The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Tuesday, January 23, 2018 — 9:15 a.m. to 12:15 p.m., only

Student Name: MR SIBOL

School Name: JMAP

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 35 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III,** and **IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...
A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

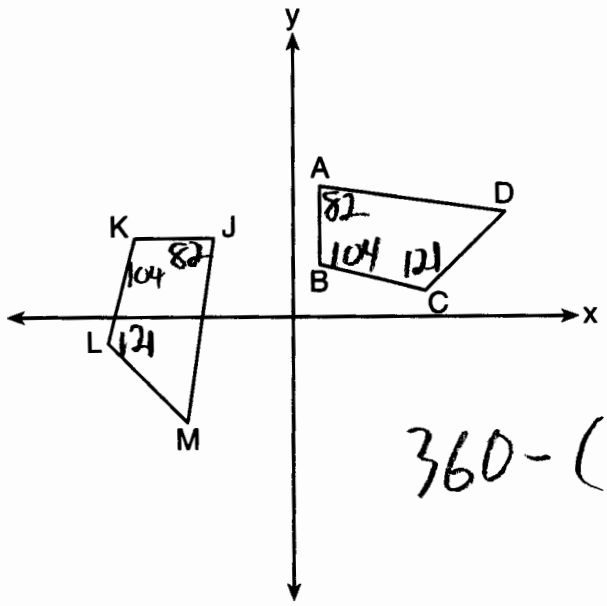
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 In the diagram below, a sequence of rigid motions maps *ABCD* onto *JKLM*.



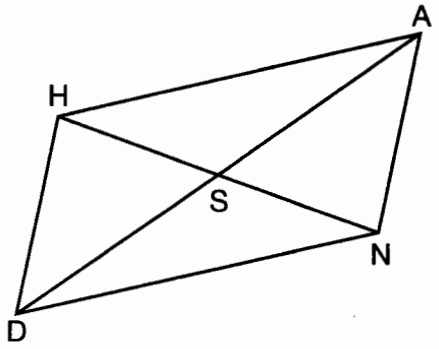
360 - (82 + 104 + 121)

If $m\angle A = 82°$, $m\angle B = 104°$, and $m\angle L = 121°$, the measure of $\angle M$ is

(1) 53°
(2) 82°
(3) 104°
(4) 121°

Use this space for computations.

2 Parallelogram $HAND$ is drawn below with diagonals $\overline{HN}$ and $\overline{AD}$ intersecting at S.



Which statement is always true?

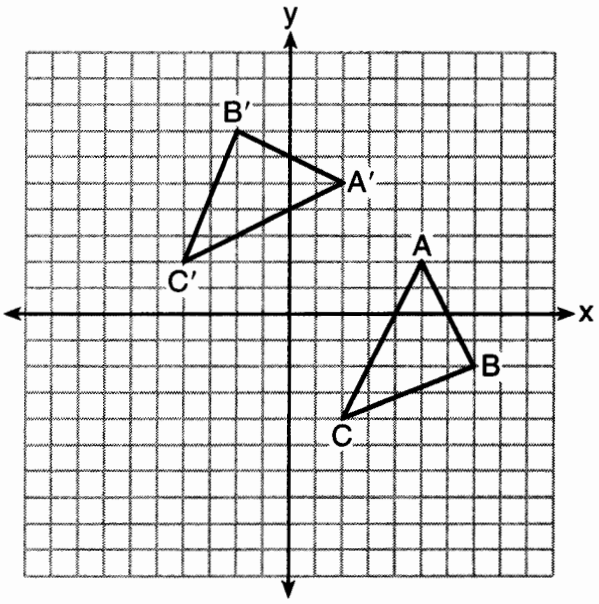
(1) $HN = \frac{1}{2}AD$

(2) $AS = \frac{1}{2}AD$

(3) $\angle AHS \cong \angle ANS$

(4) $\angle HDS \cong \angle NDS$

3 The graph below shows two congruent triangles, ABC and $A'B'C'$.



Which rigid motion would map $\triangle ABC$ onto $\triangle A'B'C'$?

