The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, January 25, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. S.601

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

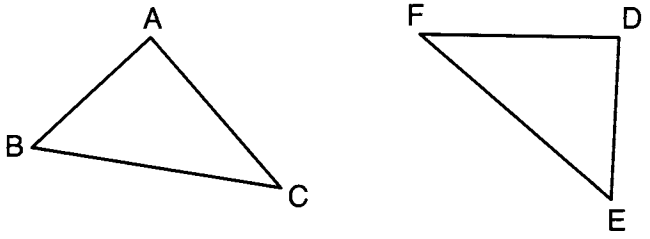
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, a line reflection followed by a rotation maps $\triangle ABC$ onto $\triangle DEF$.



Which statement is always true?

(1) $\overline{BC} \cong \overline{EF}$

(2) $\overline{AC} \cong \overline{DE}$

(3) $\angle A \cong \angle F$

(4) $\angle B \cong \angle D$

2 A circle is continuously rotated about its diameter. Which three-dimensional object will be formed?

(1) cone

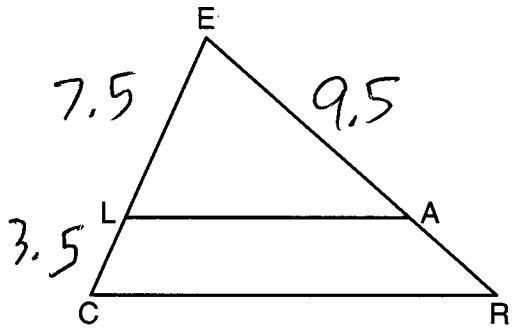
(2) prism

(3) sphere

(4) cylinder

Use this space for computations.

3 In the diagram below of $\triangle CER$, $\overline{LA} \parallel \overline{CR}$.



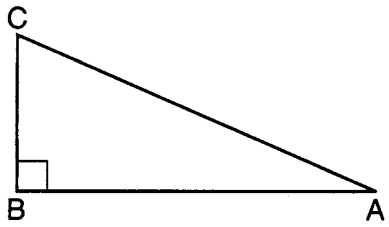
$\frac{7.5}{3.5} = \frac{9.5}{x}$

$x \approx 4.4$

If $CL = 3.5$, $LE = 7.5$, and $EA = 9.5$, what is the length of $\overline{AR}$, to the *nearest tenth*?

(1) 5.5
(2) 4.4
(3) 3.0
(4) 2.8

4 Right triangle ABC is shown below.



Which trigonometric equation is always true for triangle ABC?

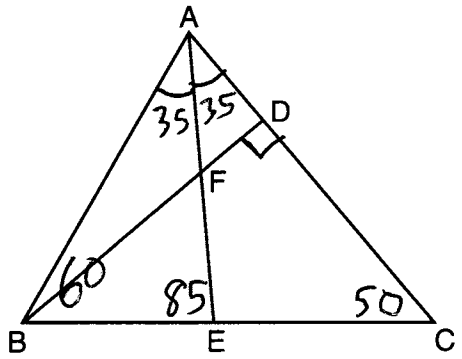
(1) $\sin A = \cos C$
(2) $\cos A = \sin A$
(3) $\cos A = \cos C$
(4) $\tan A = \tan C$

$\sin A = \frac{BC}{AC}$

$\cos C = \frac{BC}{AC}$

Use this space for computations.

5 In the diagram of $\triangle ABC$ below, $\overline{AE}$ bisects angle BAC, and altitude $\overline{BD}$ is drawn.



If $m\angle C = 50°$ and $m\angle ABC = 60°$, $m\angle FEB$ is

(1) 35°
(2) 40°
(3) 55°
(4) 85°

6 A jewelry company makes copper heart pendants. Each heart uses 0.75 in^3 of copper and there is 0.323 pound of copper per cubic inch. If copper costs \$3.68 per pound, what is the total cost for 24 copper hearts?

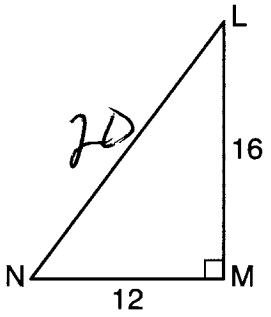
(1) \$5.81
(2) \$21.40
(3) \$66.24
(4) \$205.08

$24 \text{ ht} \cdot \frac{.75 \text{ in}^3}{\text{ht}} \cdot \frac{.323 \text{ lb}}{\text{in}^3} \cdot \frac{\$3.68}{\text{lb}} \approx$

\$21.40

Use this space for computations.

7 In right triangle LMN shown below, $m\angle M = 90°$, $MN = 12$, and $LM = 16$.



3-4-5
12-16-20

The ratio of cos N is

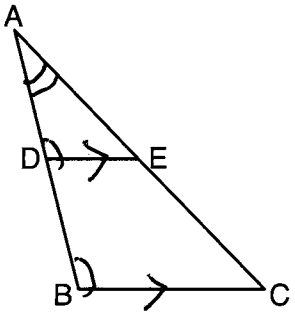
(1) $\frac{12}{20}$

(2) $\frac{16}{20}$

(3) $\frac{12}{16}$

(4) $\frac{16}{12}$

8 In $\triangle ABC$ below, $\overline{DE}$ is drawn such that D and E are on $\overline{AB}$ and $\overline{AC}$, respectively.



