The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

(Common Core)

Friday, June 16, 2017 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice…
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

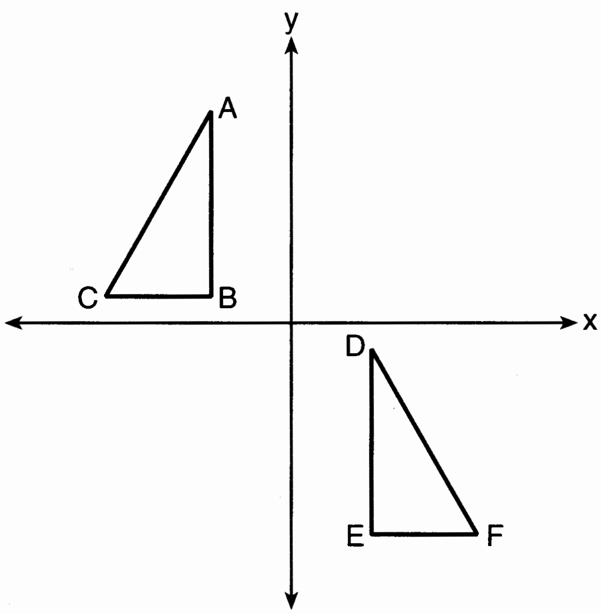
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, $\triangle ABC \cong \triangle DEF$.



Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

(1) a reflection over the x-axis followed by a translation

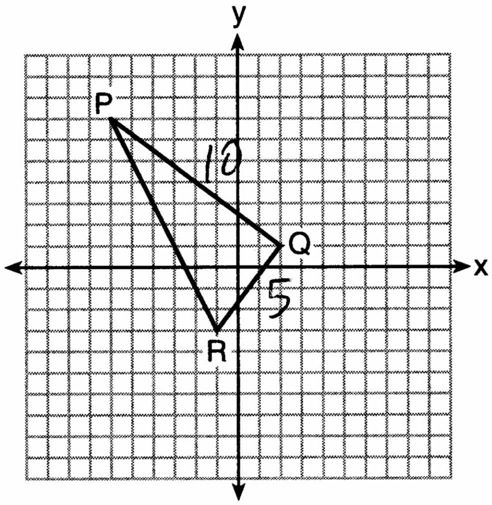
(2) a reflection over the y-axis followed by a translation

(3) a rotation of 180° about the origin followed by a translation

(4) a counterclockwise rotation of 90° about the origin followed by a translation

Use this space for computations.

2 On the set of axes below, the vertices of $\triangle PQR$ have coordinates $P(-6,7)$, $Q(2,1)$, and $R(-1,-3)$.



$\frac{1}{2}(5)(10) = 25$

What is the area of $\triangle PQR$?

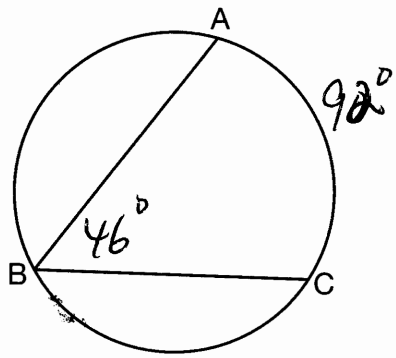
(1) 10
(2) 20
(3) 25
(4) 50

3 In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

(1) $\tan A$
(2) $\tan B$
(3) $\sin A$
(4) $\sin B$

Use this space for computations.

4 In the diagram below, $m\widehat{ABC} = 268°$.



$\frac{1}{2}(360-268)$

46

What is the number of degrees in the measure of $\angle ABC$?

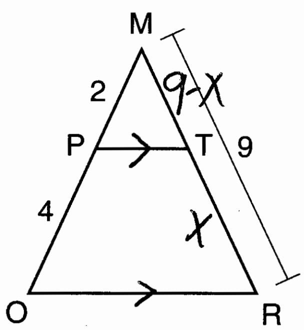
(1) 134°

(2) 92°

(3) 68°

(4) 46°

5 Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.



$\frac{2}{4} = \frac{9-x}{x}$

$2x = 36 - 4x$

$6x = 36$

$x = 6$

What is the length of $\overline{TR}$?

(1) 4.5

(2) 5

(3) 3

(4) 6

Use this space for computations.

6 A line segment is dilated by a scale factor of 2 centered at a point not on the line segment. Which statement regarding the relationship between the given line segment and its image is true?

(1) The line segments are perpendicular, and the image is one-half of the length of the given line segment.

