The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# GEOMETRY (Common Core)

**Thursday,** January 28, 2016 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

**The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.**

Print your name and the name of your school on the lines above.

A separate answer sheet for Part I has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 36 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

**Notice...**
**A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.**

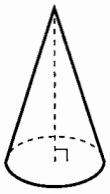
**DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.**

## Part I

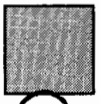
**Answer all 24 questions in this part. Each correct answer will receive 2 credits. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question.** [48]

**Use this space for computations.**

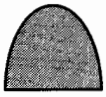
1 William is drawing pictures of cross sections of the right circular cone below.



Which drawing can *not* be a cross section of a cone?

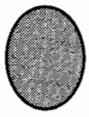


(1)



(3)

(2)



(4)

2 An equation of a line perpendicular to the line represented by the equation $y = -\frac{1}{2}x - 5$ and passing through $(6,-4)$ is

(1) $y = -\frac{1}{2}x + 4$ $m = -\frac{1}{2}$

(2) $y = -\frac{1}{2}x - 1$ $M_{\perp} = 2$

(3) $y = 2x + 14$

(4) $y = 2x - 16$

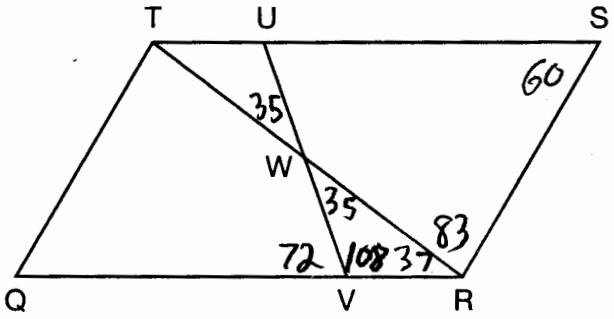
$-4 = 2(6) + b$

$-4 = 12 + b$

$-16 = b$

**Use this space for computations.**

**3** In parallelogram $QRST$ shown below, diagonal $\overline{TR}$ is drawn, $U$ and $V$ are points on $\overline{TS}$ and $\overline{QR}$, respectively, and $\overline{UV}$ intersects $\overline{TR}$ at $W$.




If $m\angle S = 60°$, $m\angle SRT = 83°$, and $m\angle TWU = 35°$, what is $m\angle WVQ$?

(1) 37°
(2) 60°
(3) 72°
(4) 83°

**4** A fish tank in the shape of a rectangular prism has dimensions of 14 inches, 16 inches, and 10 inches. The tank contains 1680 cubic inches of water. What percent of the fish tank is empty?

(1) 10
(2) 25
(3) 50
(4) 75

$14 \times 16 \times 10 = 2240$

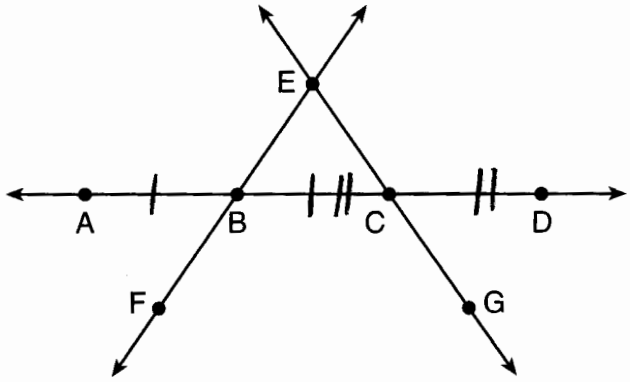
$\frac{2240 - 1680}{2240} = .25$

**5** Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?

(1) $(x,y) \rightarrow (y,x)$
(2) $(x,y) \rightarrow (x,-y)$
(3) $(x,y) \rightarrow (4x,4y)$
(4) $(x,y) \rightarrow (x + 2,y - 5)$

**Use this space for computations.**

**6** In the diagram below, $\overleftrightarrow{FE}$ bisects $\overline{AC}$ at $B$, and $\overleftrightarrow{GE}$ bisects $\overline{BD}$ at $C$.




Which statement is always true?

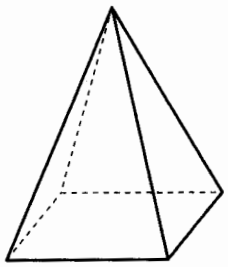
(1) $\overline{AB} \cong \overline{DC}$

(2) $\overline{FB} \cong \overline{EB}$

(3) $\overleftrightarrow{BD}$ bisects $\overline{GE}$ at $C$.

(4) $\overleftrightarrow{AC}$ bisects $\overline{FE}$ at $B$.