(1) a rotation of 90 degrees counterclockwise about the origin

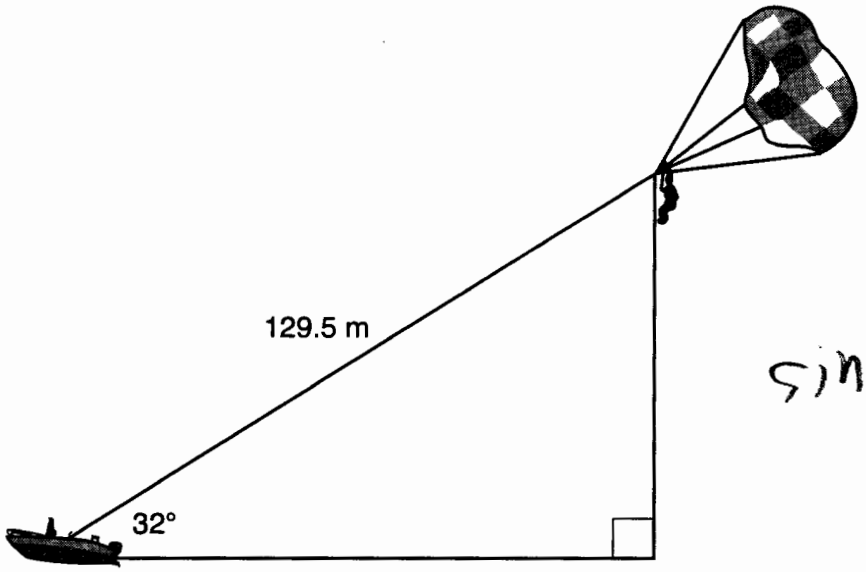
(2) a translation of three units to the left and three units up

(3) a rotation of 180 degrees about the origin

(4) a reflection over the line $y = x$

Use this space for computations.

4 A man was parasailing above a lake at an angle of elevation of 32° from a boat, as modeled in the diagram below.



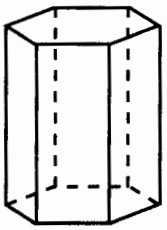
$\sin 32 = \frac{O}{129.5}$

$O \approx 68.6$

If 129.5 meters of cable connected the boat to the parasail, approximately how many meters above the lake was the man?

(1) 68.6
(2) 80.9
(3) 109.8
(4) 244.4

5 A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.

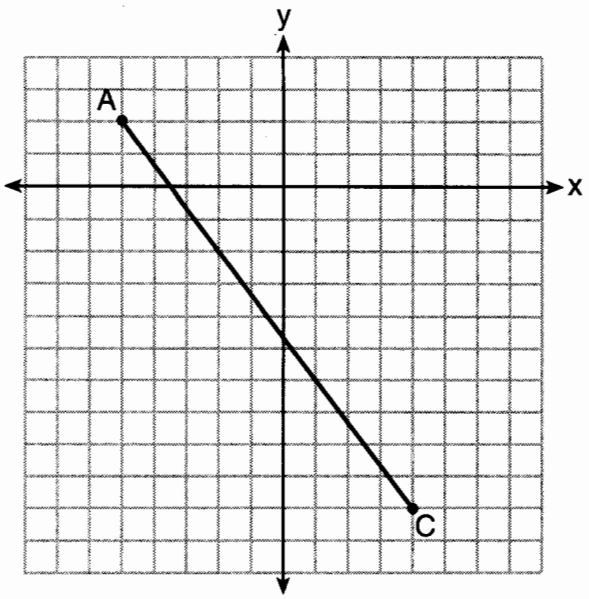


Which figure describes the two-dimensional cross section?

(1) triangle
(2) rectangle
(3) pentagon
(4) hexagon

Use this space for computations.

6 In the diagram below, $\overline{AC}$ has endpoints with coordinates $A(-5,2)$ and $C(4,-10)$.



$x = -5 + \frac{1}{3}(4 - -5)$

$= -5 + 3$

$= -2$

$y = 2 + \frac{1}{3}(-10 - 2)$

$2 - 4 = -2$

If B is a point on $\overline{AC}$ and $AB:BC = 1:2$, what are the coordinates of B?

