The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 22, 2025 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP, Inc.

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

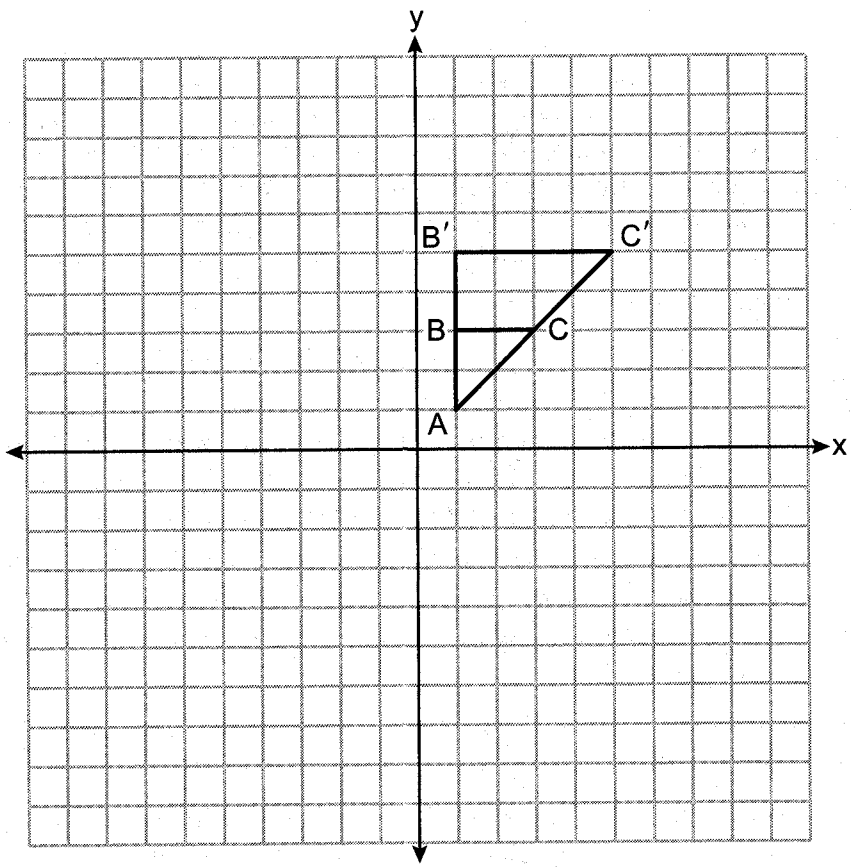
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 On the set of axes below, $\triangle AB'C'$ is the image of $\triangle ABC$.



What is the scale factor and center of dilation that maps $\triangle ABC$ onto $\triangle AB'C'$?

(1) $\frac{1}{2}$ and the origin

(2) 2 and the origin

(3) $\frac{1}{2}$ and vertex A

(4) 2 and vertex A

Use this space for computations.

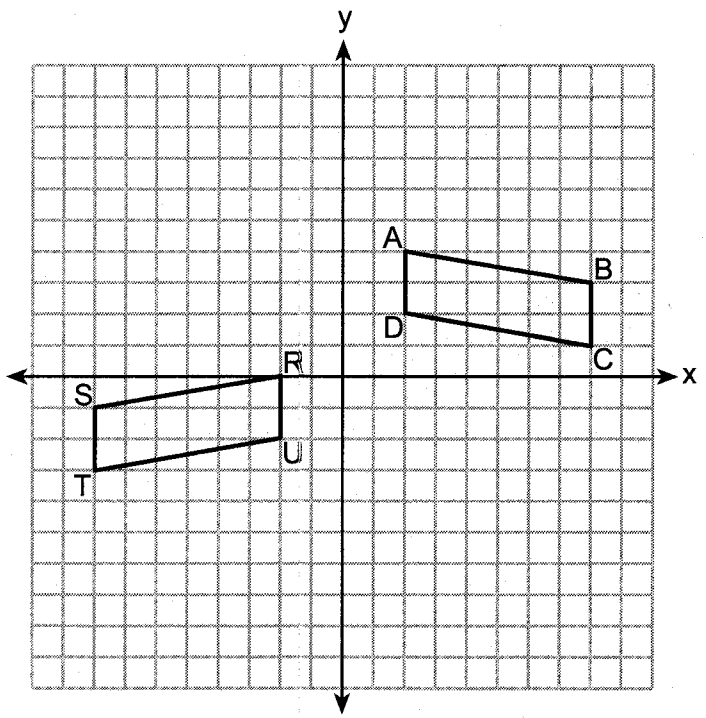
2 Line segment PAQ has endpoints whose coordinates are $P(-2,6)$ and $Q(3,-4)$. What are the coordinates of point A, such that $PA:AQ = 2:3$?