**7** As shown in the diagram below, a regular pyramid has a square base whose side measures 6 inches.




If the altitude of the pyramid measures 12 inches, its volume, in cubic inches, is

(1) 72

(2) 144

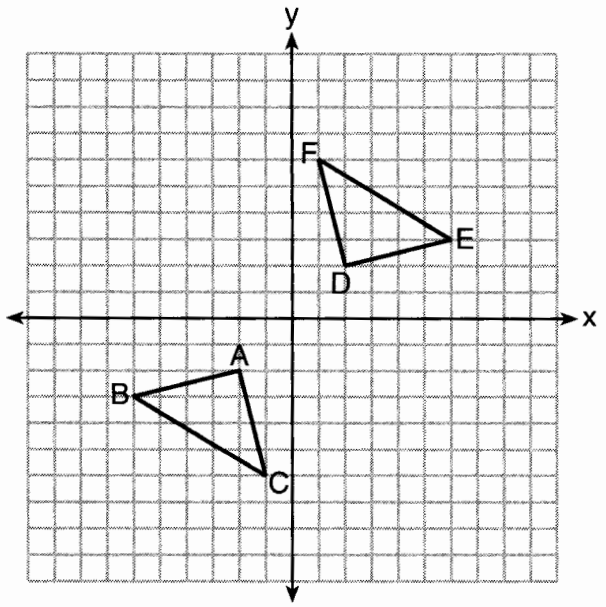
(3) 288

(4) 432

$V = \frac{1}{3} \cdot 6^2 \cdot 12 = 144$

**Use this space for computations.**

8 Triangle $ABC$ and triangle $DEF$ are graphed on the set of axes below.




Which sequence of transformations maps triangle $ABC$ onto triangle $DEF$?

(1) a reflection over the $x$-axis followed by a reflection over the $y$-axis

(2) a 180° rotation about the origin followed by a reflection over the line $y = x$

(3) a 90° clockwise rotation about the origin followed by a reflection over the $y$-axis

(4) a translation 8 units to the right and 1 unit up followed by a 90° counterclockwise rotation about the origin

**Use this space for computations.**

**9** In $\triangle ABC$, the complement of $\angle B$ is $\angle A$. Which statement is always true?

(1) $\tan \angle A = \tan \angle B$

(2) $\sin \angle A = \sin \angle B$

(3) $\cos \angle A = \tan \angle B$

(4) $\sin \angle A = \cos \angle B$

**10** A line that passes through the points whose coordinates are (1,1) and (5,7) is dilated by a scale factor of 3 and centered at the origin. The image of the line

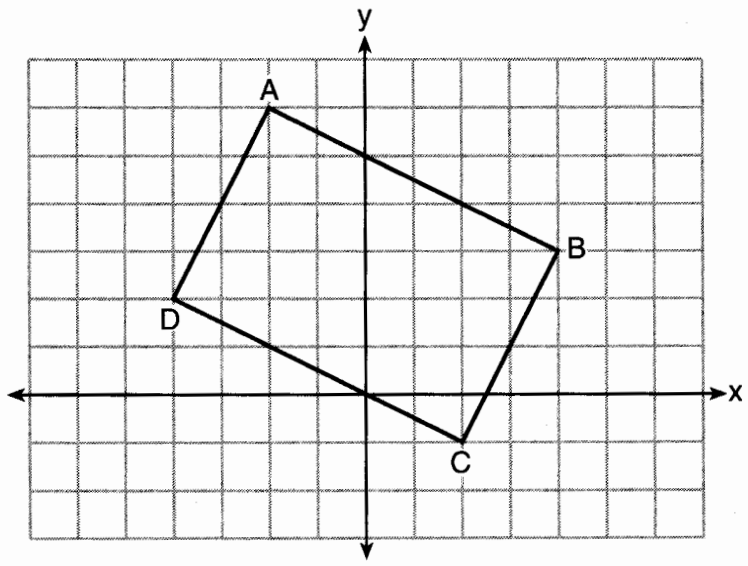
(1) is perpendicular to the original line

(2) is parallel to the original line

(3) passes through the origin

(4) is the original line

**11** Quadrilateral $ABCD$ is graphed on the set of axes below.




When $ABCD$ is rotated 90° in a counterclockwise direction about the origin, its image is quadrilateral $A'B'C'D'$. Is distance preserved under this rotation, and which coordinates are correct for the given vertex?

