engage^ny

Our Students. Their Moment.

Regents Examination in Geometry (Common Core)

Sample Items
Spring 2014

New York State Common Core Sample Questions: Regents Examination in Geometry (Common Core)

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. Educators around the state have already begun instituting Common Core instruction in their classrooms. To aid in this transition, we are providing sample Regents Examination in Geometry (Common Core) questions to help students, parents, and educators better understand the instructional shifts demanded by the Common Core and the rigor required to ensure that all students are on track to college and career readiness.

These Questions Are Teaching Tools

The sample questions emphasize the instructional shifts demanded by the Common Core. For Geometry (Common Core) we have provided fourteen questions. These questions include multiple-choice and constructed response. The sample questions are teaching tools for educators and can be shared freely with students and parents. They are designed to help clarify the way the Common Core should drive instruction and how students will be assessed on the Geometry Regents Examination in Geometry measuring CCLS beginning in June 2015. NYSED is eager for feedback on these sample questions. Your input will guide us as we develop future exams.

These Questions Are NOT Test Samplers

While educators from around the state have helped craft these sample questions, they have not undergone the same extensive review, vetting, and field testing that occurs with actual questions used on the State exams. The sample questions were designed to help educators think about content, NOT to show how operational exams look exactly or to provide information about how teachers should administer the test.

How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standards are assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting, and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.

- Notice the application of mathematical ways of thinking to real-world issues and challenges.
- Pay attention to the strong distractors in each multiple-choice question.
- **Don't** consider these questions to be the only way the standards will be assessed.
- **Don't** assume that the sample questions represent a mini-version of future State exams.

Understanding Math Sample Questions

Multiple-Choice Questions

Sample multiple-choice math questions are designed to assess CCLS math standards. Math multiple-choice questions assess procedural fluency and conceptual understanding. Unlike questions on past math exams, many require the use of multiple skills and concepts. Within the sample questions, all distractors will be based on plausible missteps.

Constructed Response Questions

Math constructed response questions are similar to past questions, asking students to show their work in completing one or more tasks or more extensive problems. Constructed response questions allow students to show their understanding of math procedures, conceptual understanding, and application.

Format of the Math Sample Questions Document

The Math Sample Questions document is formatted so that headings appear below each item to provide information for teacher use to help interpret the item, understand measurement with the CCLS, and inform instruction. A list of the headings with a brief description of the associated information is shown below.

Key: This is the correct response or, in the case of multiple-choice items, the correct option.

Measures CCLS: This item measures the knowledge, skills, and proficiencies characterized by the standards within the identified cluster.

Mathematical Practices: If applicable, this is a list of mathematical practices associated with the item.

Commentary: This is an explanation of how the item measures the knowledge, skills, and proficiencies characterized by the identified cluster.

Rationale: For multiple-choice items, this section provides the correct option and demonstrates one method for arriving at that response. For constructed response items, one possible approach to solving the item is shown followed by the scoring rubric that is specific to the item. Note that there are often multiple approaches to solving each problem. The rationale section provides only one example. The scoring rubrics should be used to evaluate the efficacy of different methods of arriving at a solution.

Common Core Sample Question #1

1 What are the coordinates of the point on the directed line segment from $K(-5,-4)$ to $L(5,1)$ that partitions the segment into a ratio of 3 to 2?

(1) $(-3,-3)$

(2) $(-1,-2)$

(3) $(0,-\frac{3}{2})$

(4) $(1,-1)$

Key: 4

Measures CCLS: G-GPE.B

Mathematical Practice: 2, 7

Commentary: This question measures G-GPE.B because the student needs to find the coordinates of a point dividing a directed line segment into the ratio of 3 to 2.

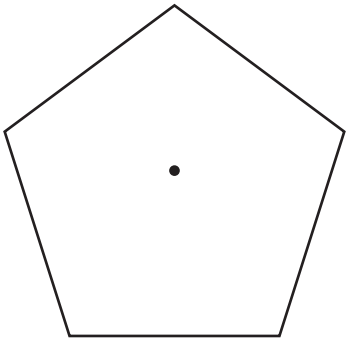
Rationale: Option 4 is correct. Since $\overline{KL}$ is a directed line segment, the point dividing $\overline{KL}$ into a ratio of 3 to 2 is $\frac{3}{5}$ the distance from point K to point L.

x	y
$-5+\frac{3}{5}(5--5)$	$-4+\frac{3}{5}(1--4)$
$-5+\frac{3}{5}(10)$	$-4+\frac{3}{5}(5)$
$-5+6$	$-4+3$
1	-1

$(1, -1)$

Common Core Sample Question #2

2 A regular pentagon is shown in the diagram below.



If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

(1) 54º
(2) 72º
(3) 108º
(4) 360º

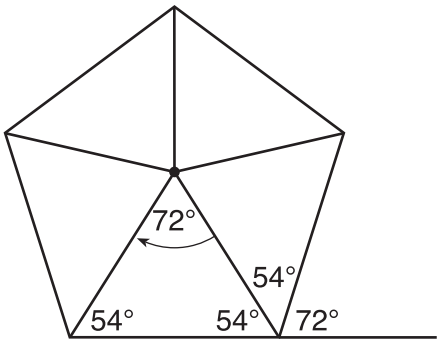
Key: 2

Measures CCLS: G-CO.A

Mathematical Practice: 2, 7

Commentary: This question measures G-CO.A because it requires the student to describe a rotation that carries a regular pentagon onto itself.