(1) (1,0)
(2) (2,−2)
(3) (−1,4)
(4) (0,2)

$-2 + \frac{2}{5}(3 - -2) = 0$

$6 + \frac{2}{5}(-4 - 6) = 2$

3 On the set of axes below, congruent parallelograms $ABCD$ and $RSTU$ are graphed.



Which sequence of transformations maps $ABCD$ onto $RSTU$?

(1) a reflection over the x-axis followed by a translation ten units to the left and one unit up

(2) a translation four units down followed by a reflection over the y-axis

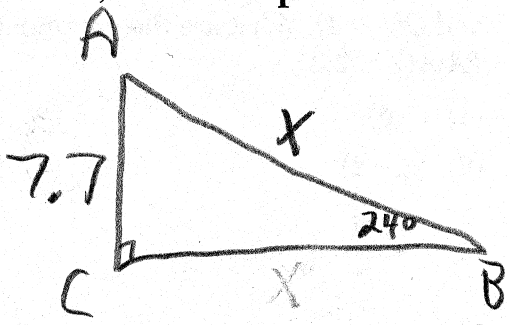
(3) a reflection over the y-axis followed by a translation of two units down

(4) a translation ten units to the left followed by a reflection over the x-axis

Use this space for computations.

4 Triangle ABC has a right angle at C. If $AC = 7.7$ and $m\angle B = 24°$, what is AB, to the *nearest tenth*?

(1) 18.9
(2) 17.3
(3) 8.4
(4) 3.1



$\sin 24 = \frac{7.7}{x}$

$x \approx 18.9$

5 Given $\triangle PQR$ and $\triangle LMN$ with $\overline{PQ} \cong \overline{LM}$, which additional statement is sufficient to always prove $\triangle PQR \cong \triangle LMN$?

(1) $\overline{QR} \cong \overline{MN}$ and $\angle R \cong \angle N$
(2) $\overline{QR} \cong \overline{MN}$ and $\angle Q \cong \angle M$
(3) $\overline{QR} \cong \overline{MN}$ and $\angle P \cong \angle L$
(4) $\overline{QR} \cong \overline{MN}$ and $\angle P \cong \angle M$

6 The equation of a circle is $x^2 + 6y = 4x - y^2 + 12$. What are the coordinates of the center and the length of the radius?

(1) center $(2,-3)$ and radius 5
(2) center $(-2,3)$ and radius 5
(3) center $(2,-3)$ and radius 25
(4) center $(-2,3)$ and radius 25

$x^2 - 4x + 4 + y^2 + 6y + 9 = 12 + 4 + 9$

$(x-2)^2 + (y+3)^2 = 25$

7 A square with a side length of 3 is continuously rotated about one of its sides. The resulting three-dimensional object is a

(1) cube with a volume of 9.
(2) cube with a volume of 27.
(3) cylinder with a volume of 27π.
(4) cylinder with a volume of 54π.

$V = \pi r^2 h$

$= \pi (3)^2 (3)$

$= 27\pi$

Use this space for computations.

8 Line k is represented by the equation $4y + 3 = 7x$. Which equation represents a line that is perpendicular to line k and passes through the point $(-5,2)$?

(1) $y + 2 = \frac{4}{7}(x - 5)$

(2) $y - 2 = \frac{4}{7}(x + 5)$

(3) $y + 2 = -\frac{4}{7}(x - 5)$

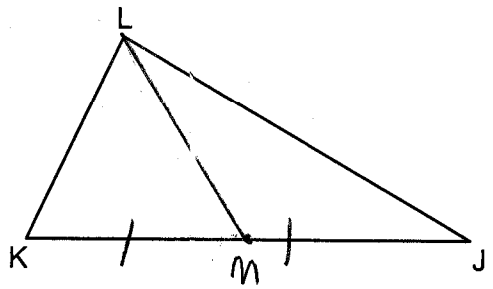
(4) $y - 2 = -\frac{4}{7}(x + 5)$

$\frac{4y}{4} = \frac{7x}{4} - \frac{3}{4}$

$m = \frac{7}{4}$ $m_\perp = -\frac{4}{7}$

$y - 2 = -\frac{4}{7}(x + 5)$

9 Scalene triangle JKL is drawn below.



If median $\overline{LM}$ is drawn to side $\overline{KJ}$, which statement is always true?