(2) The line segments are perpendicular, and the image is twice the length of the given line segment.

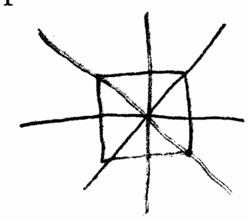
(3) The line segments are parallel, and the image is twice the length of the given line segment.

(4) The line segments are parallel, and the image is one-half of the length of the given line segment.

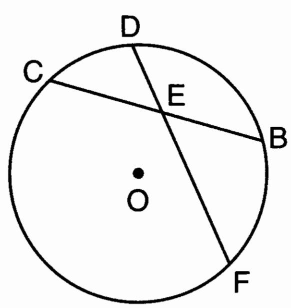
7 Which figure always has exactly four lines of reflection that map the figure onto itself?

(1) square (3) regular octagon

(2) rectangle (4) equilateral triangle



8 In the diagram below of circle *O*, chord $\overline{DF}$ bisects chord $\overline{BC}$ at *E*.



$6 \cdot 6 = x(x-5)$

$0 = x^2 - 5x - 36$

$0 = (x-9)(x+4)$

$x = 9, \cancel{-4}$

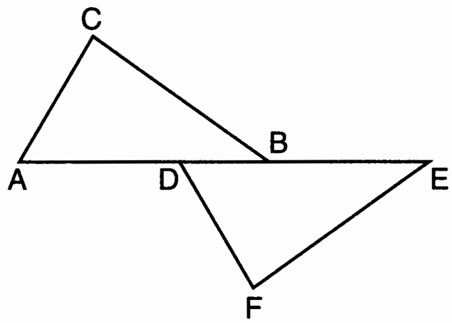
If $BC = 12$ and *FE* is 5 more than *DE*, then *FE* is

(1) 13 (3) 6

(2) 9 (4) 4

Use this space for computations.

9 Kelly is completing a proof based on the figure below.

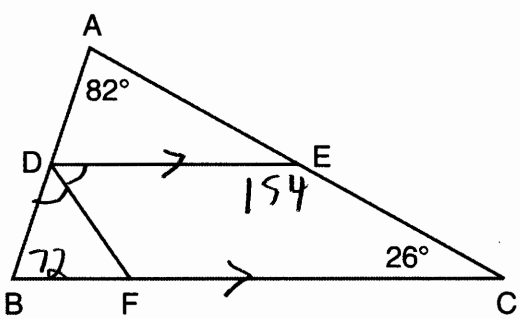


She was given that $\angle A \cong \angle EDF$, and has already proven $\overline{AB} \cong \overline{DE}$. Which pair of corresponding parts and triangle congruency method would *not* prove $\triangle ABC \cong \triangle DEF$?

(1) $\overline{AC} \cong \overline{DF}$ and SAS (3) $\angle C \cong \angle F$ and AAS

(2) $\overline{BC} \cong \overline{EF}$ and SAS (4) $\angle CBA \cong \angle FED$ and ASA

10 In the diagram below, $\overline{DE}$ divides $\overline{AB}$ and $\overline{AC}$ proportionally, $m\angle C = 26°$, $m\angle A = 82°$, and $\overline{DF}$ bisects $\angle BDE$.



$82 + 26 + 72 = 180$

$154 + 26 + 72 + 108 = 360$

$\frac{108}{2} = 54$

$54 + 54 + 72 = 180$

The measure of angle *DFB* is

(1) 36° (3) 72°

(2) 54° (4) 82°

Use this space for computations.

11 Which set of statements would describe a parallelogram that can always be classified as a rhombus?

I. Diagonals are perpendicular bisectors of each other.
II. Diagonals bisect the angles from which they are drawn.
III. Diagonals form four congruent isosceles right triangles.

(1) I and II
(2) I and III
(3) II and III
(4) I, II, and III

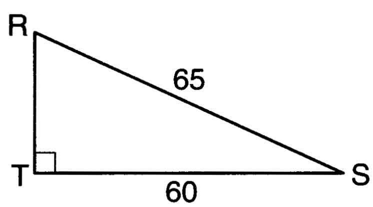
12 The equation of a circle is $x^2 + y^2 - 12y + 20 = 0$. What are the coordinates of the center and the length of the radius of the circle?

(1) center (0,6) and radius 4
(2) center (0,−6) and radius 4
(3) center (0,6) and radius 16
(4) center (0,−6) and radius 16

$x^2 + y^2 - 12y + 36 = -20 + 36$

$x^2 + (y-6)^2 = 16$

13 In the diagram of $\triangle RST$ below, $m\angle T = 90°$, $RS = 65$, and $ST = 60$.



$\cos S = \frac{60}{65}$

$S \approx 23$

What is the measure of $\angle S$, to the *nearest degree*?