If $\overline{DE} \parallel \overline{BC}$, which equation will always be true?

(1) $\frac{AD}{DE} = \frac{DB}{BC}$

(2) $\frac{AD}{DE} = \frac{AB}{BC}$

(3) $\frac{AD}{BC} = \frac{DE}{DB}$

(4) $\frac{AD}{BC} = \frac{DE}{AB}$

9 Which polygon does *not* always have congruent diagonals?

(1) square

(2) rectangle

(3) rhombus

(4) isosceles trapezoid

Use this space for computations.

10 If the circumference of a standard lacrosse ball is 19.9 cm, what is the volume of this ball, to the *nearest cubic centimeter*?

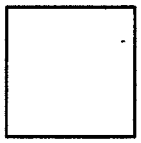
(1) 42
(2) 133
(3) 415
(4) 1065

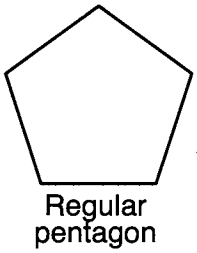
$C = \pi \cdot d$
$\frac{C}{\pi} = d = \frac{19.9}{\pi}$
$V = \frac{4}{3}\pi\left(\frac{19.9}{2\pi}\right)^3$
≈ 133

11 Which polygon always has a minimum rotation of 180° about its center to carry it onto itself?

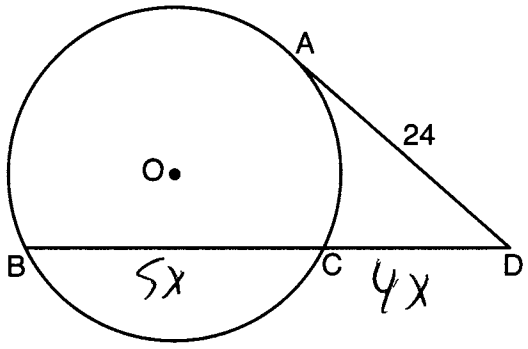

Rectangle
(1) 180°


Isosceles trapezoid
(3) 360°


Square 90°
(2)


72°
Regular pentagon
(4)

12 Circle O is drawn below with secant $\overline{BCD}$. The length of tangent $\overline{AD}$ is 24.



$24^2 = 4x \cdot 9x$
$576 = 36x^2$
$16 = x^2$
$4 = x$

If the ratio of $DC:CB$ is 4:5, what is the length of $\overline{CB}$?

(1) 36
(2) 20
(3) 16
(4) 4

$CB = 5 \cdot 4 = 20$

$-\left(\frac{3}{-5}\right) = \frac{3}{5}$

Use this space for computations.

13 The equation of a line is $3x - 5y = 8$. All lines perpendicular to this line must have a slope of

(1) $\frac{3}{5}$

(2) $\frac{5}{3}$

(3) $-\frac{3}{5}$

(4) $-\frac{5}{3}$

$m_{\perp} = -\frac{5}{3}$

14 What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + y^2 + 2x - 16y + 49 = 0$?

(1) center $(1,-8)$ and radius 4

(2) center $(-1,8)$ and radius 4

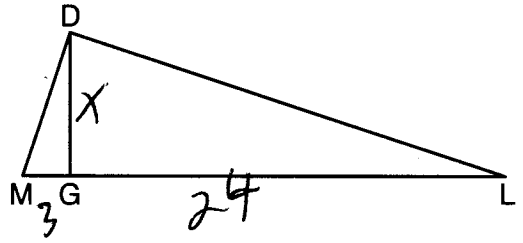
(3) center $(1,-8)$ and radius 16

(4) center $(-1,8)$ and radius 16

$x^2+2x+1+y^2-16y+64 = -49+1+64$

$(x+1)^2+(y-8)^2 = 16$

15 In the diagram below of right triangle MDL, altitude $\overline{DG}$ is drawn to hypotenuse $\overline{ML}$.



If $MG = 3$ and $GL = 24$, what is the length of $\overline{DG}$?