Rationale: Option 2 is correct. Segments drawn from the center of the regular pentagon bisect each angle of the pentagon, and create five isosceles triangles as shown in the diagram below. Since each exterior angle equals the angles formed by the segments drawn from the center of the regular pentagon, the minimum degrees necessary to carry a regular polygon onto itself are equal to the measure of an exterior angle of the regular polygon.



$\frac{360}{5} = 72.$

Common Core Sample Question #3

3 The equation of line *h* is $2x + y = 1$. Line *m* is the image of line *h* after a dilation of scale factor 4 with respect to the origin. What is the equation of the line *m*?

(1) $y = -2x + 1$
(2) $y = -2x + 4$
(3) $y = 2x + 4$
(4) $y = 2x + 1$

Key: 2

Meaures CCLS: G-SRT.A

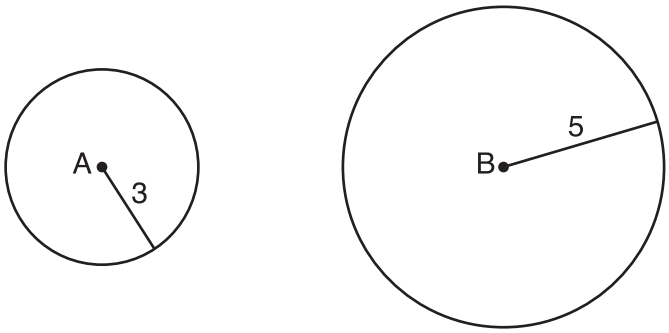
Mathematical Practice: 2

Commentary: This question measures G-SRT.A because a line that is dilated and does not pass through the center of dilation results in a parallel line.

Rationale: Option 2 is correct. The given line h, $2x + y = 1$, does not pass through the center of dilation, the origin, because the y-intercept is at (0,1). The slope of the dilated line, m, will remain the same as the slope of line h, –2. All points on line h, such as (0,1), the y-intercept, are dilated by a scale factor of 4; therefore, the y-intercept of the dilated line is (0,4) because the center of dilation is the origin, resulting in the dilated line represented by the equation $y = -2x + 4$.

Common Core Sample Question #4

4 As shown in the diagram below, circle *A* as a radius of 3 and circle *B* has a radius of 5.



Use transformations to explain why circles *A* and *B* are similar.

Key: See explanation in the rationale below. A correct explanation must include a written verbal statement.

Measures CCLS: G-C.A

Mathematical Practice: 3, 6

Commentary: This question measures G-C.A because the student must explain why two given circles are similar.

Rationale: Circle A can be mapped onto circle B by first translating circle A along vector $\overrightarrow{AB}$ such that A maps onto B, and then dilating circle A, centered at A, by a scale factor of $\frac{5}{3}$. Since there exists a sequence of transformations that maps circle A onto circle B, circle A is similar to circle B.

Rubric:

[2] A complete and correct explanation is written.

[1] An appropriate explanation is written, but one computational error is made.

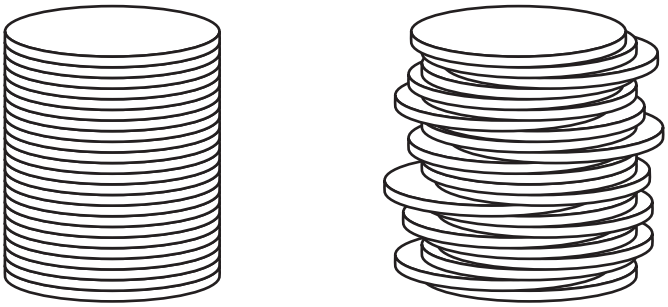
or

[1] An appropriate explanation is written, but one conceptual error is made.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #5

5 Two stacks of 23 quarters each are shown below. One stack forms a cylinder but the other stack does not form a cylinder.



Use Cavelieri's principle to explain why the volumes of these two stacks of quarters are equal.

Key: See explanation in rationale below.

Measures CCLS: G-GMD.A

Mathematical Practice: 3, 6

Commentary: This question measures G-GMD.A because the student is required to explain the relationship of the volumes of two objects using Cavelieri's principle.

Rationale: Each quarter in both stacks has the same base area. Therefore, each corresponding cross-section of the stacks will have the same area. Since the two stacks of quarters have the same height of 23 quarters, the two volumes must be the same.

Rubric:

[2] A complete and correct explanation is written.