(1) 23°
(2) 43°
(3) 47°
(4) 67°

Use this space for computations.

14 Triangle $A'B'C'$ is the image of $\triangle ABC$ after a dilation followed by a translation.

Which statement(s) would always be true with respect to this sequence of transformations?

I. $\triangle ABC \cong \triangle A'B'C'$

II. $\triangle ABC \sim \triangle A'B'C'$

III. $\overline{AB} \parallel \overline{A'B'}$

IV. $AA' = BB'$

(1) II, only
(2) I and II
(3) II and III
(4) II, III, and IV

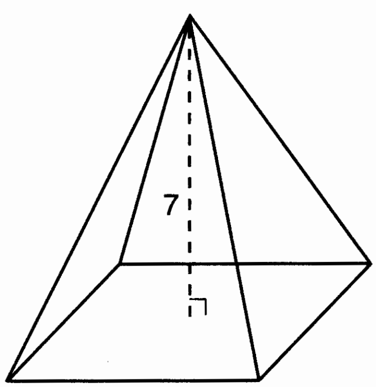
15 Line segment RW has endpoints $R(-4,5)$ and $W(6,20)$. Point P is on $\overline{RW}$ such that $RP:PW$ is 2:3. What are the coordinates of point P?

(1) (2,9)
(2) (0,11)
(3) (2,14)
(4) (10,2)

$-4+\frac{2}{5}(6--4)=0$

$5+\frac{2}{5}(20-5)=11$

16 The pyramid shown below has a square base, a height of 7, and a volume of 84.



What is the length of the side of the base?

(1) 6
(2) 12
(3) 18
(4) 36

$84=\frac{1}{3}s^2 \cdot 7$

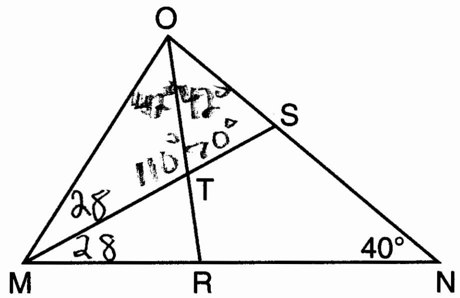
$12=\frac{1}{3}s^2$

$36=s^2$

$6=s$

Use this space for computations.

17 In the diagram below of triangle MNO, $\angle M$ and $\angle O$ are bisected by $\overline{MS}$ and $\overline{OR}$, respectively. Segments MS and OR intersect at T, and $m\angle N = 40°$.

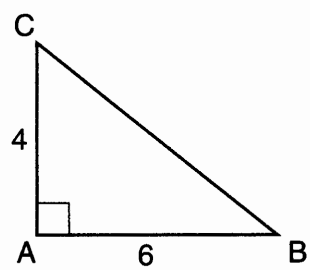


If $m\angle TMR = 28°$, the measure of angle OTS is

(1) 40° (3) 60°

(2) 50° (4) 70°

18 In the diagram below, right triangle ABC has legs whose lengths are 4 and 6.



$\frac{1}{3}\pi(4^2)6 = 32\pi$

What is the volume of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

(1) 32π (3) 96π

(2) 48π (4) 144π

Use this space for computations.

19 What is an equation of a line that is perpendicular to the line whose equation is $2y = 3x - 10$ and passes through $(-6,1)$?

(1) $y = -\frac{2}{3}x - 5$ (3) $y = \frac{2}{3}x + 1$

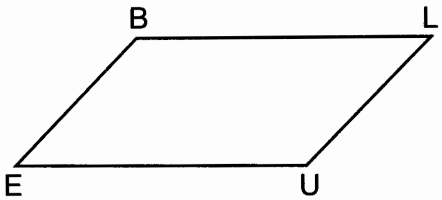
(2) $y = -\frac{2}{3}x - 3$ (4) $y = \frac{2}{3}x + 10$

$M = \frac{3}{2}$ $M_{\perp} = -\frac{2}{3}$

$1 = -\frac{2}{3}(-6) + b$

$-3 = b$

20 In quadrilateral $BLUE$ shown below, $\overline{BE} \cong \overline{UL}$.



Which information would be sufficient to prove quadrilateral $BLUE$ is a parallelogram?