(1) 8

(2) 9

(3) $\sqrt{63}$

(4) $\sqrt{72}$

$x^2 = 3 \cdot 24$

$x^2 = 72$

$x = \sqrt{72}$

16 Segment AB is the perpendicular bisector of $\overline{CD}$ at point M. Which statement is always true?

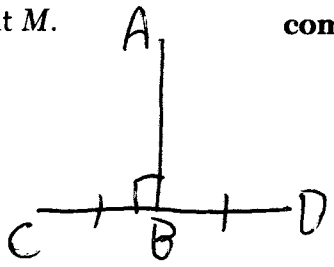
(1) $\overline{CB} \cong \overline{DB}$

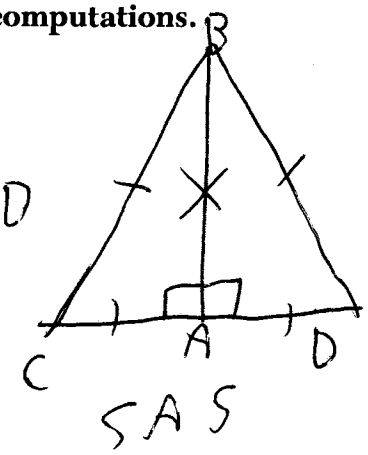
(2) $\overline{CD} \cong \overline{AB}$

(3) $\triangle ACD \sim \triangle BCD$

(4) $\triangle ACM \sim \triangle BCM$

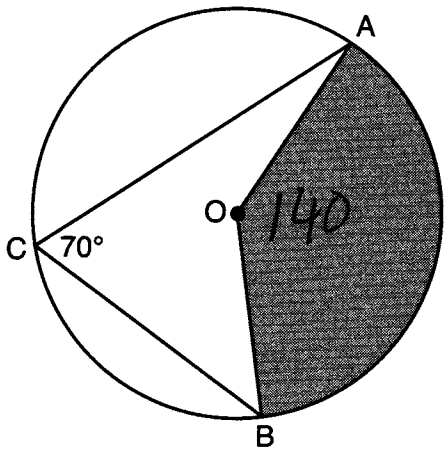
Use this space for computations.





SAS

17 In the diagram below of circle O, $\overline{AC}$ and $\overline{BC}$ are chords, and $m\angle ACB = 70°$.



$\frac{140}{360}(9^2)\pi = 31.5\pi$

If $OA = 9$, the area of the shaded sector AOB is

(1) 3.5π

(2) 7π

(3) 15.75π

(4) 31.5π

Use this space for computations.

18 Quadrilateral $BEST$ has diagonals that intersect at point D. Which statement would *not* be sufficient to prove quadrilateral $BEST$ is a parallelogram?

(1) $\overline{BD} \cong \overline{SD}$ and $\overline{ED} \cong \overline{TD}$ Diagonals bisect each other

(2) $\overline{BE} \cong \overline{ST}$ and $\overline{ES} \cong \overline{TB}$ Opposite sides are $\cong$

(3) $\overline{ES} \cong \overline{TB}$ and $\overline{BE} \parallel \overline{TS}$ Could be isosceles trapezoid

(4) $\overline{ES} \parallel \overline{BT}$ and $\overline{BE} \parallel \overline{TS}$ Opposite sides are $\parallel$

$y = 3x - 6$

19 The equation of line t is $3x - y = 6$. Line m is the image of line t after a dilation with a scale factor of $\frac{1}{2}$ centered at the origin. What is an equation of line m?

(1) $y = \frac{3}{2}x - 3$

(2) $y = \frac{3}{2}x - 6$

(3) $y = 3x + 3$

(4) $y = 3x - 3$

$-6 \cdot \frac{1}{2} = -3$

$r = 8$

20 A cylindrical pool has a diameter of 16 feet and height of 4 feet. The pool is filled to $\frac{1}{2}$ foot below the top. How much water does the pool contain, to the *nearest gallon*? [1 ft^3 = 7.48 gallons]

(1) 704

(2) 804

(3) 5264

(4) 6016

$V = \pi r^2 h$

$= \pi (8)^2 (4 - 0.5)$

$= 224\pi$

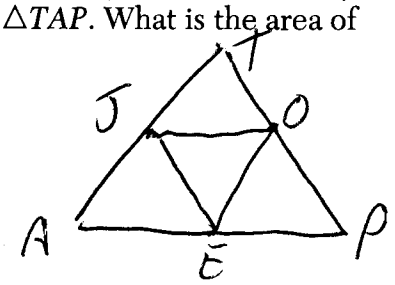
$224\pi \cdot 7.48 \approx 5264$

Use this space for computations.

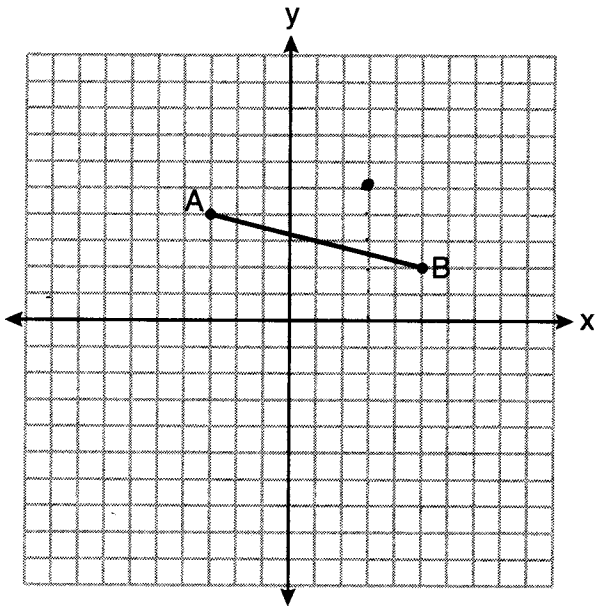
$36 \div 4 = 9$

21 The area of $\triangle TAP$ is 36 cm^2. A second triangle, JOE, is formed by connecting the midpoints of each side of $\triangle TAP$. What is the area of $\triangle JOE$, in square centimeters?