[1] An appropriate explanation is written, but one conceptual error is made.

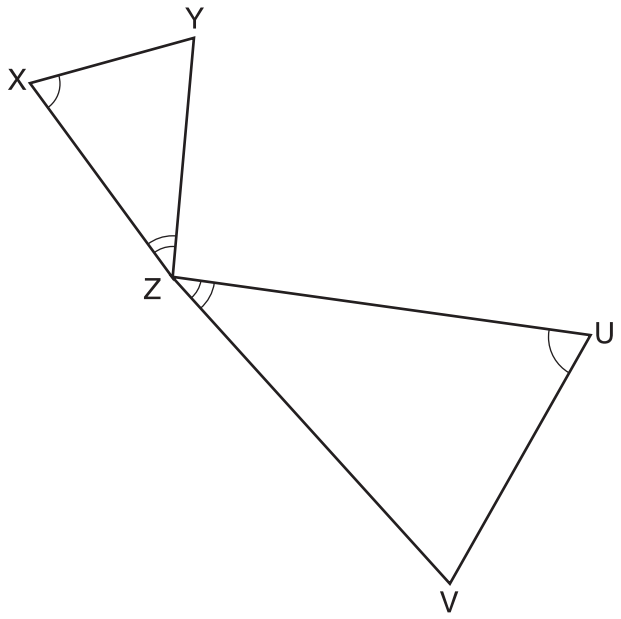
or

[1] An incomplete or partially correct explanation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #6

6 In the diagram below, triangles *XYZ* and *UVZ* are drawn such that $\angle X \cong \angle U$ and $\angle XZY \cong \angle UZV$.



Describe a sequence of similarity transformations that shows $\triangle XYZ$ is similar to $\triangle UVZ$.

Key: See the description in the rationale below.

Measures CCLS: G-SRT.A

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.A because students must describe a sequence of similarity transformations to show two triangles are similar when they have two pairs of corresponding angles congruent.

Rationale: Triangle $X'Y'Z'$ is the image of $\triangle XYZ$ after a rotation about point Z such that $\overline{ZX}$ coincides with $\overline{ZU}$. Since rotations preserve angle measure, $\overline{ZY}$ coincides with $\overline{ZV}$, and corresponding angles X and Y, after the rotation, remain congruent, so $\overline{XY} \parallel \overline{UV}$.

Then, dilate $\triangle X'Y'Z'$ by a scale factor of $\frac{ZU}{ZX}$ with its center at point Z. Since dilations preserve parallelism, $\overline{XY}$ maps onto $\overline{UV}$. Therefore, $\triangle XYZ \sim \triangle UVZ$.

Rubric:

[2] A correct description of sequence of similarity transformations is written.

[1] One conceptual error is made, but an appropriate sequence of similarity transformations is written.

or

[1] An incomplete or partially correct sequence of similarity transformations is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #7

7 Explain why $\cos(x) = \sin(90 - x)$ for x such that $0 < x < 90$.

Key: See explanation in the rationale below. A correct explanation must include a written verbal statement.

Measures CCLS: G-SRT.C

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.C because the student is required to explain why the sine and cosine of complementary angles are equal.

Rationale: The acute angles in a right triangle are always complementary. The sine of any acute angle is equal to the cosine of its complement.

Rubric:

[2] A correct and complete explanation is written.

[1] One conceptual error is made, but an appropriate explanation is written.

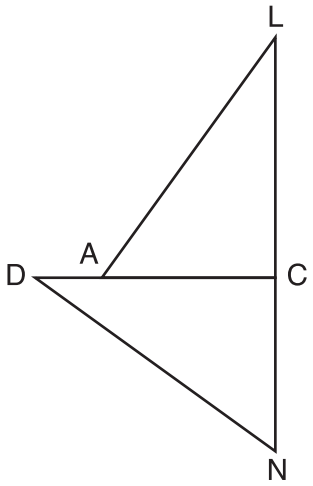
or

[1] An incomplete or partially correct explanation is written.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #8

8 In the diagram of $\triangle LAC$ and $\triangle DNC$ below, $\overline{LA} \cong \overline{DN}$, $\overline{CA} \cong \overline{CN}$, and $\overline{DAC} \perp \overline{LCN}$.



a) Prove that $\triangle LAC \cong \triangle DNC$.

b) Describe a sequence of rigid motions that will map $\triangle LAC$ onto $\triangle DNC$.

Key: See rationale below. A sequence of one transformation is acceptable.

Measures CCLS: G-SRT.B, G-CO.A

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.B and G-CO.A because students are required to prove two triangles are congruent and demonstrate congruence using rigid motion.