(1) no and $C'(1,2)$

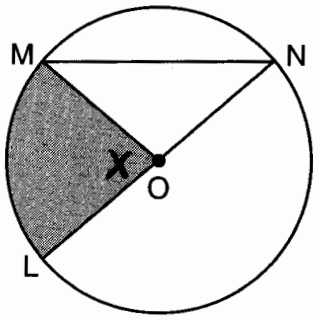
(2) no and $D'(2,4)$

(3) yes and $A'(6,2)$

(4) yes and $B'(-3,4)$

**Use this space for computations.**

**12** In the diagram below of circle $O$, the area of the shaded sector $LOM$ is $2\pi$ cm$^2$.



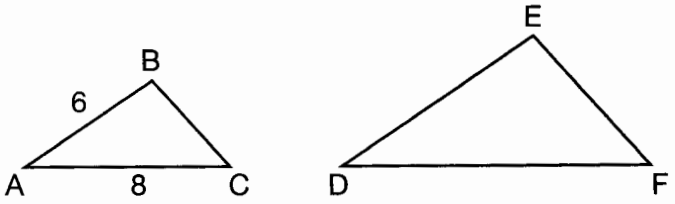

$\frac{X}{360} \cdot 3^2\pi = 2\pi$

$X = 80$

If the length of $\overline{NL}$ is 6 cm, what is $m\angle N$?

(1) 10°
(2) 20°
(3) 40°
(4) 80°

**13** In the diagram below, $\triangle ABC \sim \triangle DEF$.




$\frac{6}{8} = \frac{9}{12}$

If $AB = 6$ and $AC = 8$, which statement will justify similarity by SAS?

(1) $DE = 9$, $DF = 12$, and $\angle A \cong \angle D$
(2) $DE = 8$, $DF = 10$, and $\angle A \cong \angle D$
(3) $DE = 36$, $DF = 64$, and $\angle C \cong \angle F$
(4) $DE = 15$, $DF = 20$, and $\angle C \cong \angle F$

**14** The diameter of a basketball is approximately 9.5 inches and the diameter of a tennis ball is approximately 2.5 inches. The volume of the basketball is about how many times greater than the volume of the tennis ball?

$\frac{\frac{4}{3}\pi\left(\frac{9.5}{2}\right)^3}{\frac{4}{3}\pi\left(\frac{2.5}{2}\right)^3} \approx 55$

(1) 3591
(2) 65
(3) 55
(4) 4

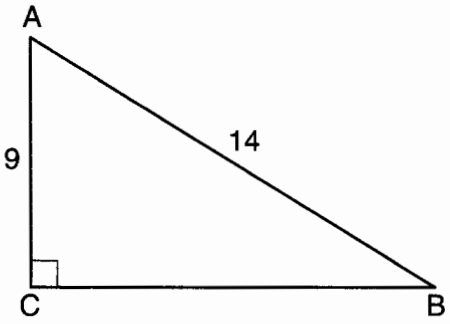
**Use this space for computations.**

**15** The endpoints of one side of a regular pentagon are $(-1,4)$ and $(2,3)$. What is the perimeter of the pentagon?

(1) $\sqrt{10}$
(2) $5\sqrt{10}$
(3) $5\sqrt{2}$
(4) $25\sqrt{2}$

$\sqrt{(-1-2)^2+(4-3)^2} = \sqrt{10}$

**16** In the diagram of right triangle $ABC$ shown below, $AB = 14$ and $AC = 9$.




$\cos A = \frac{9}{14}$

$A \approx 50^\circ$

What is the measure of $\angle A$, to the *nearest degree*?

(1) 33
(2) 40
(3) 50
(4) 57

**17** What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + 6x + y^2 - 4y = 23$?

(1) (3,−2) and 36
(2) (3,−2) and 6
(3) (−3,2) and 36
(4) (−3,2) and 6

$x^2+6x+9+y^2-4x+4 = 23+9+4$

$(x+3)^2+(y-2)^2 = 36$

**18** The coordinates of the vertices of $\triangle RST$ are $R(-2,-3)$, $S(8,2)$, and $T(4,5)$. Which type of triangle is $\triangle RST$?

(1) right
(2) acute
(3) obtuse
(4) equiangular

$m_{\overline{RT}} = \frac{5--3}{4--2} = \frac{8}{6} = \frac{4}{3}$

$m_{\overline{ST}} = \frac{5-2}{4-8} = \frac{3}{-4}$

Slopes are opposite reciprocals, so lines form a right angle

**Use this space for computations.**

**19** Molly wishes to make a lawn ornament in the form of a solid sphere. The clay being used to make the sphere weighs .075 pound per cubic inch. If the sphere's radius is 4 inches, what is the weight of the sphere, to the *nearest pound*?