(1) 9
(3) 18
(2) 12
(4) 27



22 On the set of axes below, the endpoints of $\overline{AB}$ have coordinates $A(-3,4)$ and $B(5,2)$.



If $\overline{AB}$ is dilated by a scale factor of 2 centered at (3,5), what are the coordinates of the endpoints of its image, $\overline{A'B'}$?

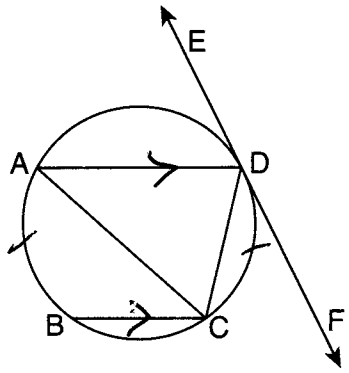
(1) $A'(-7,5)$ and $B'(9,1)$
(3) $A'(-6,8)$ and $B'(10,4)$
(2) $A'(-1,6)$ and $B'(7,4)$
(4) $A'(-9,3)$ and $B'(7,-1)$

$A(-3,4) \rightarrow (-6,-1) \rightarrow (-12,-2) \rightarrow (-9,3)$

$B(5,2) \rightarrow (2,-3) \rightarrow (4,-6) \rightarrow (7,-1)$

Use this space for computations.

23 In the circle below, $\overline{AD}$, $\overline{AC}$, $\overline{BC}$, and $\overline{DC}$ are chords, $\overleftrightarrow{EDF}$ is tangent at point D, and $\overline{AD} \parallel \overline{BC}$.



Since $\overline{AD} \parallel \overline{BC}$, $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

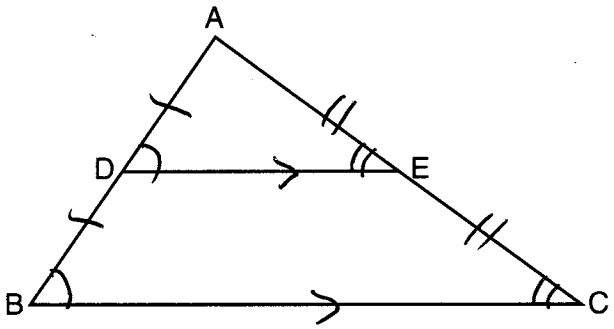
$\angle ACB = \frac{1}{2} m\overset{\frown}{AB}$

$\angle CDF = \frac{1}{2} m\overset{\frown}{CD}$

Which statement is always true?

(1) $\angle ADE \cong \angle CAD$
(2) $\angle CDF \cong \angle ACB$
(3) $\angle BCA \cong \angle DCA$
(4) $\angle ADC \cong \angle ADE$

24 In the diagram below of $\triangle ABC$, D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, respectively, and $\overline{DE}$ is drawn.



I. AA similarity see above
II. SSS similarity the 3 corresponding sides are proportional
III. SAS similarity 2 corresponding sides are proportional & 1 angle is equal

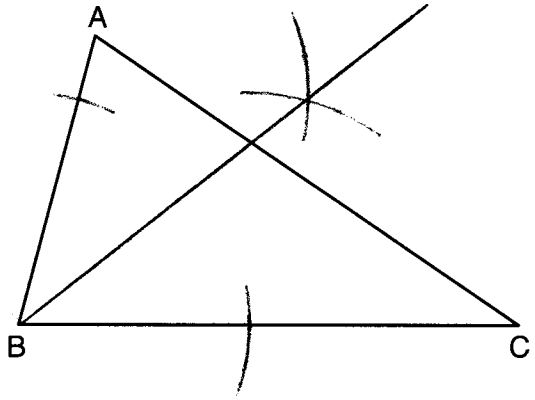
Which methods could be used to prove $\triangle ABC \sim \triangle ADE$?