Rationale:

Statement	Reasons
1. $\overline{LA} \cong \overline{DN}, \overline{CA} \cong \overline{CN},$ and $\overline{DC} \perp \overline{LN}$	1. Given
2. $\angle LCA$ and $\angle DCN$ are right angles	2. Definition of perpendicular lines
3. $\triangle LAC$ and $\triangle DNC$ are right triangles	3. Definition of a right triangle
4. $\triangle LAC \cong \triangle DNC$	4. H.L. Theorem

Triangle *LAC* will map onto $\triangle DNC$ after rotating $\triangle LAC$ counterclockwise 90 degrees about point *C* such that point *L* maps onto point *D*.

Rubric:

Part a)

[2] A complete and correct proof that includes a conclusion is written.

[1] Only one correct statement and reason are written.

or

[1] One conceptual error is made.

[0] The "given" and/or the "prove" statements are written, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Part b)

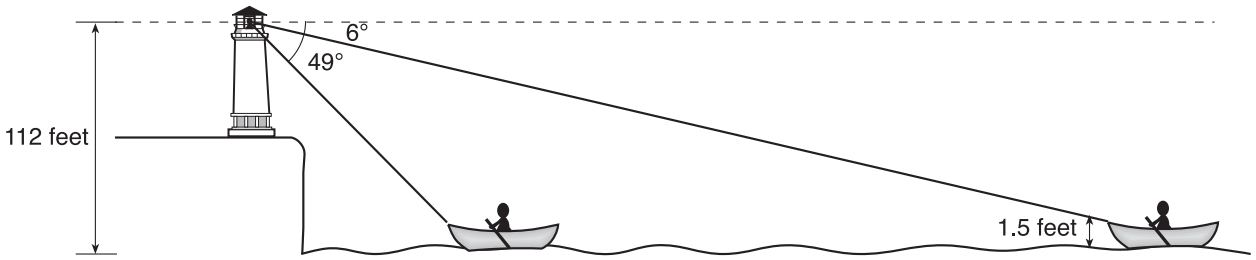
[2] A complete and correct description mapping $\triangle LAC$ onto $\triangle DNC$ is written.

[1] An appropriate description is written, but one conceptual error is made.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #9

9 As shown below, a canoe is approaching a lighthouse on the coastline of a lake. The front of the canoe is 1.5 feet above the water and an observer in the lighthouse is 112 feet above the water.



(Not drawn to scale)

At 5:00, the observer in the lighthouse measured the angle of depression to the front of the canoe to be 6°. Five minutes later, the observer measured and saw the angle of depression to the front of the canoe had increased by 49°. Determine and state, to the *nearest foot per minute*, the average speed at which the canoe traveled toward the lighthouse.

Key: 195

Measures CCLS: G-SRT.C

Mathematical Practice: 1, 4

Commentary: This question measures G-SRT.C because students need to use modeling and trigonometric ratios to find the average speed.

Rationale: x represents the distance between the lighthouse and the canoe at 5:00.
y represents the distance between the lighthouse and the canoe at 5:05.

$$\tan 6 = \frac{112 - 1.5}{x} \qquad \tan (6 + 49) = \frac{112 - 1.5}{y}$$

$$x = \frac{110.5}{\tan 6} \qquad y = \frac{110.5}{\tan 55}$$

$$x = 1051.337272 \qquad y = 77.37293297$$

$$\text{Average speed} = \frac{973.964339 \text{ ft}}{5 \text{ min}} = 194.7928678$$

Average speed = 195 ft/min

Rubric:

[4] 195, and correct work is shown.

[3] Appropriate work is shown, but one computational or rounding error is made.

or

[3] Appropriate work is shown to find the distance traveled, but no further correct work is shown.

[2] Appropriate work is shown, but two computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find the distance from the lighthouse at 5:00 and at 5:05, but no further correct work is shown.

[1] Appropriate work is shown, but one computational or rounding error and one conceptual error are made.

or

[1] Appropriate work is shown to find the distance from the lighthouse at either 5:00 or at 5:05, but no further correct work is shown.

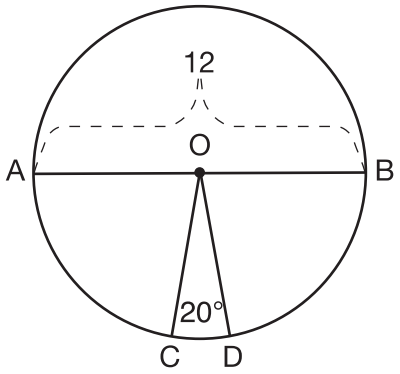
or

[1] 195, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #10

10 In the diagram below of circle *O*, diameter $\overline{AB}$ and radii $\overline{OC}$ and $\overline{OD}$ are drawn. The length of $\overline{AB}$ is 12 and the measure of $\angle COD$ is 20 degrees.



If $\overset{\frown}{AC} \cong \overset{\frown}{BD}$, find the area of sector *BOD* in terms of π.

Key: 8π

Measures CCLS: G-C.B

Mathematical Practice: 2

Commentary: This question measures G-C.B because students are required to find the area of a sector.

Rationale: A_o represents the area of circle *O* and A_s represents the area of sector *BOD*.