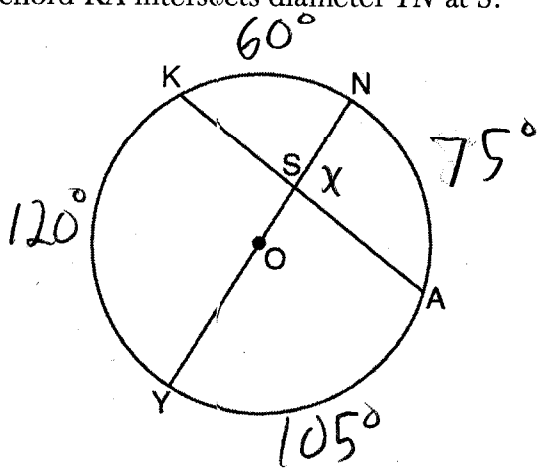
(1) $LM = KM$

(2) $KM = \frac{1}{2}KJ$

(3) $\overline{LM} \perp \overline{KJ}$

(4) $\angle KLM \cong \angle JLM$

10 In circle O, chord $\overline{KA}$ intersects diameter $\overline{YN}$ at S.



$x = \frac{120 + 75}{2}$

If $m\overset{\frown}{YK} = 120°$ and $m\overset{\frown}{YA} = 105°$, what is $m\angle ASN$?

(1) 22.5°

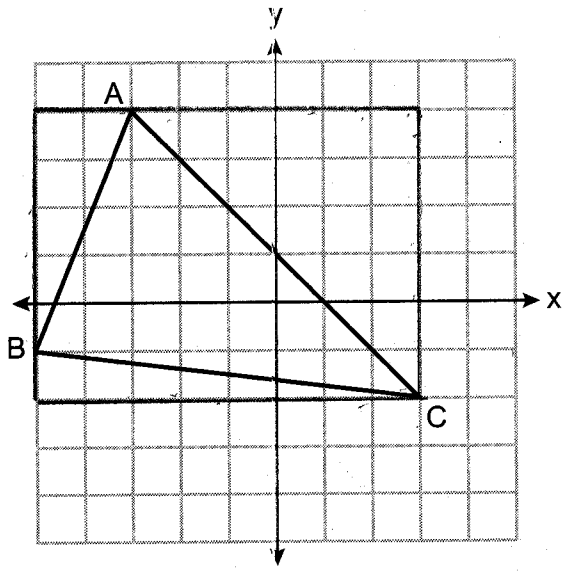
(2) 75°

(3) 97.5°

(4) 120°

Use this space for computations.

11 Triangle ABC is graphed on the set of axes below. The vertices of $\triangle ABC$ have coordinates $A(-3,4)$, $B(-5,-1)$, and $C(3,-2)$.



$8 \cdot 6 - \frac{1}{2}(8 \cdot 1 + 5 \cdot 2 + 6 \cdot 6)$

$48 - \frac{1}{2}(54)$

21

What is the area of $\triangle ABC$?

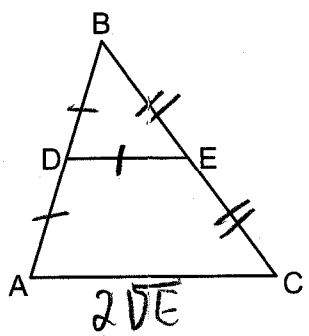
(1) 16

(2) 20

(3) 21

(4) 24

12 In $\triangle ABC$ below, $\overline{DE}$ is a midsegment, and $\overline{BD} \cong \overline{DE}$.



Which statement is always true?

(1) $\triangle ABC$ is isosceles

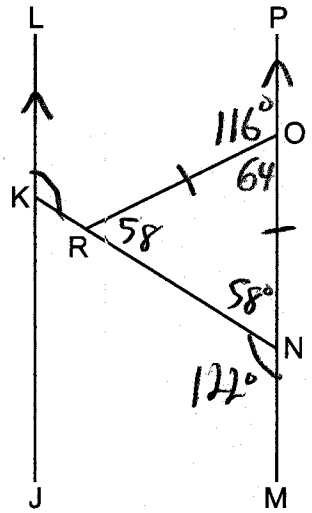
(2) $\triangle ABC$ is scalene

(3) $\overline{BD} \cong \overline{BE}$

(4) $\overline{DA} \cong \overline{EC}$

Use this space for computations.