(1) 34
(2) 20
(3) 15
(4) 4

$\frac{4}{3}\pi \times 4^3 \times 0.075 \approx 20$

**20** The ratio of similarity of $\triangle BOY$ to $\triangle GRL$ is 1:2. If $BO = x + 3$ and $GR = 3x - 1$, then the length of $\overline{GR}$ is

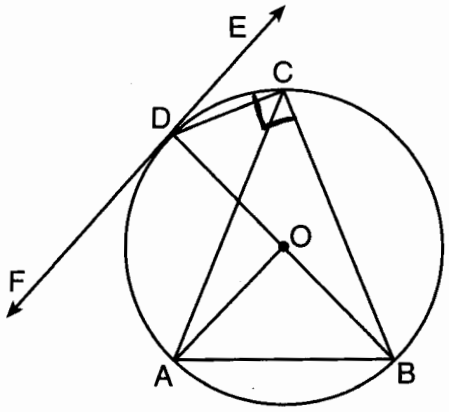
(1) 5
(2) 7
(3) 10
(4) 20

$\frac{1}{2} = \frac{x+3}{3x-1}$

$3x - 1 = 2x + 6$

$x = 7 \quad GR = 3(7) - 1 = 20$

**21** In the diagram below, $\overline{DC}$, $\overline{AC}$, $\overline{DOB}$, $\overline{CB}$, and $\overline{AB}$ are chords of circle $O$, $\overleftrightarrow{FDE}$ is tangent at point $D$, and radius $\overline{AO}$ is drawn. Sam decides to apply this theorem to the diagram: "An angle inscribed in a semi-circle is a right angle."

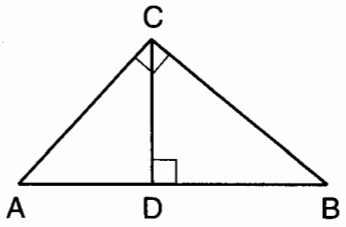



Which angle is Sam referring to?

(1) $\angle AOB$
(2) $\angle BAC$
(3) $\angle DCB$
(4) $\angle FDB$

**Use this space for computations.**

**22** In the diagram below, $\overline{CD}$ is the altitude drawn to the hypotenuse $\overline{AB}$ of right triangle $ABC$.




Which lengths would *not* produce an altitude that measures $6\sqrt{2}$?

(1) $AD = 2$ and $DB = 36$
(2) $AD = 3$ and $AB = 24$
(3) $AD = 6$ and $DB = 12$
(4) $AD = 8$ and $AB = 17$

① $\sqrt{2 \cdot 36} = \sqrt{72} = 6\sqrt{2}$ ③ $\sqrt{6 \cdot 12} = \sqrt{72} = 6\sqrt{2}$

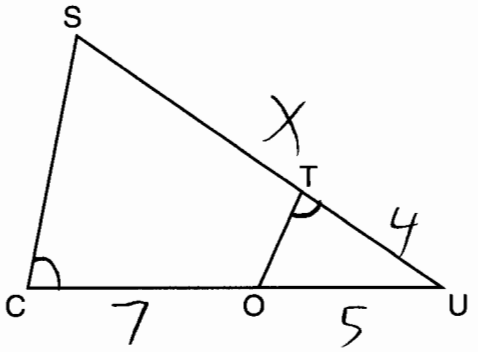
② $\sqrt{3 \cdot 21} = \sqrt{63} = 3\sqrt{7}$ ④ $\sqrt{8 \cdot 9} = \sqrt{72} = 6\sqrt{2}$

**23** A designer needs to create perfectly circular necklaces. The necklaces each need to have a radius of 10 cm. What is the largest number of necklaces that can be made from 1000 cm of wire?

(1) 15
(2) 16
(3) 31
(4) 32

$\frac{1000}{20\pi} \approx 15.9$

**24** In $\triangle SCU$ shown below, points $T$ and $O$ are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment $OT$ is drawn so that $\angle C \cong \angle OTU$.