$A_o = \pi r^2$ $\quad$ $m\angle BOD = \frac{180 - 20}{2}$ $\quad$ $\frac{A_s}{36\pi} = \frac{80}{360}$

$A_o = \pi(6)^2$ $\quad$ $m\angle BOD = 80$ $\quad$ $360(A_s) = 2880\pi$

$A_o = 36\pi$ $\quad$ $A_s = 8\pi$

Rubric:

[4] 8π, and correct work is shown.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Appropriate work is shown, but the area of the sector is written as an appropriate decimal.

[2] Appropriate work is shown, but two computational errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find 36π, the area of the circle, and 80, the measure of angle *BOD*, but no further correct work is shown.

[1] Appropriate work is shown, but one computational error and one conceptual error are made.

or

[1] Appropriate work is shown to find either 36π, the area of the circle, or 80, the measure of angle *BOD*, but no further correct work is shown.

or

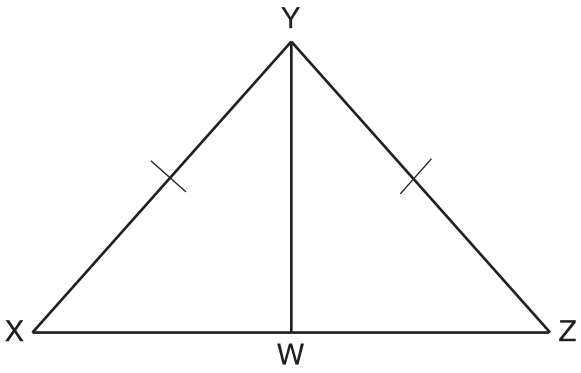
[1] 8π, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #11

11 Given: $\triangle XYZ$, $\overline{XY} \cong \overline{ZY}$, and $\overline{YW}$ bisects $\angle XYZ$

Prove that $\angle YWZ$ is a right angle.



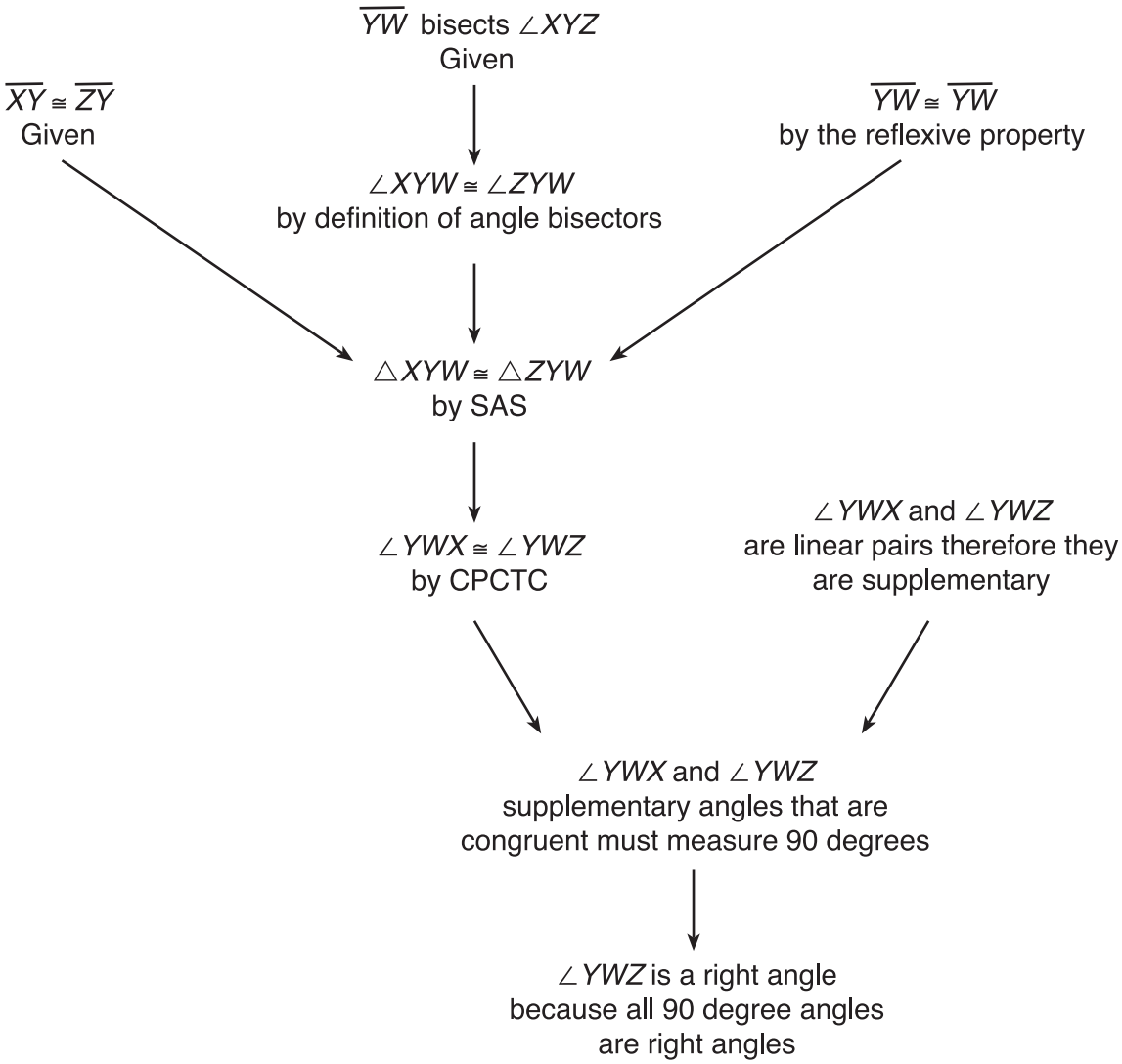
Key: See proof in the rationale below.