13 As shown in the diagram below, $\overline{JKL} \parallel \overline{MNOP}$, $\overline{KRN}$, and $\overline{OR} \cong \overline{ON}$.



If m∠*POR* = 116°, what is m∠*LKN*?

(1) 58°
(2) 116°
(3) 122°
(4) 128°

14 The ratio of similarity of square *ABCD* to square *WXYZ* is 2:5. If $AB = x + 3$ and $WX = 3x + 5$, then the perimeter of *ABCD* is

(1) 8
(2) 20
(3) 32
(4) 80

$\frac{2}{5}(x+3) = 3x+5$

$5x+15 = 6x+10$

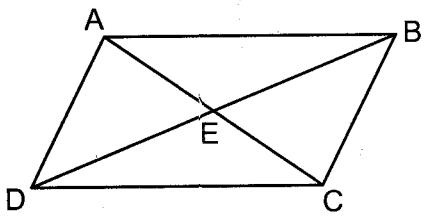
$5 = x$

$AB = 5+3 = 8$

$8 \times 4 = 32$

Use this space for computations.

15 In parallelogram $ABCD$ below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E.



Which transformation would map $\triangle ABC$ onto $\triangle CDA$?

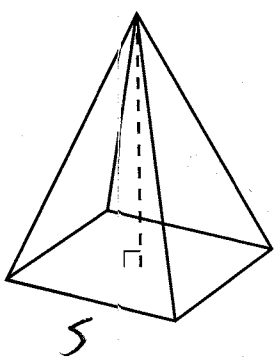
(1) a reflection over $\overline{AC}$

(2) a reflection over $\overline{DB}$

(3) a clockwise rotation of 90° about point E

(4) a clockwise rotation of 180° about point E

16 The square pyramid drawn below has a volume of 175.



If the height of the pyramid is 21, what is the perimeter of the base?

(1) 5

(2) 10

(3) 20

(4) 25

$175 = \frac{1}{3}(s^2)(21)$

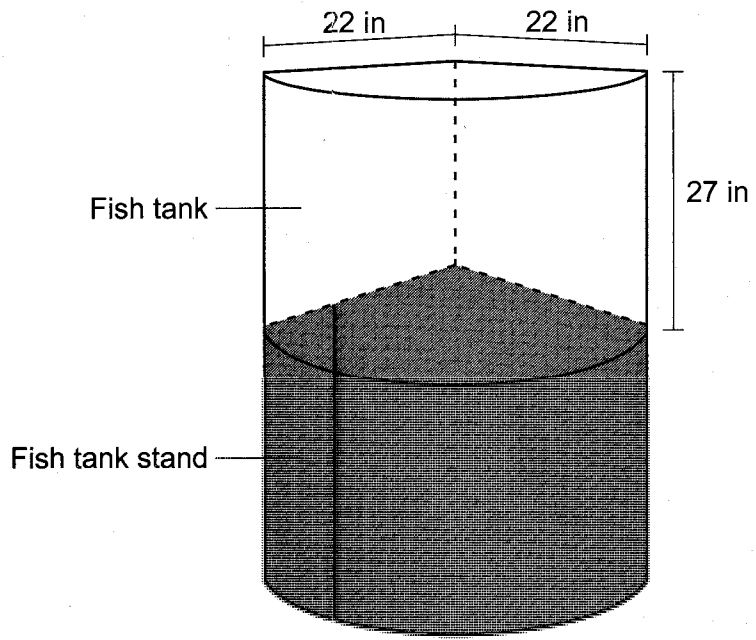
$25 = s^2$

$5 = s$

$20 = p$

Use this space for computations.

17 A glass fish tank is designed to be placed on a stand in the corner of a room with perpendicular walls. The tank can be modeled using part of a cylinder, as shown below. The inner length of the fish tank along the wall is 22 inches, and the height of the tank is 27 inches.