(1) $\overline{BL} \parallel \overline{EU}$ (3) $\overline{BE} \cong \overline{BL}$

(2) $\overline{LU} \parallel \overline{BE}$ (4) $\overline{LU} \cong \overline{EU}$

21 A ladder 20 feet long leans against a building, forming an angle of 71° with the level ground. To the *nearest foot*, how high up the wall of the building does the ladder touch the building?

(1) 15 (3) 18

(2) 16 (4) 19

$\sin 71 = \frac{x}{20}$

$x \approx 19$

22 In the two distinct acute triangles ABC and DEF, $\angle B \cong \angle E$. Triangles ABC and DEF are congruent when there is a sequence of rigid motions that maps

(1) $\angle A$ onto $\angle D$, and $\angle C$ onto $\angle F$

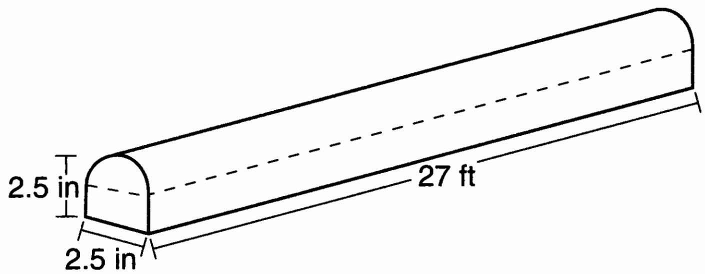
(2) $\overline{AC}$ onto $\overline{DF}$, and $\overline{BC}$ onto $\overline{EF}$

(3) $\angle C$ onto $\angle F$, and $\overline{BC}$ onto $\overline{EF}$

(4) point A onto point D, and $\overline{AB}$ onto $\overline{DE}$

Use this space for computations.

23 A fabricator is hired to make a 27-foot-long solid metal railing for the stairs at the local library. The railing is modeled by the diagram below. The railing is 2.5 inches high and 2.5 inches wide and is comprised of a rectangular prism and a half-cylinder.

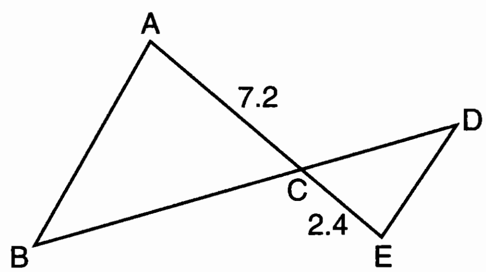


$2.5 \cdot 1.25(27 \times 12) +$
$\frac{1}{2}\pi(1.25)^2(27 \times 12) \approx$
1808

How much metal, to the *nearest cubic inch*, will the railing contain?

(1) 151
(2) 795
(3) 1808
(4) 2025

24 In the diagram below, $AC = 7.2$ and $CE = 2.4$.



Which statement is *not* sufficient to prove $\triangle ABC \sim \triangle EDC$?

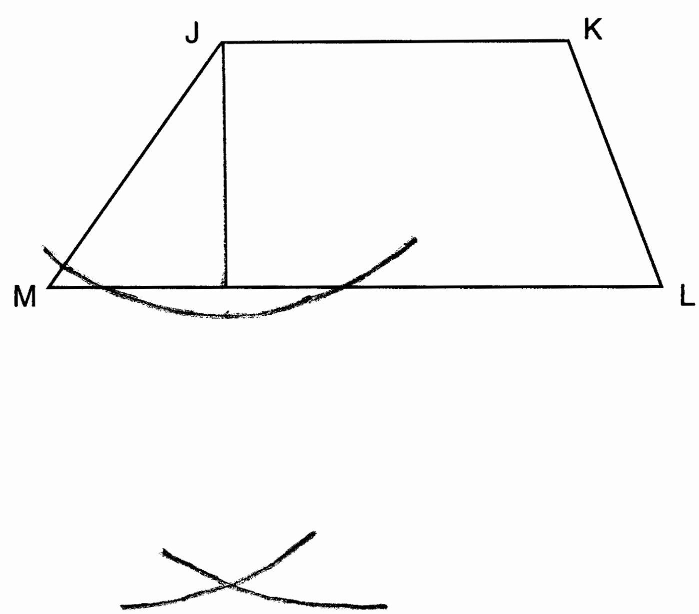
(1) $\overline{AB} \parallel \overline{ED}$ AA
(2) $DE = 2.7$ and $AB = 8.1$
(3) $CD = 3.6$ and $BC = 10.8$ SAS
(4) $DE = 3.0$, $AB = 9.0$, $CD = 2.9$, and $BC = 8.7$ SSS