Measures CCLS: G-CO.C

Mathematical Practice: 3, 6

Commentary: This question measures G-CO.C because students are required to prove that the altitude of an isosceles triangle forms right angles.

Rationale: Multiple methods of proof are acceptable.



or

Statements	Reasons
1. $\triangle XYZ$, $\overline{XY} \cong \overline{ZY}$, $\overline{YW}$ bisects $\angle XYZ$	1. Given
2. $\triangle XYZ$ is isosceles	2. Definition of isosceles triangle
3. $\overline{YW}$ is an altitude of $\triangle XYZ$	3. The angle bisector of the vertex of an isosceles triangle is also the altitude of that triangle.
4. $\overline{YW} \perp \overline{XZ}$	4. Definition of altitude
5. $\angle YWZ$ is a right angle	5. Definition of perpendicular lines

Rubric:

[4] A complete and correct proof that includes a concluding statement is written.

[3] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect, or the concluding statement is missing.

[2] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect.

or

[2] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.

[1] Only one correct relevant statement and reason are written.

[0] The "given" and/or the "prove" statements are written, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #12

12 Trees that are cut down and stripped of their branches for timber are approximately cylindrical. A timber company specializes in a certain type of tree that has a typical diameter of 50 cm and a typical height of about 10 meters. The density of the wood is 380 kilograms per cubic meter, and the wood can be sold by mass at a rate of $4.75 per kilogram. Determine and state the minimum number of whole trees that must be sold to raise at least $50,000.

Key: 15

Measures CCLS: G-MG.A

Mathematical Practice: 1, 4

Commentary: This question measures G-MG.A because a cylinder is used to model a tree trunk to solve the problem. This problem requires students to navigate multiple steps and develop an appropriate model.

Rationale:

Volume of one tree	Weight of one tree (x)	Whole trees needed (n)
$V = \pi r^2 h$	$\frac{380 \text{ K}}{1 \text{ m}^3} = \frac{x}{0.625\pi}$	$n = \frac{50,000}{(4.75)(746.1282552)}$
$V = \pi(0.25)^2(10)$	$x = 746.1282552$ K	$n = 14.10791739$
$V = 0.625\pi$		
		15 whole trees

Rubric:

[4] 15, and correct work is shown.

[3] Appropriate work is shown, but one computational error is made.

or

[3] Appropriate work is shown, but 15 is not identified as the answer.

[3] Appropriate work is shown to find the volume and weight of one tree and amount of money for the sale of one tree. No further correct work is shown.

[2] Appropriate work is shown, but two or more computational or rounding errors are made.

or

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] Appropriate work is shown to find the volume and weight of one tree, but no further correct work is shown.

[1] Appropriate work is shown, but one conceptual and one computational or rounding error are made.

or

[1] Appropriate work is shown to find the volume of one tree, but no further correct work is shown.

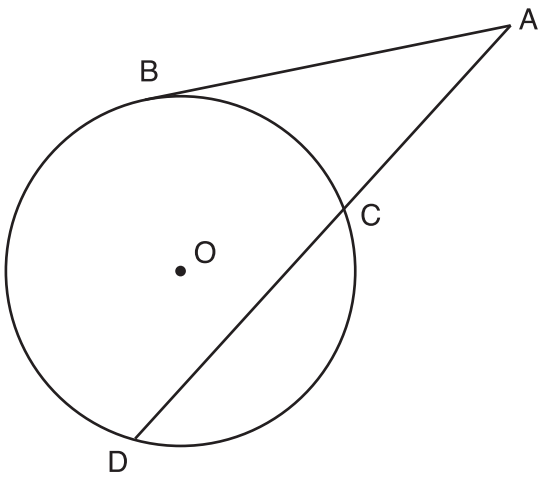
or

[1] 15, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #13

13 In the diagram below, secant $\overline{ACD}$ and tangent $\overline{AB}$ are drawn from external point *A* to circle *O*.



Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. ($AC \cdot AD = AB^2$)

Key: See proof in the rationale below.