$\frac{1}{4}\pi(22)^2(27) \approx$

231

44

How much water, to the *nearest gallon*, does the fish tank hold? [1 gal = 231 in^3]

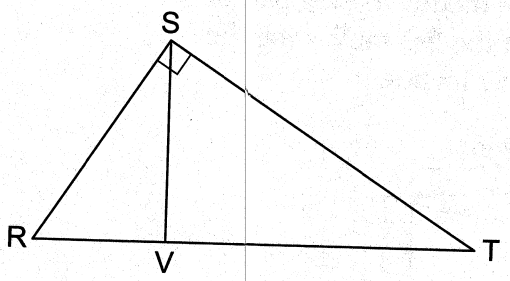
(1) 44
(2) 59
(3) 89
(4) 178

18 Line m, whose equation is $y = -2x + 8$, is dilated by a scale factor of $\frac{1}{2}$ centered at the origin. Which equation represents the image of line m?

(1) $y = -x + 4$
(2) $y = -2x + 4$
(3) $y = -x + 8$
(4) $y = -2x + 8$

19 In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$.

Use this space for computations.



Which statement is always true?

(1) $\frac{RT}{ST} = \frac{ST}{VT}$

(2) $\frac{VR}{VT} = \frac{VT}{VS}$

(3) $\frac{RV}{SV} = \frac{SV}{RT}$

(4) $\frac{TR}{VR} = \frac{VR}{SR}$

20 What is the measure, in radians, of a central angle that intercepts an arc length of 12π cm in a circle with a diameter of 36 cm?

(1) $\frac{\pi}{6}$

(2) $\frac{\pi}{3}$

(3) $\frac{2\pi}{3}$

(4) $\frac{3\pi}{2}$

$\frac{x}{2\pi} \cdot \frac{18}{36\pi} = \frac{12\pi}{18}$

21 A regular nonagon has a center point, P. What degree of rotation about point P will carry the nonagon onto itself?

(1) 60°

(2) 90°

(3) 180°

(4) 200°

Must be a multiple of $\frac{360}{9} = 40$

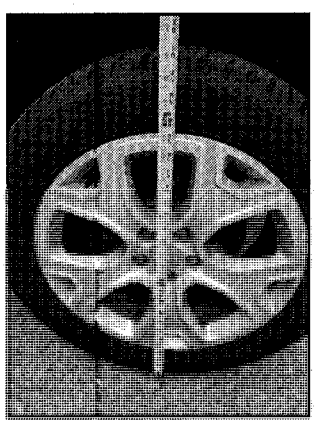
Use this space for computations.

22 If two sides of a triangle have lengths of 2 and 7, the length of the third side could be

(1) 9
(2) 8
(3) 5
(4) 4

$2+7>8$

23 The car tire shown in the photograph below has a diameter of $2\frac{1}{4}$ feet.



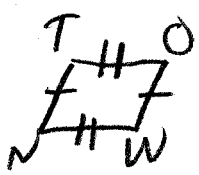
$\frac{5280}{2.25\pi} \approx 747$

Approximately how many rotations will the tire make in one mile?

(1) 373
(2) 747
(3) 1328
(4) 2347

24 In quadrilateral *TOWN*, $\overline{OW} \cong \overline{TN}$ and $\overline{OT} \cong \overline{WN}$. Which additional information is sufficient to prove quadrilateral *TOWN* is a rhombus?

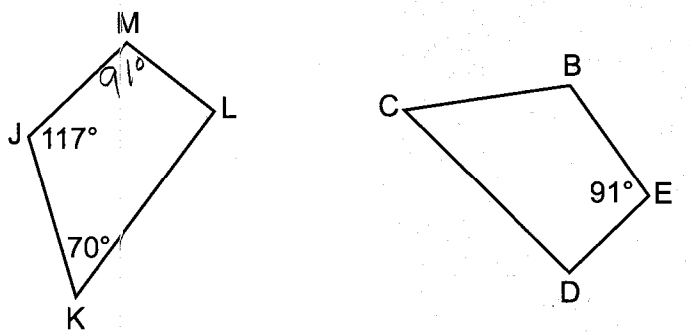
(1) $\overline{ON} \perp \overline{TW}$
(2) $\overline{TO} \perp \overline{OW}$
(3) $\overline{OW} \parallel \overline{TN}$
(4) $\overline{ON}$ and $\overline{TW}$ bisect each other.



Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

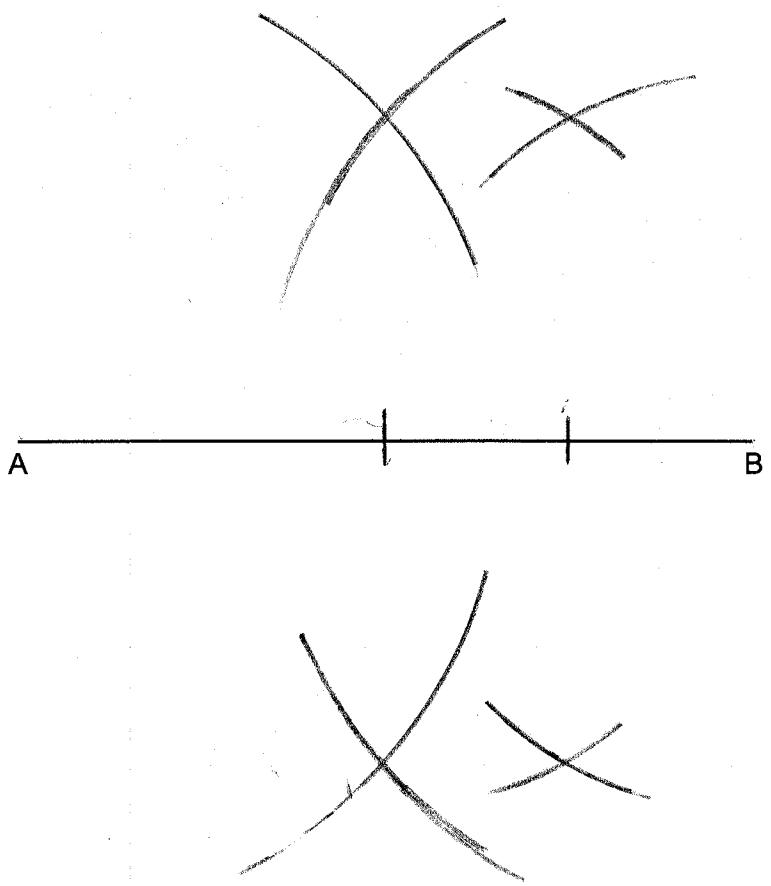
25 In the diagram below, quadrilateral *BCDE* maps onto quadrilateral *JKLM* using a sequence of rigid motions.



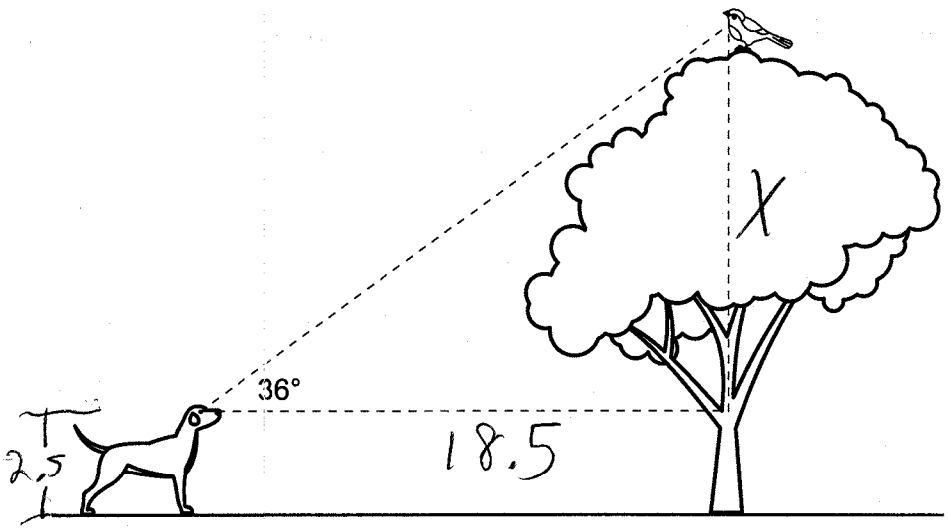
Determine and state the degree measure of angle *D*.

$360-(117+70+91)=82$

26 Given $\overline{AB}$ below, use a compass and a straightedge to construct a segment that is $\frac{1}{4}AB$. [Leave all construction marks.]



27 A dog sees a bird in a tree. The angle of elevation from the dog's eyes to the bird is 36°, as modeled below.



The dog is 18.5 feet away from the base of the tree, and his eyes are 2.5 feet above the ground. Determine and state how high the bird is above the ground, to the *nearest foot*.