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Given: Trapezoid $JKLM$ with $\overline{JK} \parallel \overline{ML}$

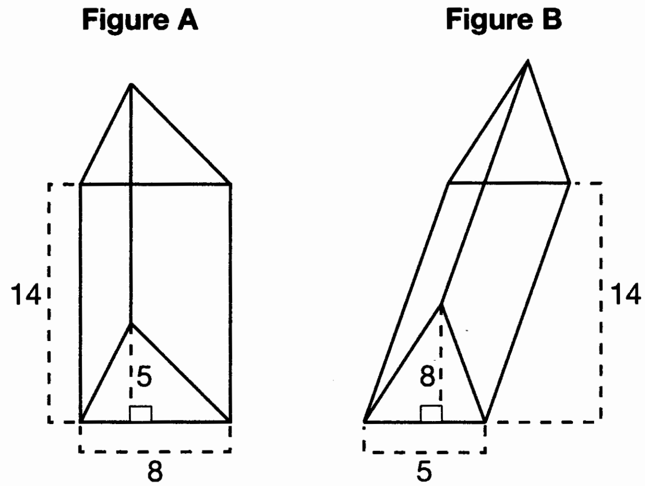
Using a compass and straightedge, construct the altitude from vertex J to $\overline{ML}$.
[Leave all construction marks.]



26 Determine and state, in terms of π, the area of a sector that intercepts a 40° arc of a circle with a radius of 4.5.

$$\frac{40}{360}(\pi)(4.5)^2 = 2.25\pi$$

27 The diagram below shows two figures. Figure *A* is a right triangular prism and figure *B* is an oblique triangular prism. The base of figure *A* has a height of 5 and a length of 8 and the height of prism *A* is 14. The base of figure *B* has a height of 8 and a length of 5 and the height of prism *B* is 14.



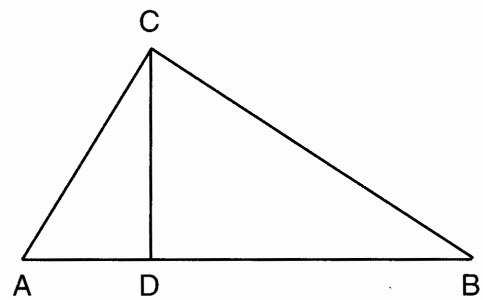
Use Cavalieri's Principle to explain why the volumes of these two triangular prisms are equal.

Each prism has the same base area. Each corresponding cross-section of the prisms will have the same area. Since the prisms have the same height, the volumes must be the same.

28 When volleyballs are purchased, they are not fully inflated. A partially inflated volleyball can be modeled by a sphere whose volume is approximately 180 in^3. After being fully inflated, its volume is approximately 294 in^3. To the *nearest tenth of an inch*, how much does the radius increase when the volleyball is fully inflated?

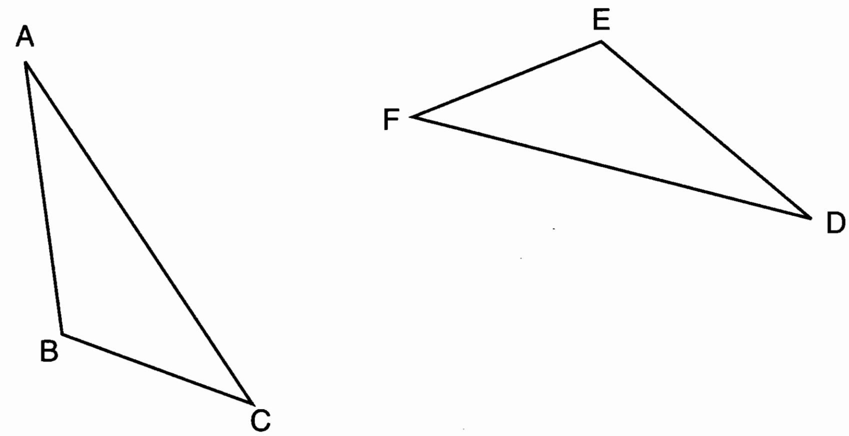
$$\sqrt[3]{\frac{3V_f}{4\pi}} - \sqrt[3]{\frac{3V_p}{4\pi}} = \sqrt[3]{\frac{3(294)}{4\pi}} - \sqrt[3]{\frac{3(180)}{4\pi}} \approx 0.6$$

29 In right triangle ABC shown below, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$. Explain why $\triangle ABC \sim \triangle ACD$.