$\triangle SCU \sim \triangle OTU$

$\frac{12}{4} = \frac{x}{5}$

$4x = 60$

$x = 15$

$15 - 4 = 11$

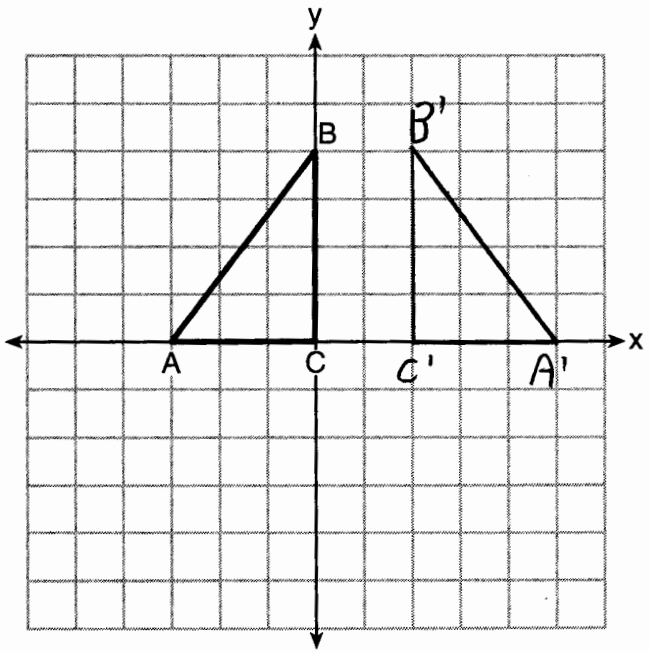
If $TU = 4$, $OU = 5$, and $OC = 7$, what is the length of $\overline{ST}$?

(1) 5.6
(2) 8.75
(3) 11
(4) 15

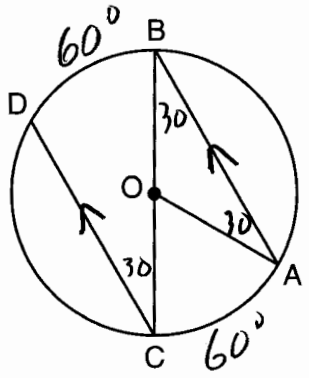
## Part II

**Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [14]

**25** Triangle $ABC$ is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$.



**26** In the diagram below of circle $O$ with diameter $\overline{BC}$ and radius $\overline{OA}$, chord $\overline{DC}$ is parallel to chord $\overline{BA}$.




If $m\angle BCD = 30°$, determine and state $m\angle AOB$.

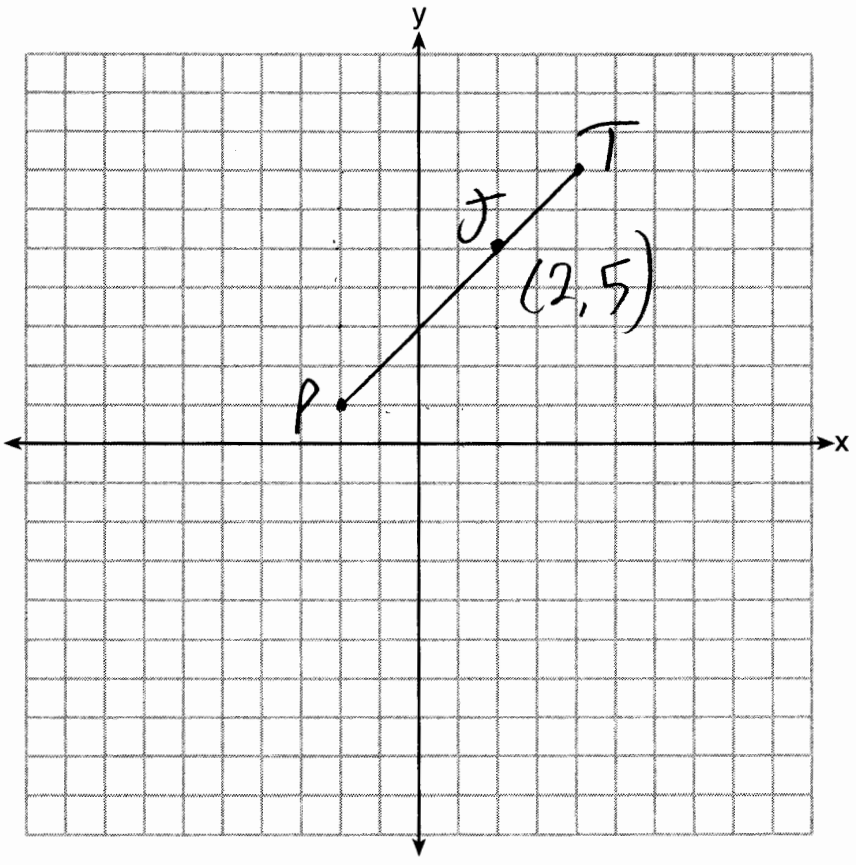
$180 - 2(30) = 120°$

**27** Directed line segment $PT$ has endpoints whose coordinates are $P(-2,1)$ and $T(4,7)$. Determine the coordinates of point $J$ that divides the segment in the ratio 2 to 1.