(1) $(-2,-2)$

(2) $\left(-\frac{1}{2},-4\right)$

(3) $\left(0,-\frac{14}{3}\right)$

(4) $(1,-6)$

7 An ice cream waffle cone can be modeled by a right circular cone with a base diameter of 6.6 centimeters and a volume of 54.45π cubic centimeters. What is the number of centimeters in the height of the waffle cone?

(1) $3\frac{3}{4}$

(2) 5

(3) 15

(4) $24\frac{3}{4}$

$54.45\pi = \frac{1}{3}\pi(3.3)^2 h$

$15 = h$

8 The vertices of $\triangle PQR$ have coordinates $P(2,3)$, $Q(3,8)$, and $R(7,3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?

(1) $(x,y) \rightarrow (2x, 3y)$

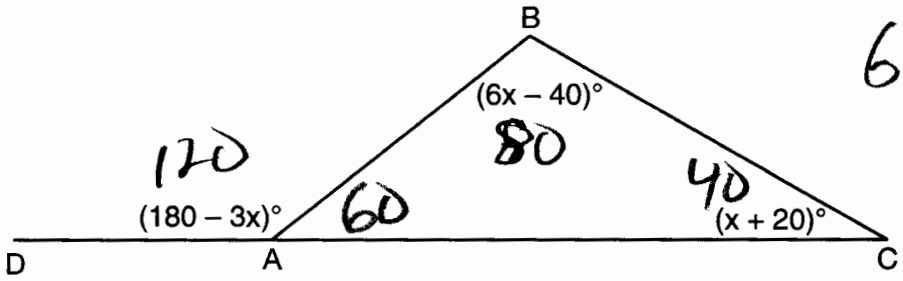
(2) $(x,y) \rightarrow (x + 2, 3y)$

(3) $(x,y) \rightarrow (2x, y + 3)$

(4) $(x,y) \rightarrow (x + 2, y + 3)$

Use this space for computations.

9 In $\triangle ABC$ shown below, side $\overline{AC}$ is extended to point D with $m\angle DAB = (180 - 3x)°$, $m\angle B = (6x - 40)°$, and $m\angle C = (x + 20)°$.



$60x - 40 + x + 20 = 180 - 3x$

$10x = 200$

$x = 20$

What is $m\angle BAC$?

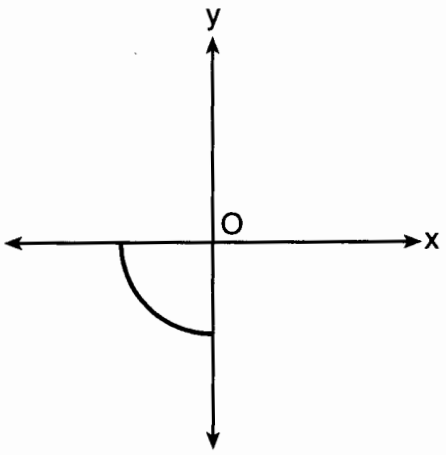
(1) 20°

(2) 40°

(3) 60°

(4) 80°

10 Circle O is centered at the origin. In the diagram below, a quarter of circle O is graphed.



Which three-dimensional figure is generated when the quarter circle is continuously rotated about the y-axis?

(1) cone

(2) sphere

(3) cylinder

(4) hemisphere

Use this space for computations.

11 Rectangle $A'B'C'D'$ is the image of rectangle $ABCD$ after a dilation centered at point A by a scale factor of $\frac{2}{3}$. Which statement is correct?

(1) Rectangle $A'B'C'D'$ has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle $ABCD$.

(2) Rectangle $A'B'C'D'$ has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle $ABCD$.

(3) Rectangle $A'B'C'D'$ has an area that is $\frac{2}{3}$ the area of rectangle $ABCD$.

(4) Rectangle $A'B'C'D'$ has an area that is $\frac{3}{2}$ the area of rectangle $ABCD$.