(1) I and II, only
(2) II and III, only
(3) I and III, only
(4) I, II, and III

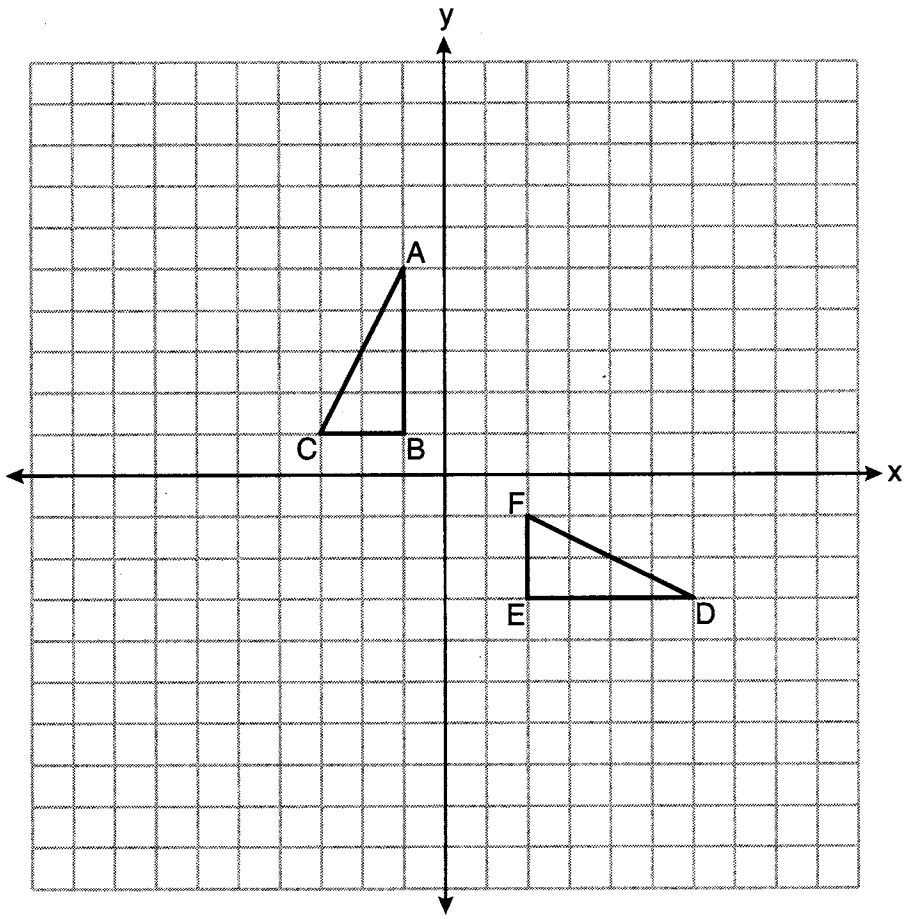
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Using a compass and straightedge, construct the angle bisector of $\angle ABC$.
[Leave all construction marks.]



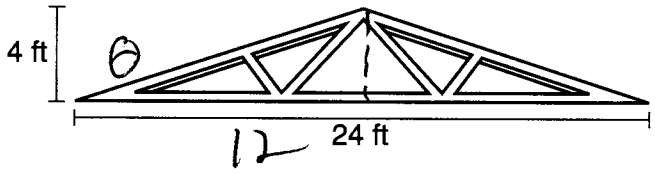
26 On the set of axes below, $\triangle ABC$ and $\triangle DEF$ are graphed.



Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle DEF$.

Rotate 90° clockwise about B & then translate down 4 & right 3.

27 As shown in the diagram below, a symmetrical roof frame rises 4 feet above a house and has a width of 24 feet.



Determine and state, to the *nearest degree*, the angle of elevation of the roof frame.

$\tan \theta = \frac{4}{12}$

$\theta \approx 18$

28 Directed line segment AB has endpoints whose coordinates are $A(-2,5)$ and $B(8,-1)$. Determine and state the coordinates of P, the point which divides the segment in the ratio 3:2.

[The use of the set of axes below is optional.]

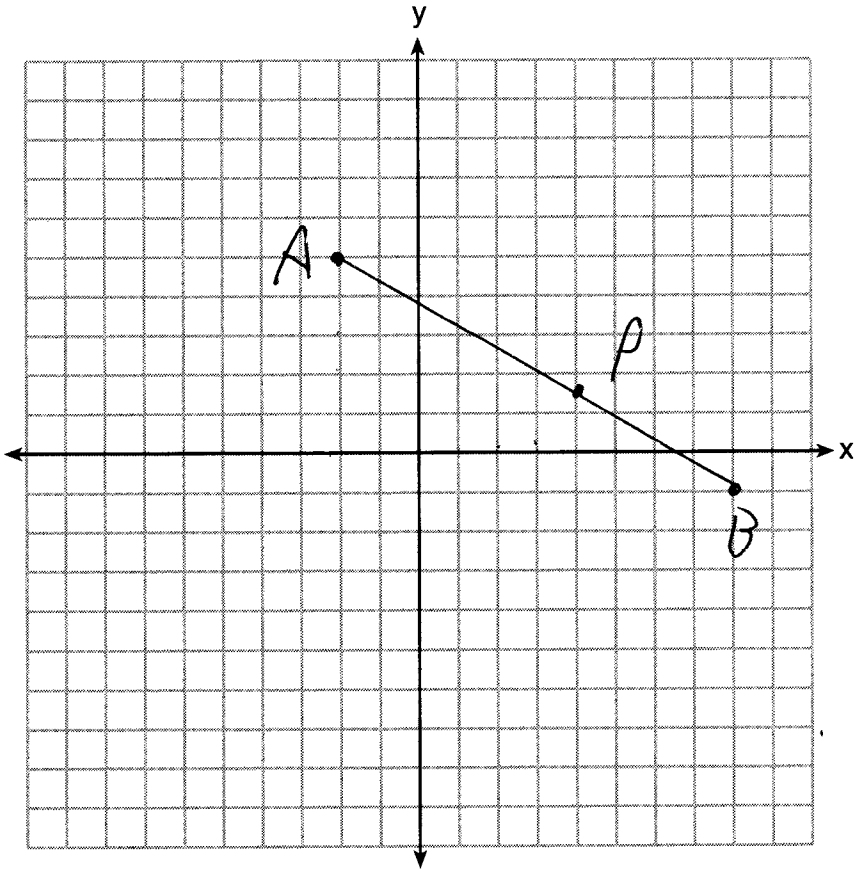
$-2+\frac{3}{5}(8+2) = -2+\frac{3}{5}(10) = -2+6 = 4$