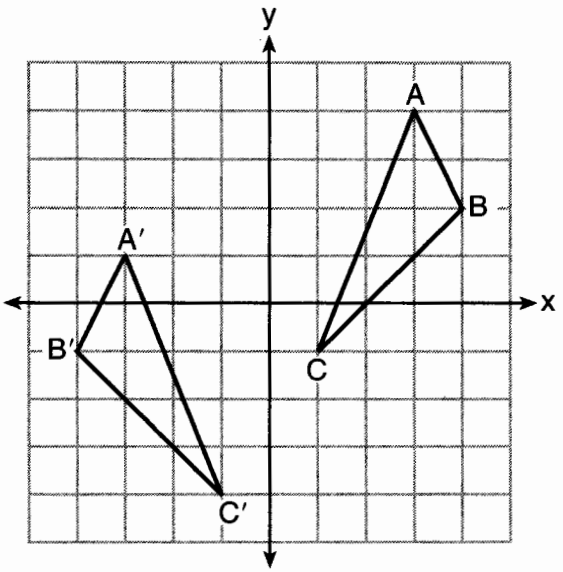
[The use of the set of axes below is optional.]

X $\frac{2}{3} \cdot 4 - -2 = 4$

Y $\frac{2}{3} \cdot 7 - 1 = 4$



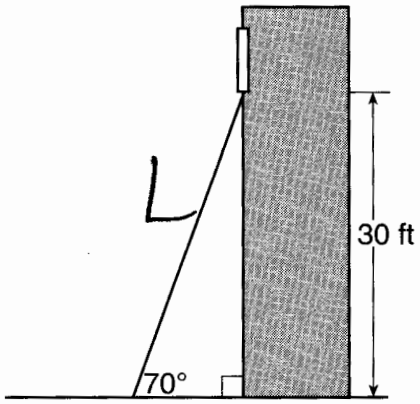
**28** As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.




Is $\triangle A'B'C'$ congruent to $\triangle ABC$? Use the properties of rigid motion to explain your answer.

Yes. The sequence of transformations consists of a reflection & a translation, which are isometrics which preserve distance & congruency.

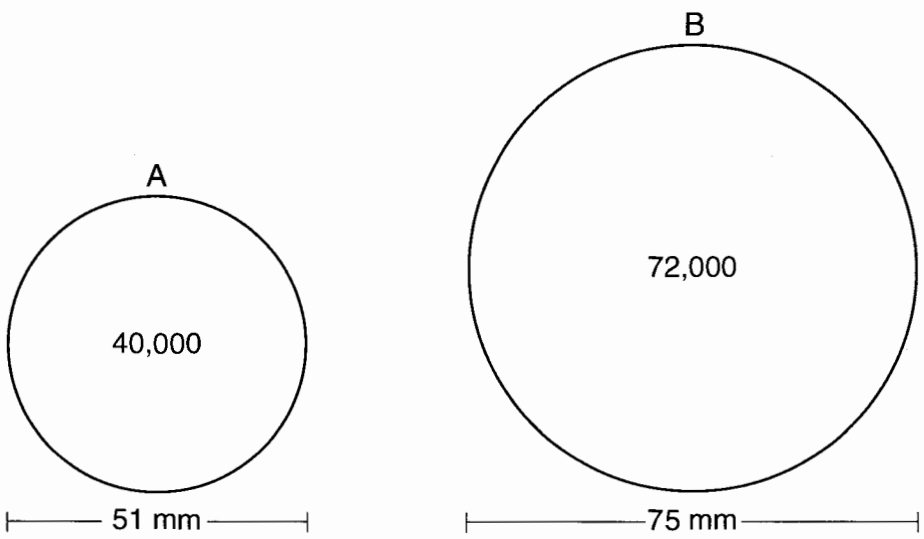
**29** A carpenter leans an extension ladder against a house to reach the bottom of a window 30 feet above the ground. As shown in the diagram below, the ladder makes a 70° angle with the ground. To the *nearest foot*, determine and state the length of the ladder.




$\sin 70 = \frac{30}{L}$

$L \approx 32$

**30** During an experiment, the same type of bacteria is grown in two petri dishes. Petri dish *A* has a diameter of 51 mm and has approximately 40,000 bacteria after 1 hour. Petri dish *B* has a diameter of 75 mm and has approximately 72,000 bacteria after 1 hour.




Determine and state which petri dish has the greater population density of bacteria at the end of the first hour.

$\frac{40,000}{\pi(\frac{51}{2})^2}$ $\frac{72,000}{\pi(\frac{75}{2})^2}$

19.6 16.3

A

**31** Line $\ell$ is mapped onto line $m$ by a dilation centered at the origin with a scale factor of 2. The equation of line $\ell$ is $3x - y = 4$. Determine and state an equation for line $m$.