Measures CCLS: G-SRT.B, G-C.A

Mathematical Practice: 3, 6

Commentary: This question measures G-SRT.B because the student is required to use similarity criteria to prove relationships in a geometric figure. It also aligns to G-C.A because the student would use angles formed by chords to prove the triangle are similar.

Rationale:

Statements	Reasons
1. Circle O, Secant $\overline{ACD}$, Tangent $\overline{AB}$	1. Given
2. Chords $\overline{BC}$ and $\overline{BD}$ are drawn	2. Auxiliary lines
3. $\angle A \cong \angle A$, $\overset{\frown}{BC} \cong \overset{\frown}{BC}$	3. Reflexive property
4. $\text{m}\angle BDC = \frac{1}{2}\text{m}\overset{\frown}{BC}$	4. The measure of an inscribed angle is half the measure of the intercepted arc.
5. $\text{m}\angle CBA = \frac{1}{2}\text{m}\overset{\frown}{BC}$	5. The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.
6. $\angle BDC \cong \angle CBA$	6. Angles equal to half of the same arc are congruent.
7. $\triangle ABC \sim \triangle ADB$	7. AA
8. $\frac{AB}{AC} = \frac{AD}{AB}$	8. Corresponding sides of similar triangles are proportional.
9. $AC \bullet AD = AB^2$	9. In a proportion, the product of the means equals the product of the extremes.

Rubric:

[6] A complete and correct proof that includes a concluding statement is written.

[5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect.

[4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect.

or

[4] $\triangle ABC \sim \triangle ADB$, but no further correct work is shown.

[3] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.

[2] A proof is written that demonstrates a method of proof, but one conceptual error is made, and one statement and/or reason is missing or is incorrect.

or

[2] Some correct relevant statements about the proof are made, but three or four statements or reasons are missing or are incorrect.

[1] Only one correct relevant statement and reason are written.

[0] The "given" and/or the "prove" statements are rewritten in the style of a formal proof, but no further correct relevant statements are written.

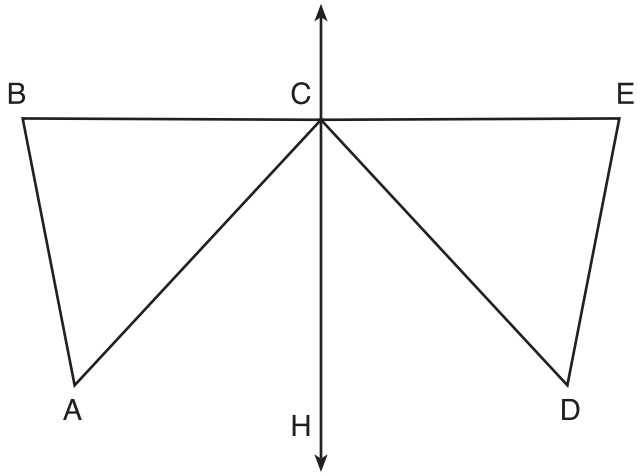
or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

Common Core Sample Question #14

14 Given: D is the image of A after a reflection over $\overleftrightarrow{CH}$.
$\overleftrightarrow{CH}$ is the perpendicular bisector of $\overline{BCE}$
$\triangle ABC$ and $\triangle DEC$ are drawn

Prove: $\triangle ABC \cong \triangle DEC$



Key: See proof in the rationale below.

Measures CCLS: G-CO.B

Mathematical Practice: 3, 6

Commentary: This question measures G-CO.B because the student is required to prove that two triangles are congruent using the definition of congruence in terms of rigid motion.

Rationale:

It is given that point *D* is the image of point *A* after a reflection in line *CH*.

It is given that $\overleftrightarrow{CH}$ is the perpendicular bisector of $\overline{BE}$ at point *C*. Since a bisector divides a segment into two congruent segments at its midpoint, $\overline{BC} \cong \overline{EC}$. Point *E* is the image of point *B* after a reflection over the line *CH*, since points *B* and *E* are equidistant from point *C* and it is given that $\overleftrightarrow{CH}$ is perpendicular to $\overline{BE}$.

Point *C* is on $\overleftrightarrow{CH}$ therefore, point *C* maps to itself after the reflection over $\overleftrightarrow{CH}$.

Since all three vertices of triangle *ABC* map to all three vertices of triangle *DEC* under the same line reflection, then $\triangle ABC \cong \triangle DEC$ because a line reflection is a rigid motion and triangles are congruent when one can be mapped onto the other using a sequence of rigid motions.

Rubric:

[6] A complete and correct proof that includes a concluding statement is written.

[5] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one supporting statement and/or reason is missing or is incorrect.

[4] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two supporting statements and/or reasons are missing or are incorrect.

[3] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.

[2] A proof is written that demonstrates a method of proof, but one conceptual error is made, and one supporting statement and/or reason is missing or is incorrect.

[1] Only one correct relevant statement and reason are written.

[0] The “given” and/or the “prove” statements are rewritten, but no further correct relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

engage^ny

Our Students. Their Moment.