$\tan 36° = \frac{X}{18.5}$

$X \approx 13.44$

$+ 2.5$

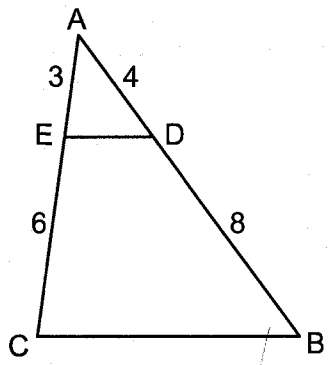
~ 16

28 Pure silver has a density of 10.5 g/cm^3. Samantha has a pure silver charm on her necklace in the shape of a sphere. The radius of the charm is 0.5 cm.

Determine and state the mass of the charm, to the *nearest tenth of a gram.*

$\frac{4}{3}\pi(.5)^3 \cdot 10.5 \approx 5.5$

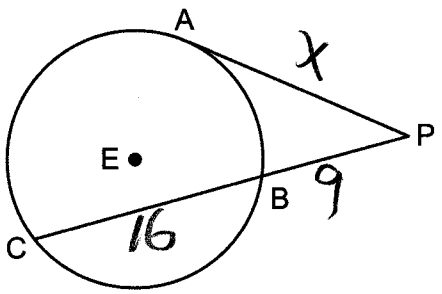
29 In $\triangle ABC$ below, $\overline{DE}$ is drawn such that $AD = 4$, $DB = 8$, $AE = 3$, and $EC = 6$.



Explain why $\triangle ADE \sim \triangle ABC$.

Because $\overline{ED}$ divides $\overline{AC}$ & $\overline{AB}$ proportionally ($\frac{3}{6} = \frac{4}{8}$), $\overline{ED}$ is a side splitter, & $\overline{ED} \parallel \overline{CB}$. Therefore $\angle AED \cong \angle ACB$ & $\angle ADE \cong \angle ABC$ as corresponding angles. $\triangle ADE \sim \triangle ABC$ by AA.

30 In circle E below, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn.



If $PB = 9$ and $BC = 16$, determine and state the length of $\overline{PA}$.

$x^2 = 9 \cdot 25$

$x^2 = 225$

$x = 15$

31 In a right triangle, $\sin(4x + 3)° = \cos(2x - 9)°$. Determine and state the value of x.

$4x+3+2x-9=90$

$6x-6=90$

$6x=96$

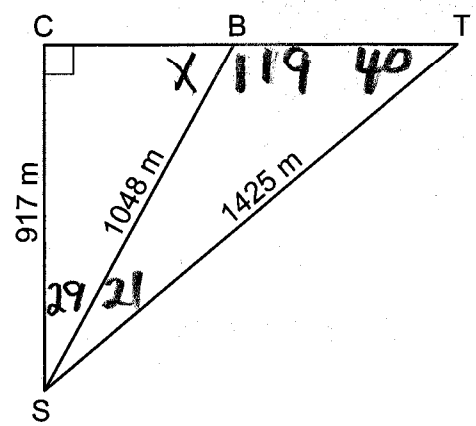
$x=16$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Modeled by right triangles below, a surveyor (S) is taking land measurements using a cabin (C), a boulder (B), and a tree (T) as fixed points of reference. The cabin, boulder, and tree are collinear.

The surveyor is 917 meters from the cabin, 1048 meters from the boulder, and 1425 meters from the tree.



Determine and state, to the *nearest degree*, the measure of $\angle BST$.