$\ell \quad y = 3x - 4$

$\times 2$

$m \quad y = 3x - 8$

## Part III

**Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

**32** The aspect ratio (the ratio of screen width to height) of a rectangular flat-screen television is 16:9. The length of the diagonal of the screen is the television's screen size. Determine and state, to the *nearest inch*, the screen size (diagonal) of this flat-screen television with a screen height of 20.6 inches.

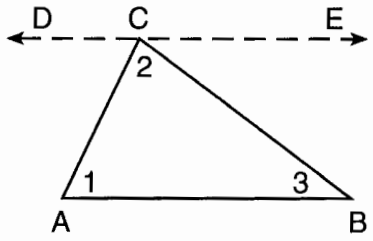
$\frac{16}{9} = \frac{x}{20.6}$

$x \approx 36.6$

$D = \sqrt{36.6^2 + 20.6^2}$

$\approx 42$

**33** Given the theorem, "The sum of the measures of the interior angles of a triangle is 180°," complete the proof for this theorem.




Given: $\triangle ABC$

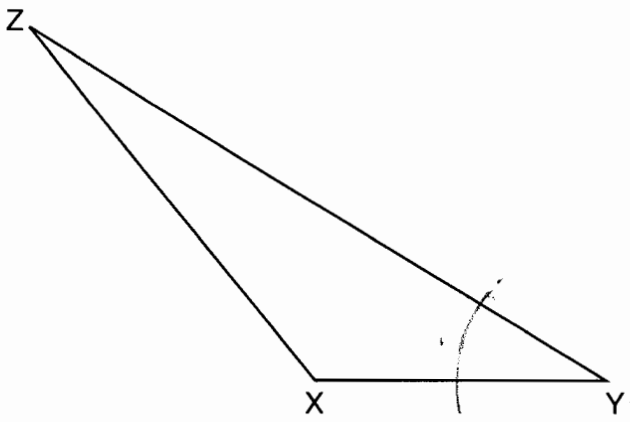
Prove: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$

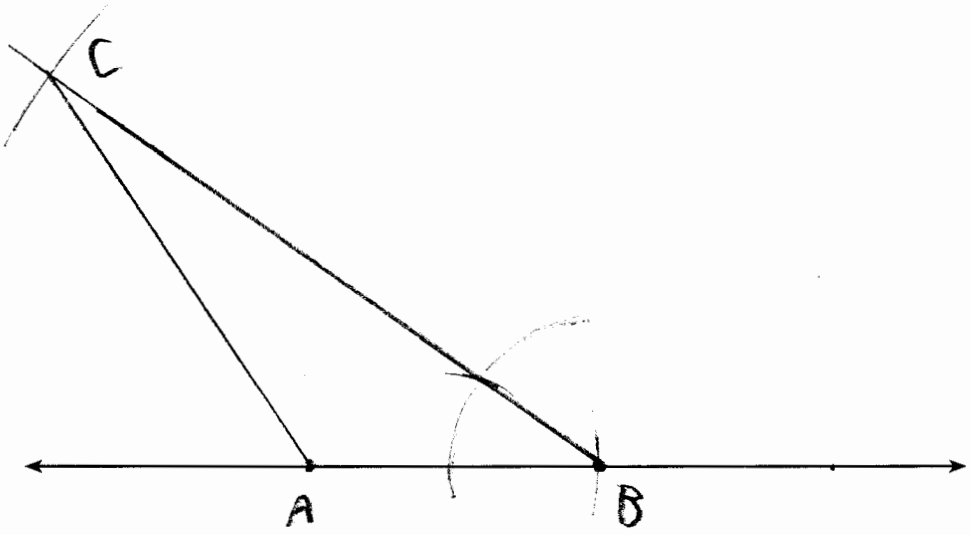
Fill in the missing reasons below.

| Statements | Reasons |
|---|---|
| (1) $\triangle ABC$ | (1) Given |
| (2) Through point $C$, draw $\overleftrightarrow{DCE}$ parallel to $\overline{AB}$. | (2) Euclid's Parallel Postulate |
| (3) $m\angle 1 = m\angle ACD$, $m\angle 3 = m\angle BCE$ | (3) Alternate interior angles formed by parallel lines & a transversal are $\cong$ |
| (4) $m\angle ACD + m\angle 2 + m\angle BCE = 180°$ | (4) Angles forming a line are supplementary |
| (5) $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ | (5) Substitution |

**34** Triangle $XYZ$ is shown below. Using a compass and straightedge, on the line below, construct and label $\triangle ABC$, such that $\triangle ABC \cong \triangle XYZ$. [Leave all construction marks.]