12 The equation of a circle is $x^2 + y^2 - 6x + 2y = 6$. What are the coordinates of the center and the length of the radius of the circle?

(1) center $(-3,1)$ and radius 4

(2) center $(3,-1)$ and radius 4

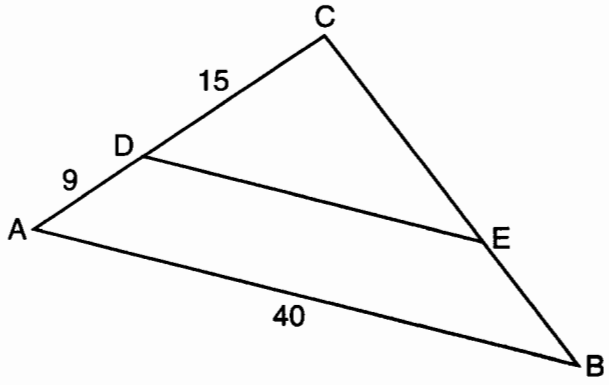
(3) center $(-3,1)$ and radius 16

(4) center $(3,-1)$ and radius 16

$x^2-6x+9+y^2+2y+1=6+9+1$

$(x-3)^2+(y+1)^2=16$

13 In the diagram of $\triangle ABC$ below, $\overline{DE}$ is parallel to $\overline{AB}$, $CD = 15$, $AD = 9$, and $AB = 40$.



$\frac{24}{40} = \frac{15}{x}$

$x = 25$

The length of $\overline{DE}$ is

(1) 15

(2) 24

(3) 25

(4) 30

Use this space for computations.

14 The line whose equation is $3x - 5y = 4$ is dilated by a scale factor of $\frac{5}{3}$ centered at the origin. Which statement is correct?

(1) The image of the line has the same slope as the pre-image but a different y-intercept.

(2) The image of the line has the same y-intercept as the pre-image but a different slope.

(3) The image of the line has the same slope and the same y-intercept as the pre-image.

(4) The image of the line has a different slope and a different y-intercept from the pre-image.

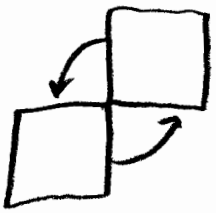
15 Which transformation would *not* carry a square onto itself?

(1) a reflection over one of its diagonals

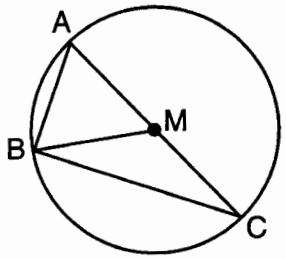
(2) a 90° rotation clockwise about its center

(3) a 180° rotation about one of its vertices

(4) a reflection over the perpendicular bisector of one side



16 In circle M below, diameter $\overline{AC}$, chords $\overline{AB}$ and $\overline{BC}$, and radius $\overline{MB}$ are drawn.



Which statement is *not* true?

(1) $\triangle ABC$ is a right triangle.

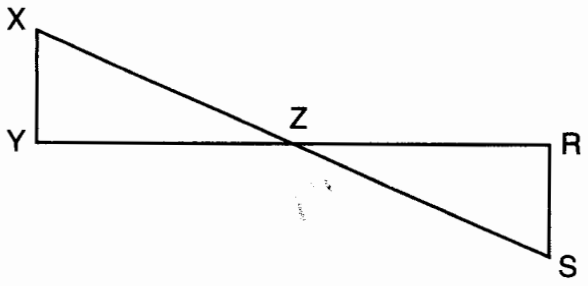
(2) $\triangle ABM$ is isosceles.

(3) $m\overset{\frown}{BC} = m\angle BMC$

(4) $m\overset{\frown}{AB} = \frac{1}{2}m\angle ACB$

Use this space for computations.

17 In the diagram below, $\overline{XS}$ and $\overline{YR}$ intersect at Z. Segments XY and RS are drawn perpendicular to $\overline{YR}$ to form triangles XYZ and SRZ.



Which statement is always true?