$\sin X = \frac{917}{1048}$

$X \approx 61$

$\sin T = \frac{917}{1425}$

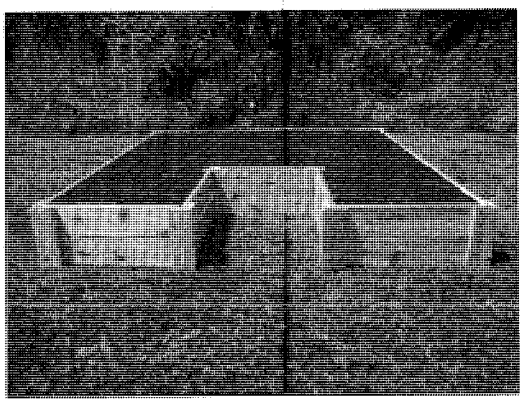
$T \approx 40$

$180 - (119 + 40) = 21$

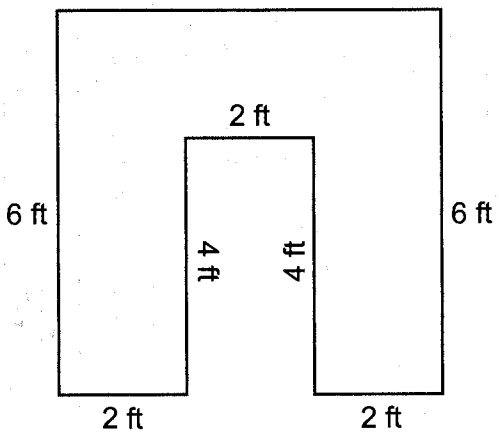
33 A garden bed, pictured below, is a square prism with a rectangular prism taken out. The inside length of the square prism is 6 feet. The rectangular prism taken out has a width of 2 feet and a length of 4 feet.

The diagram below shows the top view of the garden bed with its inside measurements.

Garden Bed



Top View of Garden Bed



The garden bed is filled with topsoil to a uniform height of 1.25 feet.

Determine and state the volume of the topsoil, in cubic feet.

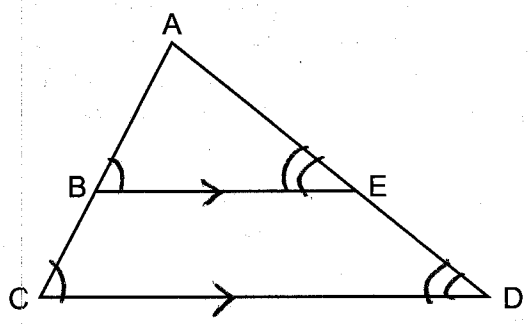
$((6 \cdot 6) - (4 \cdot 2)) \cdot 1.25 = 35$

Each bag of topsoil sells for $3.68 and contains 2 cubic feet of topsoil.

Determine and state the total cost of the bags of topsoil that must be purchased to fill the garden.

$18 \cdot 3.68 = \$66.24$

34 Given: $\triangle ACD$ with $\overline{ABC}$, $\overline{AED}$, and $\overline{BE} \parallel \overline{CD}$



Prove: $AB \cdot AD = AE \cdot AC$

Statement	Reason
① $\triangle ACD$ with $\overline{ABC}$, $\overline{AED}$ & $\overline{BE} \parallel \overline{CD}$	① Given
② $\angle ABE \cong \angle ACD$ $\angle ABB \cong \angle ADC$	② A transversal crossing parallel lines creates congruent corresponding angles.
③ $\triangle ABE \sim \triangle ACD$	③ AA
④ $\frac{\overline{AB}}{\overline{AC}} = \frac{\overline{AE}}{\overline{AD}}$	④ Corresponding sides of similar triangles are proportional
⑤ $AB \cdot AD = AE \cdot AC$	⑤ Product of the means equals the product of the extremes

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Triangle PET has vertices with coordinates $P(-6,4)$, $E(6,8)$, and $T(-4,-2)$.

Prove $\triangle PET$ is a right triangle.

[The use of the set of axes on the next page is optional.]

The slope of $\overline{PE} = \frac{8-4}{6--6} = \frac{4}{12} = \frac{1}{3}$

The slope of $\overline{PT} = \frac{4--2}{-6--4} = \frac{6}{-2} = -3$

Since the slopes are opposite reciprocals, the lines are perpendicular. Perpendicular lines form right angles. $\triangle PET$ is a right triangle because it has a right angle, $\angle P$.

State the coordinates of N, the image of P, after a 180° rotation centered at (1,3).

(8, 2)

Question 35 is continued on the next page.

Question 35 continued

Prove *PENT* is a rectangle.

[The use of the set of axes below is optional.]

The slope of $\overline{TN} = \frac{2--2}{8--4} = \frac{4}{12} = \frac{1}{3}$

The slope of $\overline{EN} = \frac{8-2}{6-8} = \frac{6}{-2} = -3$

Because the slopes of $\overline{PE}$ & $\overline{TN}$ are equal, $\overline{PE} \parallel \overline{TN}$

Because the slopes of $\overline{PT}$ & $\overline{EN}$ are equal, $\overline{PT} \parallel \overline{EN}$

Because opposite sides are ||, PENT is a parallelogram

Because $\angle P$ is a right angle, PENT is a rectangle.