If an altitude is drawn to the hypotenuse of a triangle, it divides the triangle into two right triangles similar to each other & the original triangle

30 Triangle *ABC* and triangle *DEF* are drawn below.



If $\overline{AB} \cong \overline{DE}$, $\overline{AC} \cong \overline{DF}$, and $\angle A \cong \angle D$, write a sequence of transformations that maps triangle *ABC* onto triangle *DEF*.

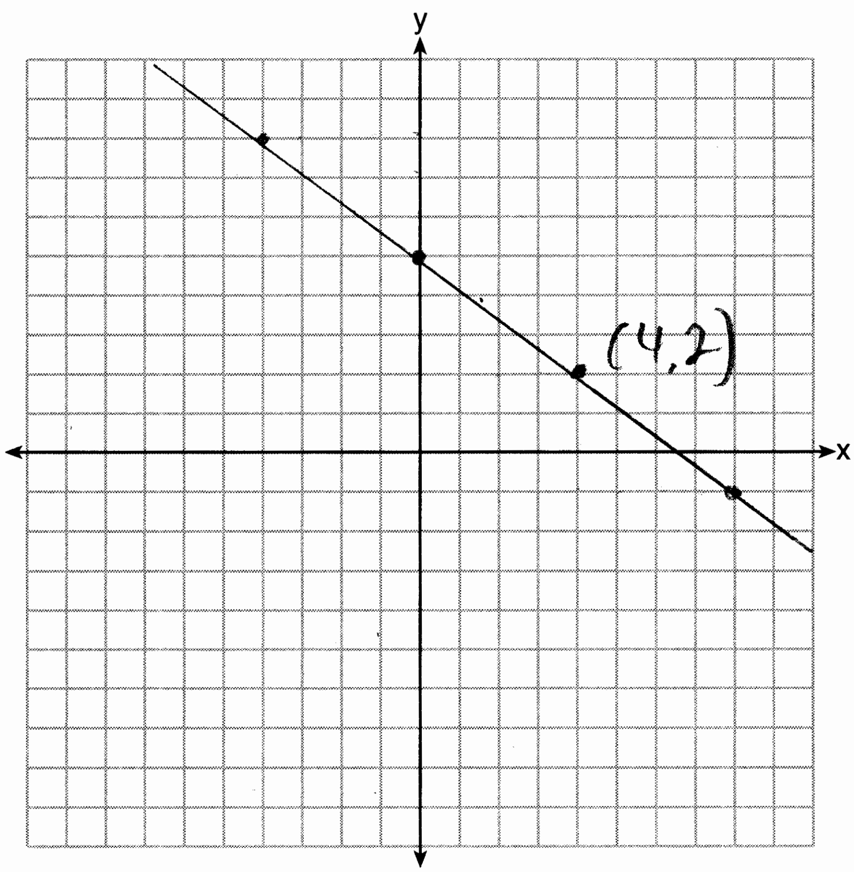
Rotate $\triangle ABC$ clockwise about point C until $\overline{DF} \parallel \overline{AC}$. Translate $\triangle ABC$ along $\overline{CF}$ so that C maps onto F.

31 Line n is represented by the equation $3x + 4y = 20$. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point $(4,2)$. [The use of the set of axes below is optional.]

Explain your answer.

The line is on the center of dilation, so the line does not change.