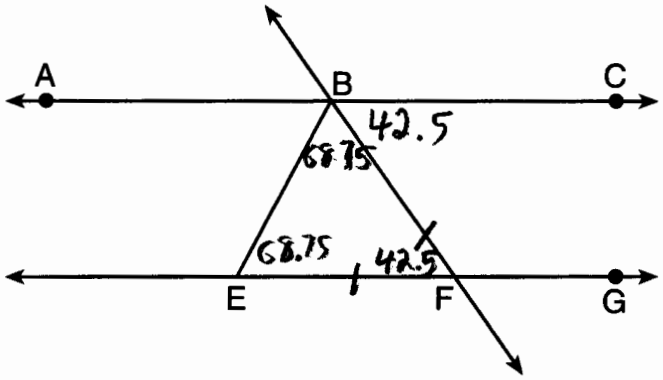
(1) $(XY)(SR) = (XZ)(RZ)$

(2) $\triangle XYZ \cong \triangle SRZ$

(3) $\overline{XS} \cong \overline{YR}$

(4) $\frac{XY}{SR} = \frac{YZ}{RZ}$

18 As shown in the diagram below, $\overleftrightarrow{ABC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$.



If $m\angle CBF = 42.5°$, then $m\angle EBF$ is

(1) 42.5°

(2) 68.75°

(3) 95°

(4) 137.5°

19 A parallelogram must be a rhombus if its diagonals

(1) are congruent

(2) bisect each other

(3) do not bisect its angles

(4) are perpendicular to each other

Use this space for computations.

20 What is an equation of a line which passes through (6,9) and is perpendicular to the line whose equation is $4x - 6y = 15$?

(1) $y - 9 = -\frac{3}{2}(x - 6)$

(2) $y - 9 = \frac{2}{3}(x - 6)$

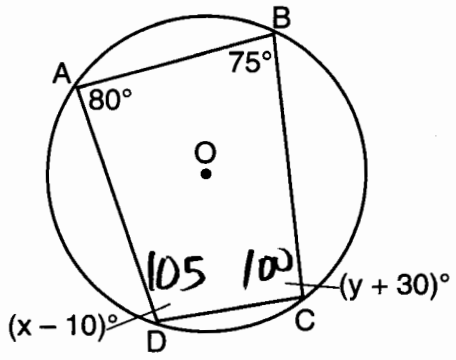
(3) $y + 9 = -\frac{3}{2}(x + 6)$

(4) $y + 9 = \frac{2}{3}(x + 6)$

$m = \frac{-4}{-6} = \frac{2}{3}$

$m_1 = -\frac{3}{2}$

21 Quadrilateral $ABCD$ is inscribed in circle O, as shown below.



Opposite ∠s are supplementary

If $m\angle A = 80°$, $m\angle B = 75°$, $m\angle C = (y + 30)°$, and $m\angle D = (x - 10)°$, which statement is true?

(1) $x = 85$ and $y = 50$

(2) $x = 90$ and $y = 45$

(3) $x = 110$ and $y = 75$

(4) $x = 115$ and $y = 70$

22 A regular pyramid has a square base. The perimeter of the base is 36 inches and the height of the pyramid is 15 inches. What is the volume of the pyramid in cubic inches?

(1) 180

(2) 405

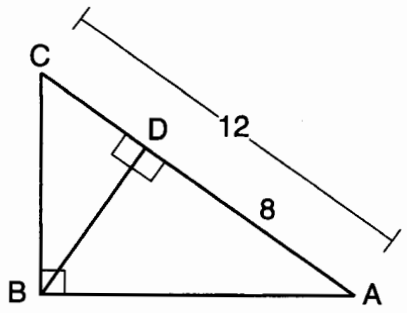
(3) 540

(4) 1215

$V = \frac{1}{3}\left(\frac{36}{4}\right)^2 \cdot 15 = 405$

Use this space for computations.

23 In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn.



$x^2 = 12(12-8)$

$x^2 = 48$

$x = 4\sqrt{3}$

What is the length of $\overline{BC}$?