$5+\frac{3}{5}(-1-5) = 5+\frac{3}{5}(-6) = 5-\frac{18}{5}$

$= \frac{25}{5} - \frac{18}{5}$

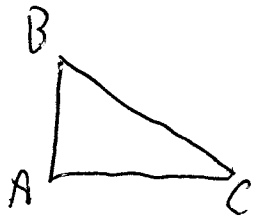
$= \frac{7}{5}$

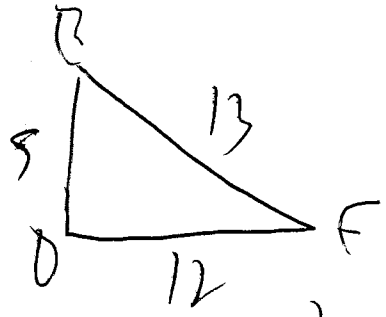
$(4, \frac{7}{5})$



29 In $\triangle ABC$, $AB = 5$, $AC = 12$, and $m\angle A = 90°$. In $\triangle DEF$, $m\angle D = 90°$, $DF = 12$, and $EF = 13$. Brett claims $\triangle ABC \cong \triangle DEF$ and $\triangle ABC \sim \triangle DEF$.

Is Brett correct? Explain why.



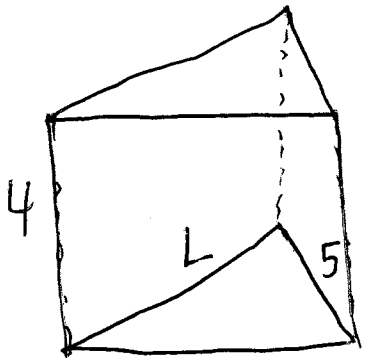


5-12-13 is a Pythagorean Triple

$\triangle ABC \cong \triangle DEF$ } SSS

$\triangle ABC \sim \triangle DBF$ }

30 The volume of a triangular prism is 70 in^3. The base of the prism is a right triangle with one leg whose measure is 5 inches. If the height of the prism is 4 inches, determine and state the length, in inches, of the other leg of the triangle.

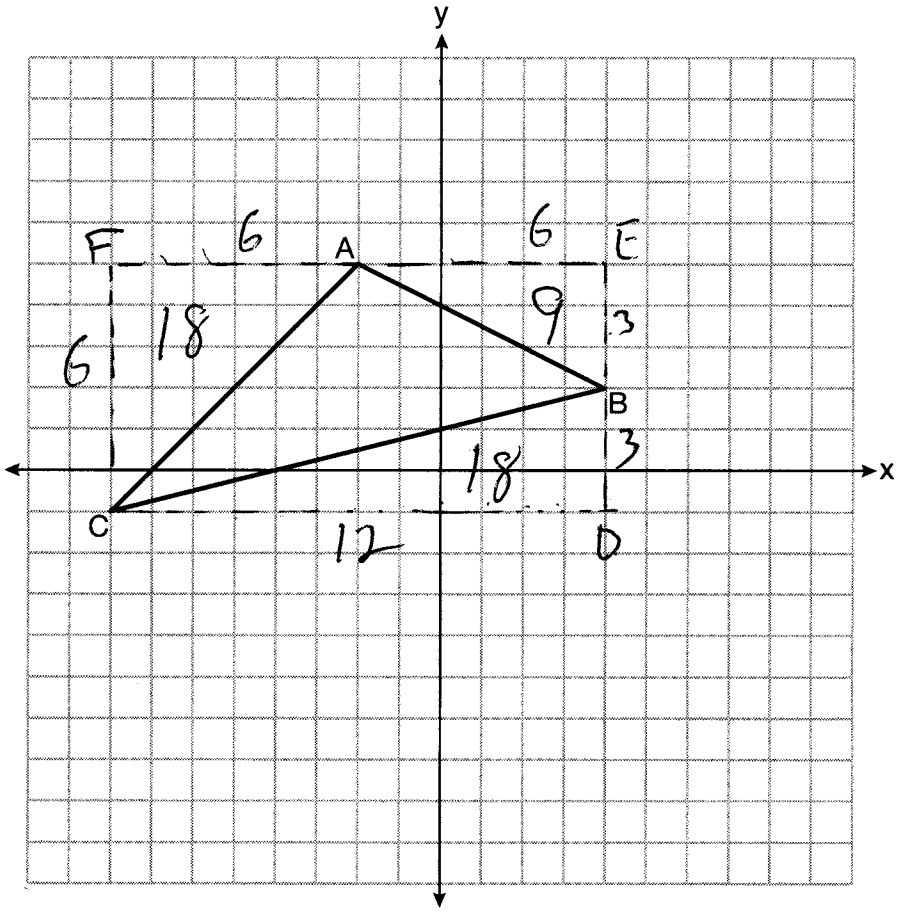


$70 = \frac{1}{2}(5)(L)(4)$

$70 = 10L$

$7 = L$

31 Triangle ABC with coordinates $A(-2,5)$, $B(4,2)$, and $C(-8,-1)$ is graphed on the set of axes below.



Determine and state the area of $\triangle ABC$.