$p: 3x+4y=20$



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

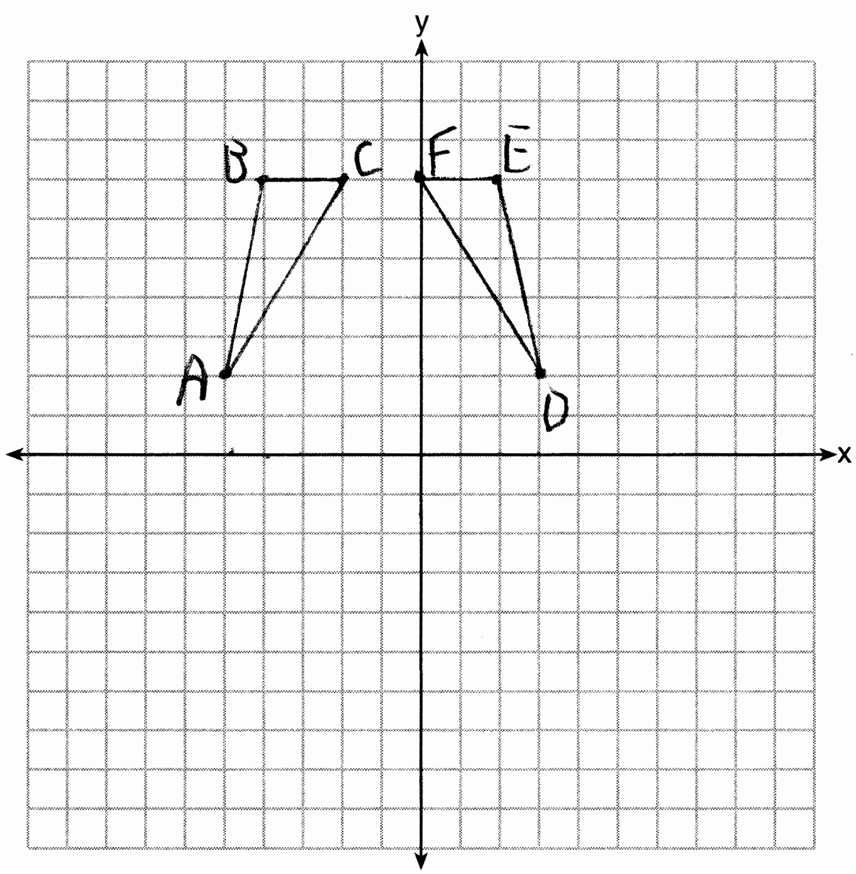
32 Triangle ABC has vertices at $A(-5,2)$, $B(-4,7)$, and $C(-2,7)$, and triangle DEF has vertices at $D(3,2)$, $E(2,7)$, and $F(0,7)$. Graph and label $\triangle ABC$ and $\triangle DEF$ on the set of axes below.

Determine and state the single transformation where $\triangle DEF$ is the image of $\triangle ABC$.

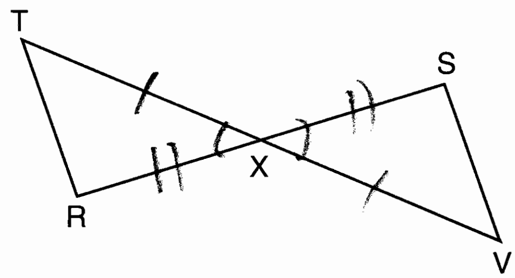
$r_{x=-1}$

Use your transformation to explain why $\triangle ABC \cong \triangle DEF$.

Reflections are rigid motions that preserve distance, so $\triangle ABC \cong \triangle DEF$



33 Given: $\overline{RS}$ and $\overline{TV}$ bisect each other at point X
$\overline{TR}$ and $\overline{SV}$ are drawn



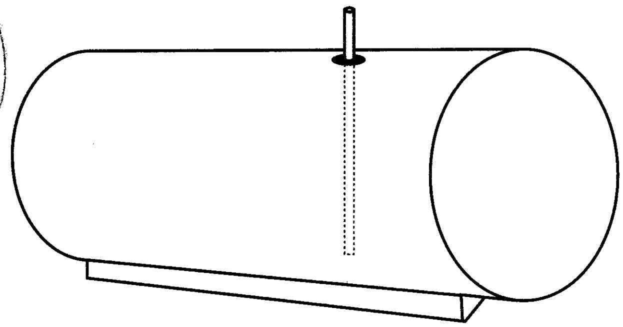
Prove: $\overline{TR} \parallel \overline{SV}$

Statement	Reason
(1) $\overline{RS}$ & $\overline{TV}$ bisect each other at point X; $\overline{TR}$ & $\overline{SV}$ are drawn	(1) Given
(2) $\overline{TX} \cong \overline{XV}$; $\overline{RX} \cong \overline{XS}$	(2) Segment bisectors create 2 congruent segments
(3) $\angle TXR \cong \angle VXS$	(3) Vertical angles are congruent
(4) $\triangle TXR \cong \triangle VXS$	(4) SAS
(5) $\angle T \cong \angle V$	(5) CPCTC
(6) $\overline{TR} \parallel \overline{SV}$	(6) A transversal that creates congruent alternate interior angles cuts parallel lines

34 A gas station has a cylindrical fueling tank that holds the gasoline for its pumps, as modeled below. The tank holds a maximum of 20,000 gallons of gasoline and has a length of 34.5 feet.

$20{,}000g\left(\frac{1\text{ ft}^3}{7.48g}\right)$

2673.8 ft^3



A metal pole is used to measure how much gas is in the tank. To the *nearest tenth of a foot*, how long does the pole need to be in order to reach the bottom of the tank and still extend one foot outside the tank? Justify your answer. [1 ft^3 = 7.48 gallons]

$2673.8 = \pi r^2(34.5)$

$r \approx 4.967$

$d \approx 9.9$

$+1$

10.9

Part IV

Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$.