Based on your construction, state the theorem that justifies why $\triangle ABC$ is congruent to $\triangle XYZ$.



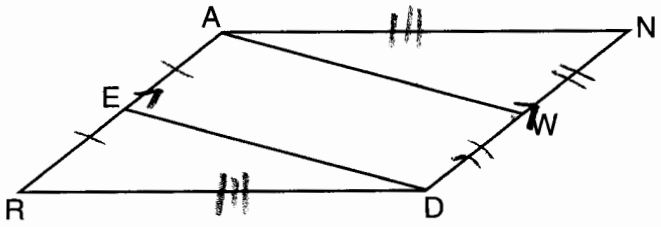




SAS ≅ SAS

## Part IV

**Answer the 2 questions in this part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil.** [12]

**35** Given: Parallelogram $ANDR$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points $W$ and $E$, respectively

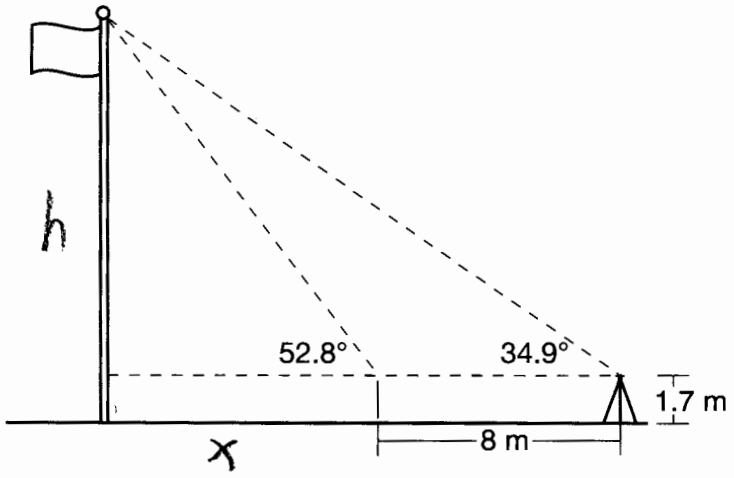



Prove that $\triangle ANW \cong \triangle DRE$.

Prove that quadrilateral $AWDE$ is a parallelogram.

| STATEMENT | REASON |
|---|---|
| (1) Parallelogram ANDR with $\overline{AW}$ & $\overline{DE}$ bisecting $\overline{NWD}$ & $\overline{REA}$ at points W & E | (1) Given |
| (2) $\overline{AN} \cong \overline{RD}$, $\overline{AR} \cong \overline{DN}$ | (2) Opposite sides of a parallelogram are $\cong$ |
| (3) $AE = \frac{1}{2}AR$, $WD = \frac{1}{2}DN$, so $\overline{AE} \cong \overline{WD}$ | (3) Definition of bisect & division property of = |
| (4) $\overline{AR} \parallel \overline{DN}$ | (4) Opposite sides of a parallelogram are $\parallel$ |
| (5) ANDR is a parallelogram | (5) Defition of parallelogram |
| (6) $RE = \frac{1}{2}AR$, $NW = \frac{1}{2}DN$, so $\overline{RE} \cong \overline{NW}$ | (6) Reason 3 |
| (7) $\overline{ED} \cong \overline{AN}$ | (7) Reason 2 |
| (8) $\triangle ANW \cong \triangle DRE$ | (8) SSS |

**36** Cathy wants to determine the height of the flagpole shown in the diagram below. She uses a survey instrument to measure the angle of elevation to the top of the flagpole, and determines it to be 34.9°. She walks 8 meters closer and determines the new measure of the angle of elevation to be 52.8°. At each measurement, the survey instrument is 1.7 meters above the ground.




Determine and state, to the *nearest tenth of a meter*, the height of the flagpole.

$\tan 52.8 = \frac{h}{x}$ $\quad \tan 34.9 = \frac{h}{x+8}$

$h = x \tan 52.8$ $\quad h = (x+8)\tan 34.9$

$x \tan 52.8 = x \tan 34.9 + 8 \tan 34.9$

$x \tan 52.8 - x \tan 34.9 = 8 \tan 34.9$

$x(\tan 52.8 - \tan 34.9) = 8 \tan 34.9$

$x = \frac{8 \tan 34.9}{\tan 52.8 - \tan 34.9}$

$x \approx 9$

$\tan 52.8 = \frac{h}{9}$

$h \approx 11.86 + 1.7 \approx 13.6$