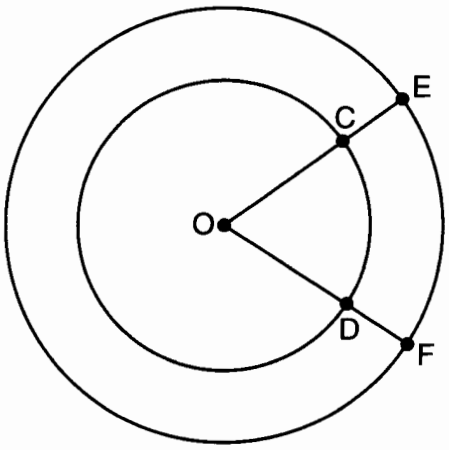
(1) $4\sqrt{2}$

(2) $4\sqrt{3}$

(3) $4\sqrt{5}$

(4) $4\sqrt{6}$

24 In the diagram below, two concentric circles with center O, and radii $\overline{OC}$, $\overline{OD}$, $\overline{OCE}$, and $\overline{ODF}$ are drawn.



If $OC = 4$ and $OE = 6$, which relationship between the length of arc EF and the length of arc CD is always true?

(1) The length of arc EF is 2 units longer than the length of arc CD.

(2) The length of arc EF is 4 units longer than the length of arc CD.

(3) The length of arc EF is 1.5 times the length of arc CD.

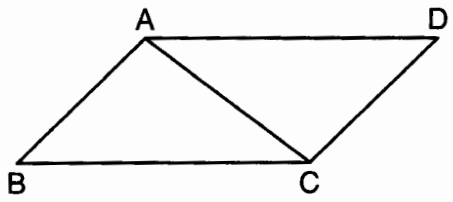
(4) The length of arc EF is 2.0 times the length of arc CD.

$\frac{5\pi}{5} : \frac{6\pi}{4\pi} = 1.5$

Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

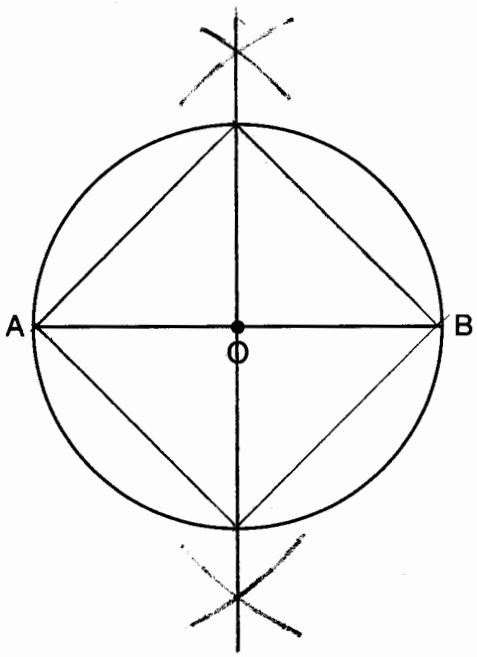
25 Given: Parallelogram $ABCD$ with diagonal $\overline{AC}$ drawn



Prove: $\triangle ABC \cong \triangle CDA$

STATEMENT	REASON
(1) Parallelogram ABCD with diagonal $\overline{AC}$	(1) Given
(2) $\overline{AC} \cong \overline{CA}$	(2) Reflexive Property
(3) $\overline{AD} \cong \overline{CB}$; $\overline{BA} \cong \overline{DC}$	(3) Opposite sides of a parallelogram are $\cong$
(4) $\triangle ABC \cong \triangle CDA$	(4) SSS

26 The diagram below shows circle O with diameter $\overline{AB}$. Using a compass and straightedge, construct a square that is inscribed in circle O. [Leave all construction marks.]

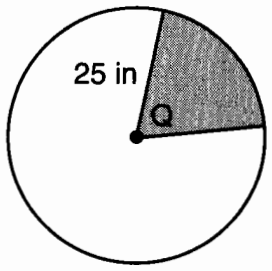


27 Given: Right triangle ABC with right angle at C

If sin A increases, does cos B increase or decrease? Explain why.

