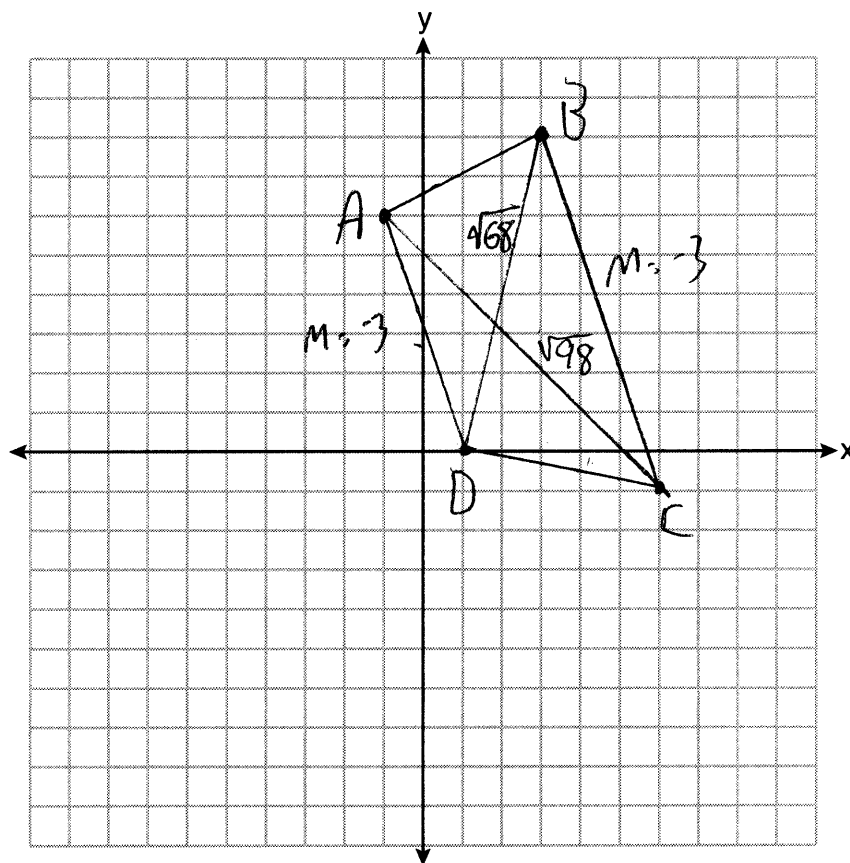
Question 32 continued

Riley defines an isosceles trapezoid as a trapezoid with congruent diagonals. Use Riley's definition to prove that $ABCD$ is *not* an isosceles trapezoid.

$$AC = \sqrt{(-1-6)^2 + (6--1)^2} = \sqrt{98}$$

$$BD = \sqrt{(8-0)^2 + (3-1)^2} = \sqrt{68}$$

$ABCD$ is not an isosceles trapezoid because its diagonals are not congruent.



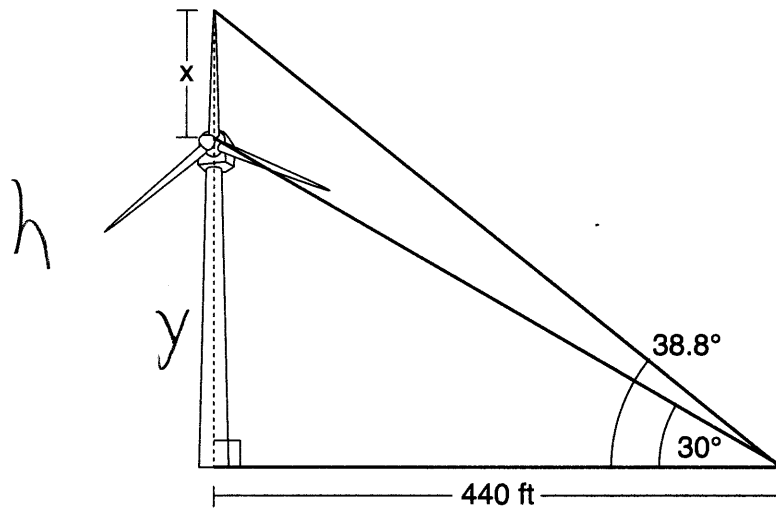
- 33 A child-sized swimming pool can be modeled by a cylinder. The pool has a diameter of $6\frac{1}{2}$ feet and a height of ~~12~~¹⁸ inches. The pool is filled with water to $\frac{2}{3}$ of its height. Determine and state the volume of the water in the pool, to the *nearest cubic foot*.

$$V = \frac{2}{3} \pi \left(\frac{6.5}{2} \right)^2 \cdot 1 \approx 22$$

One cubic foot equals 7.48 gallons of water. Determine and state, to the *nearest gallon*, the number of gallons of water in the pool.

$$22 \cdot 7.48 \approx 165$$

34 Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor's tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8° . He also measured the angle between the ground and the lowest point of the top blade, and found it was 30° .



Determine and state a blade's length, x , to the *nearest foot*.

$$\tan 30^\circ = \frac{y}{440} \qquad \tan 38.8^\circ = \frac{h}{440}$$

$$y \approx 254$$

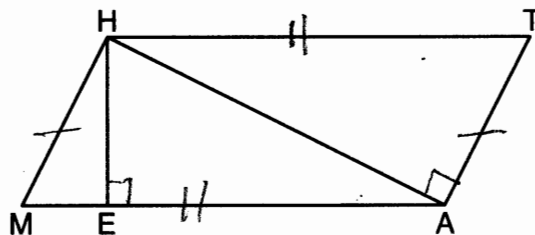
$$h \approx 353.8$$

$$h - y \approx 100$$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for the question to determine your answer. Note that diagrams are not necessarily drawn to scale. For the question in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Quadrilateral $MATH$, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$



Prove: $TA \cdot HA = HE \cdot TH$

| Statement | Reason |
|--|---|
| ① Quad $MATH$, $\overline{HM} \cong \overline{AT}$, $\overline{HT} \cong \overline{AM}$, $\overline{HE} \perp \overline{MEA}$, and $\overline{HA} \perp \overline{AT}$ | ① Given |
| ② $\angle HEA$ & $\angle TAH$ are rt angles | ② Perpendicular lines form rt angles |
| ③ $\angle HEA \cong \angle TAH$ | ③ Right angles are congruent |
| ④ $MATH$ is a parallelogram | ④ A quadrilateral with 2 pairs of congruent opposite sides is a parallelogram |
| ⑤ $\overline{MA} \parallel \overline{TH}$ | ⑤ Opposite sides of a parallelogram are parallel |
| ⑥ $\angle THA \cong \angle EAH$ | ⑥ alternate interior angles of parallel lines & a transversal are congruent |
| ⑦ $\triangle HEA \sim \triangle TAH$ | ⑦ AA |
| ⑧ $\frac{HA}{TH} = \frac{HE}{TA}$ | ⑧ corresponding sides of similar Δ s are in proportion |
| ⑨ $TA \cdot HA = HE \cdot TH$ | ⑨ product of means = product of extremes |

Work space for question 35 is continued on the next page.