Area of □ CDEF : 6X12 = 72

Area of △ CDB : $\frac{12 \times 3}{2}$ = (18)

Area of △ FAC : $\frac{6 \times 6}{2}$ = (18)

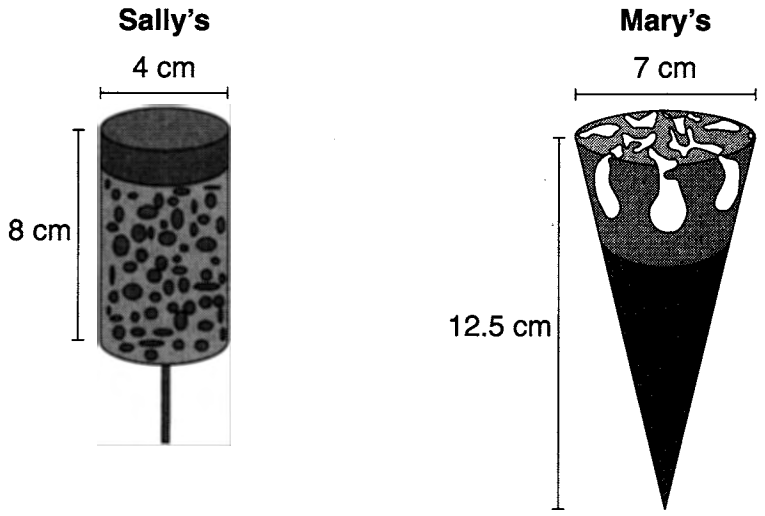
Area of △ AEB : $\frac{6 \times 3}{2}$ = (9)

27

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Sally and Mary both get ice cream from an ice cream truck. Sally's ice cream is served as a cylinder with a diameter of 4 cm and a total height of 8 cm. Mary's ice cream is served as a cone with a diameter of 7 cm and a total height of 12.5 cm. Assume that ice cream fills Sally's cylinder and Mary's cone.



Who was served more ice cream, Sally or Mary? Justify your answer.

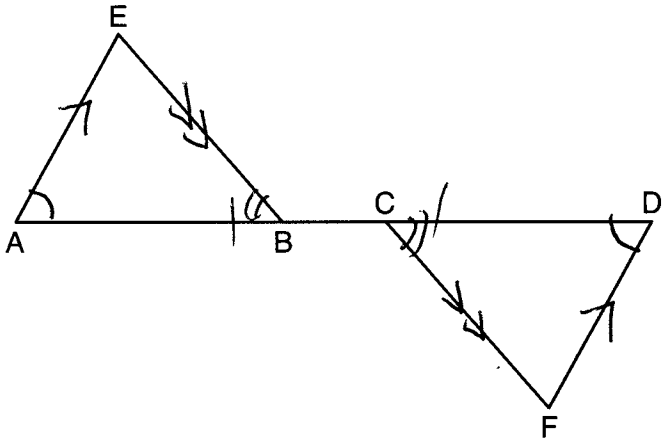
$\pi(2)^2(8) \approx 100.5 \quad \frac{1}{3}(\pi)(3.5)^2(12.5) \approx 160.4$

Mary

Determine and state how much more is served in the larger ice cream than the smaller ice cream, to the *nearest cubic centimeter*.

$160.4 - 100.5 \approx 60$

33 Given: $\triangle AEB$ and $\triangle DFC$, $\overline{ABCD}$, $\overline{AE} \parallel \overline{DF}$, $\overline{EB} \parallel \overline{FC}$, $\overline{AC} \cong \overline{DB}$