Cos B increases because ∠A & ∠B are complementary & sin A = cos B

28 In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.



Determine and state the degree measure of angle Q, the central angle of the shaded sector.

$$\frac{Q}{360}(\pi)(25^2) = \pi(25^2) - 500\pi$$

$$Q = \frac{125\pi(360)}{625\pi}$$

$$Q = 72$$

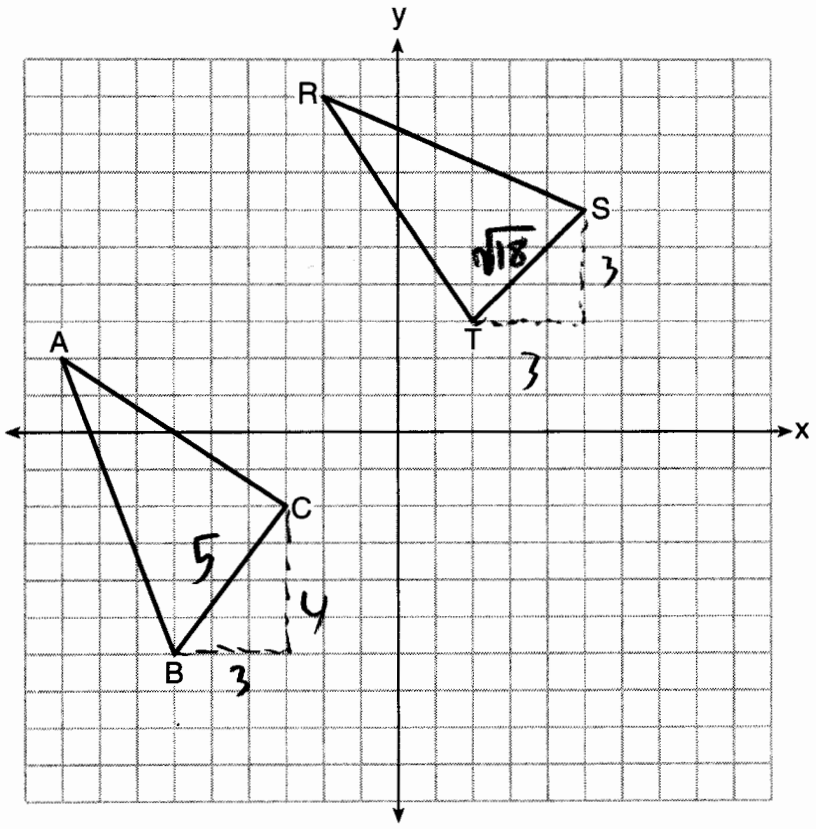
29 A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per kilogram, and has a density of 7.95 g/cm^3.

If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

$500 \times 1015 \text{ cc} \times \frac{\$0.29}{\text{kg}} \times \frac{7.95 \text{ g}}{\text{cc}} \times \frac{1 \text{ kg}}{1000 \text{ g}}$

\$1170

30 In the graph below, $\triangle ABC$ has coordinates $A(-9,2)$, $B(-6,-6)$, and $C(-3,-2)$, and $\triangle RST$ has coordinates $R(-2,9)$, $S(5,6)$, and $T(2,3)$.



Is $\triangle ABC$ congruent to $\triangle RST$? Use the properties of rigid motions to explain your reasoning.

No. Since $\overline{BC}=5$ & $\overline{ST}=\sqrt{18}$ are not congruent, the triangles are not congruent. Since rigid motions preserve distance, there is no rigid motion that maps $\triangle ABC$ onto $\triangle RST$.

31 Bob places an 18-foot ladder 6 feet from the base of his house and leans it up against the side of his house. Find, to the *nearest degree*, the measure of the angle the bottom of the ladder makes with the ground.