Prove that $PQRS$ is a rhombus.
[The use of the set of axes on the next page is optional.]

$\overline{PQ}$ $\sqrt{(8-3)^2+(3--2)^2} = \sqrt{50}$

$\overline{QR}$ $\sqrt{(1-8)^2+(4-3)^2} = \sqrt{50}$

$\overline{RS}$ $\sqrt{(-4-1)^2+(-1-4)^2} = \sqrt{50}$

$\overline{PS}$ $\sqrt{(-4-3)^2+(-1--2)^2} = \sqrt{50}$

PQRS is a rhombus because all sides are congruent.

Question 35 is continued on the next page.

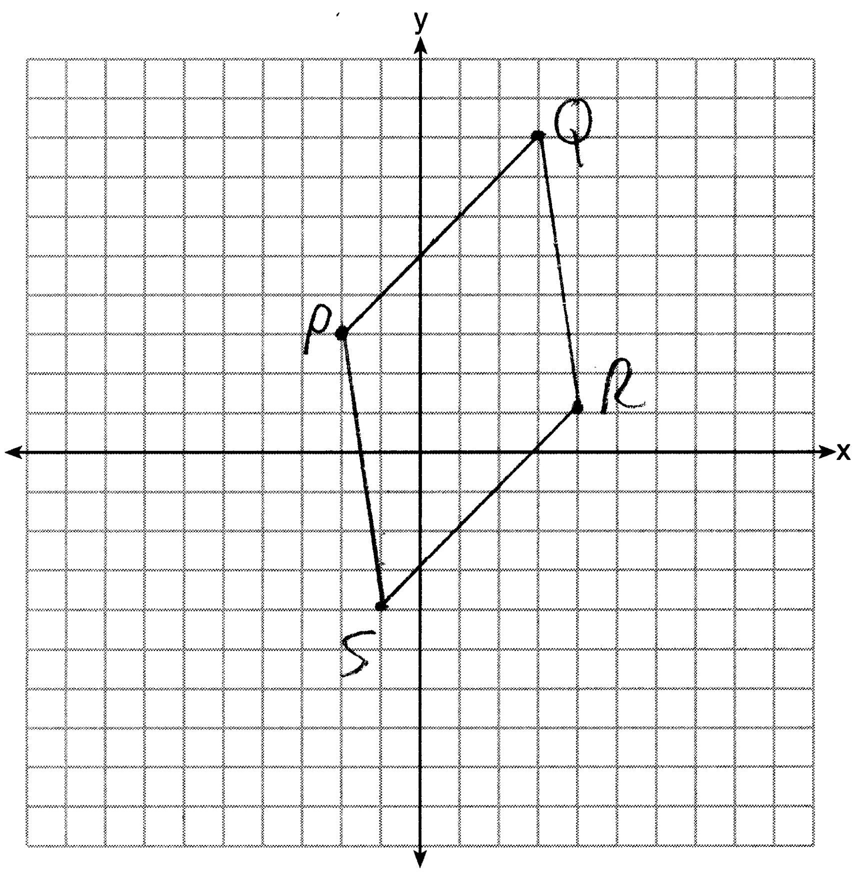
Question 35 continued.

Prove that *PQRS* is *not* a square.
[The use of the set of axes below is optional.]

$m_{\overline{PQ}} = \frac{8-3}{3--2} = \frac{5}{5} = 1$

$m_{\overline{QR}} = \frac{1-8}{4-3} = -7$

Because the slopes of adjacent sides are not opposite reciprocals, they are not $\perp$ and do not form a right angle. Therefore PQRS is not a square.



36 Freda, who is training to use a radar system, detects an airplane flying at a constant speed and heading in a straight line to pass directly over her location. She sees the airplane at an angle of elevation of 15° and notes that it is maintaining a constant altitude of 6250 feet. One minute later, she sees the airplane at an angle of elevation of 52°. How far has the airplane traveled, to the *nearest foot*?

$\tan 15 = \frac{6250}{x}$

$y \approx 23325.3 - 4883 = 18442$

$\tan 52 = \frac{6250}{y}$

$y \approx 4883$

Determine and state the speed of the airplane, to the *nearest mile per hour*.

$\frac{18442 \text{ ft}}{1 \text{ min}} \cdot \frac{1 \text{ mi}}{5280 \text{ ft}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} \approx 210$