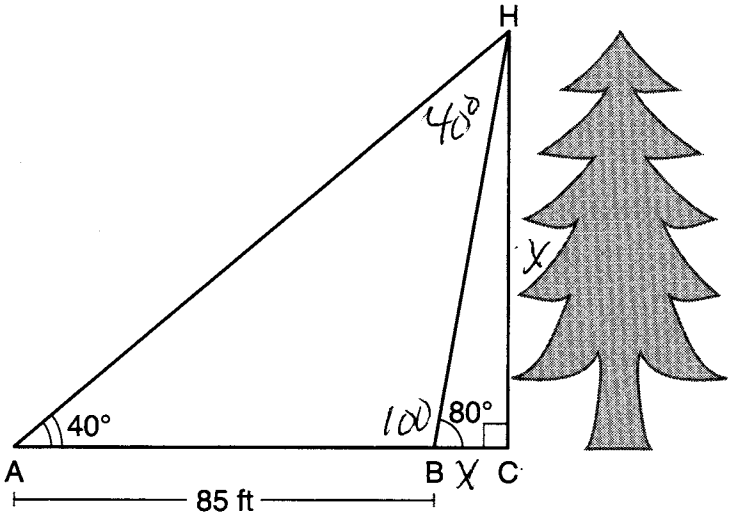


Prove: $\triangle EAB \cong \triangle FDC$

Statement	Reason
① $\triangle AEB$, $\triangle DFC$, $\overline{ABCD}$, $\overline{AE} \parallel \overline{DF}$, $\overline{EB} \parallel \overline{FC}$, $\overline{AC} \cong \overline{DB}$	① Given
② $\angle A \cong \angle D$	② Alternate interior angles Formed by $\parallel$ lines & a transversal are congruent
③ $\angle EBA \cong \angle FCD$	③ Alternate exterior angles formed by $\parallel$ lines & a transversal are $\cong$.
④ $\overline{BC} \cong \overline{BC}$	④ reflexive
⑤ $\overline{AB} \cong \overline{CD}$	⑤ subtraction postulate
⑥ $\triangle EAB \cong \triangle FDC$	⑥ ASA

[20]

34 Barry wants to find the height of a tree that is modeled in the diagram below, where $\angle C$ is a right angle. The angle of elevation from point A on the ground to the top of the tree, H, is 40°. The angle of elevation from point B on the ground to the top of the tree, H, is 80°. The distance between points A and B is 85 feet.



Barry claims that $\triangle ABH$ is isosceles. Explain why Barry is correct.

Since $\angle ABH$ is 100°, $\angle AHB$ is 40°. An isosceles $\triangle$ has two congruent angles.

Determine and state, to the *nearest foot*, the height of the tree.

$\cos 80 = \frac{X}{85}$

$X \approx 14.8$

$\tan 40 = \frac{Y}{99.8}$

$Y \approx 84$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 Given: Triangle DUC with coordinates $D(-3,-1)$, $U(-1,8)$, and $C(8,6)$

Prove: $\triangle DUC$ is a right triangle

[The use of the set of axes on the next page is optional.]

$m_{\overline{DU}} = \frac{8-(-1)}{-1-(-3)} = \frac{9}{2}$

$m_{\overline{UC}} = \frac{8-6}{-1-8} = \frac{2}{-9}$

opposite reciprocals

$\overline{DU} \perp \overline{UC}$

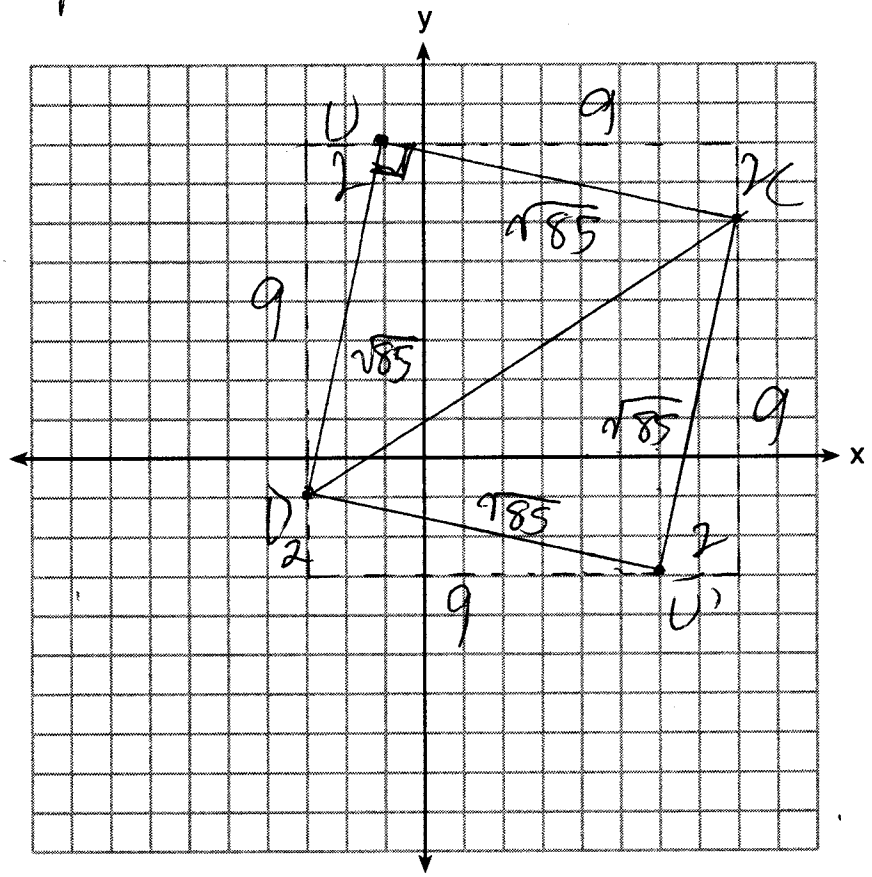
$\triangle DUC$ is a right $\triangle$ because $\angle DUC$ is a right $\angle$.

Question 35 is continued on the next page.

Question 35 continued

Point U is reflected over $\overline{DC}$ to locate its image point, U′, forming quadrilateral $DUCU'$. Prove quadrilateral $DUCU'$ is a square.

Since all four sides are congruent & $\angle DUC$ is a right angle, DUCU' is a square



$\sqrt{9^2+2^2}$

$\sqrt{85}$