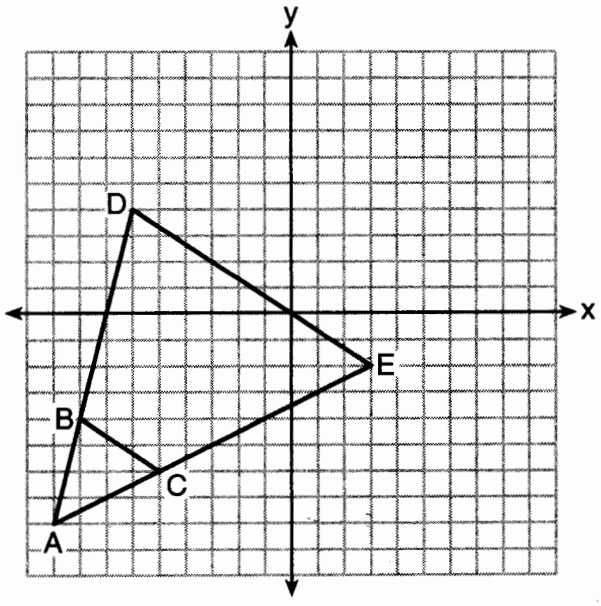
$\cos W = \frac{6}{18}$

$W \approx 71$

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Triangle *ABC* and triangle *ADE* are graphed on the set of axes below.



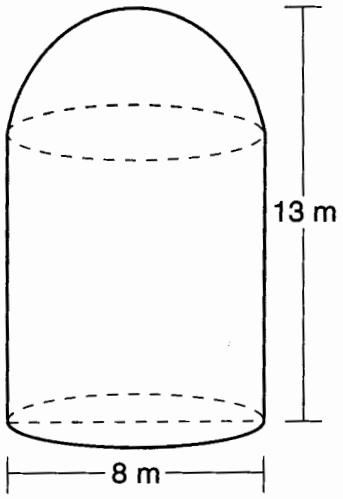
Describe a transformation that maps triangle *ABC* onto triangle *ADE*.

A dilation of 3 centered at A

Explain why this transformation makes triangle *ADE* similar to triangle *ABC*.

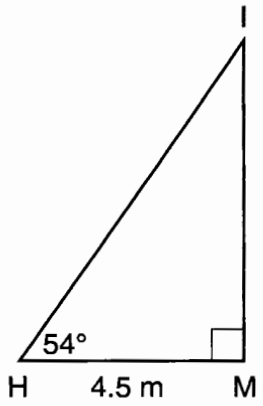
A dilation preserves angle measure, so the triangles are similar.

33 A storage tank is in the shape of a cylinder with a hemisphere on the top. The highest point on the inside of the storage tank is 13 meters above the floor of the storage tank, and the diameter inside the cylinder is 8 meters. Determine and state, to the *nearest cubic meter,* the total volume inside the storage tank.



$V = \pi(4^2)(9) + \left(\frac{1}{2}\right)\left(\frac{4}{3}\right)(\pi)(4^3) \approx 586$

34 As shown in the diagram below, an island (I) is due north of a marina (M). A boat house (H) is 4.5 miles due west of the marina. From the boat house, the island is located at an angle of 54° from the marina.



Determine and state, to the *nearest tenth of a mile,* the distance from the boat house (H) to the island (I).

$\cos 54 = \frac{4.5}{m}$

$m \approx 7.7$

Determine and state, to the *nearest tenth of a mile,* the distance from the island (I) to the marina (M).

$\tan 54 = \frac{h}{4.5}$

$h \approx 6.2$

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [6]

35 In the coordinate plane, the vertices of triangle *PAT* are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes on the next page is optional.]

$\triangle PAT$ is isosceles because sides $\overline{AP}$ & $\overline{AT}$ are congruent $\sqrt{3^2+11^2} = \sqrt{7^2+9^2}$

State the coordinates of R so that quadrilateral *PART* is a parallelogram.

$R(2,9)$

Question 35 is continued on the next page.

Question 35 continued

Prove that quadrilateral *PART* is a parallelogram.

Opposite sides are parallel because they have equal slopes

$m_{\overline{AR}} = m_{\overline{PT}} = \frac{2}{3}$

$m_{\overline{PA}} = m_{\overline{RT}} = -\frac{11}{3}$