Regents Examination in Geometry (Common Core)

Sample Items
Fall 2014

New York State Common Core Sample Questions: Regents Examination in Geometry (Common Core)

With the adoption of the New York P-12 Common Core Learning Standards (CCLS) in ELA/Literacy and Mathematics, the Board of Regents signaled a shift in both instruction and assessment. Educators around the state have already begun instituting Common Core instruction in their classrooms. To aid in this transition, we are providing sample Regents Examination in Geometry (Common Core) questions to help students, parents, and educators better understand the instructional shifts demanded by the Common Core and the rigor required to ensure that all students are on track to college and career readiness.

These Questions Are Teaching Tools

The sample questions emphasize the instructional shifts demanded by the Common Core. For Geometry (Common Core) we have provided thirteen questions. These questions include multiple choice and constructed response. The sample questions are teaching tools for educators and can be shared freely with students and parents. They are designed to help clarify the way the Common Core should drive instruction and how students will be assessed on the Regents Examination in Geometry measuring CCLS beginning in June 2015. NYSED is eager for feedback on these sample questions. Your input will guide us as we develop future exams.

These Questions Are NOT Test Samplers

While educators from around the state have helped craft these sample questions, they have not undergone the same extensive review, vetting, and field testing that occurs with actual questions used on the State exams. The sample questions were designed to help educators think about content, NOT to show how operational exams look exactly or to provide information about how teachers should administer the test.

How to Use the Sample Questions

- Interpret how the standards are conceptualized in each question.
- Note the multiple ways the standard is assessed throughout the sample questions.
- Look for opportunities for mathematical modeling, i.e., connecting mathematics with the real world by conceptualizing, analyzing, interpreting, and validating conclusions in order to make decisions about situations in everyday life, society, or the workplace.
- Consider the instructional changes that will need to occur in your classroom.
- Notice the application of mathematical ways of thinking to real-world issues and challenges.

- Pay attention to the strong distractors in each multiple-choice question.
- **Don't** consider these questions to be the only way the standard will be assessed.
- **Don't** assume that the sample questions represent a mini-version of future State exams.

Understanding Math Sample Questions

Multiple-Choice Questions

Sample multiple-choice math questions are designed to assess CCLS math standards. Math multiple-choice questions assess procedural fluency and conceptual understanding. Unlike questions on past math exams, many require the use of multiple skills and concepts. Within the sample questions, all distractors will be based on plausible missteps.

Constructed-Response Questions

Math constructed-response questions are similar to past questions, asking students to show their work in completing one or more tasks or more extensive problems. Constructed-response questions allow students to show their understanding of math procedures, conceptual understanding, and application.

Format of the Math Sample Questions Document

The Math Sample Questions document is formatted so that headings appear below each item to provide information for teacher use to help interpret the item, understand alignment with the CCLS, and inform instruction. A list of the headings with a brief description of the associated information is shown below.

Key: This is the correct response or, in the case of multiple-choice items, the correct option.

Measures CCLS: This item measures the knowledge, skills, and proficiencies characterized by the standards within the identified cluster.

Mathematical Practices: If applicable, this is a list of mathematical practices associated with the item.

Commentary: This is an explanation of how the item measures the knowledge, skills, and proficiencies characterized by the identified cluster.

Rationale: For multiple-choice items, this section provides the correct option and demonstrates one method for arriving at that response. For constructed-response items, one possible approach to solving the item is shown followed by the scoring rubric that is specific to the item. Note that there are often multiple approaches to solving each problem. The rationale section provides only one example. The scoring rubrics should be used to evaluate the efficacy of different methods of arriving at a solution.

1 A man who is 5 feet 9 inches tall casts a shadow of 8 feet 6 inches. Assuming that the man is standing perpendicular to the ground, what is the angle of elevation from the end of the shadow to the top of the man's head, to the *nearest tenth of a degree*?

(1) 34.1
(2) 34.5
(3) 42.6
(4) 55.9

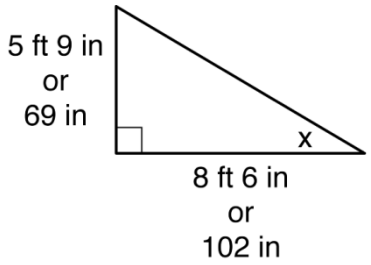
Key: 1

Measures CCLS: G-SRT.C

Mathematical Practice: 4

Commentary: This question measures G-SRT.C because students are required to find the angle of elevation using right triangle trigonometry.

Rationale: Option 1 is correct. The man's height, 69 inches, is opposite to the angle of elevation, and the shadow length, 102 inches, is adjacent to the angle of elevation. Therefore, tangent must be used to find the angle of elevation.



$\tan x = \frac{69}{102}$

$x = \tan^{-1}\left(\frac{69}{102}\right)$

$x = 34.07719528$

Angle of elevation is 34.1°

or

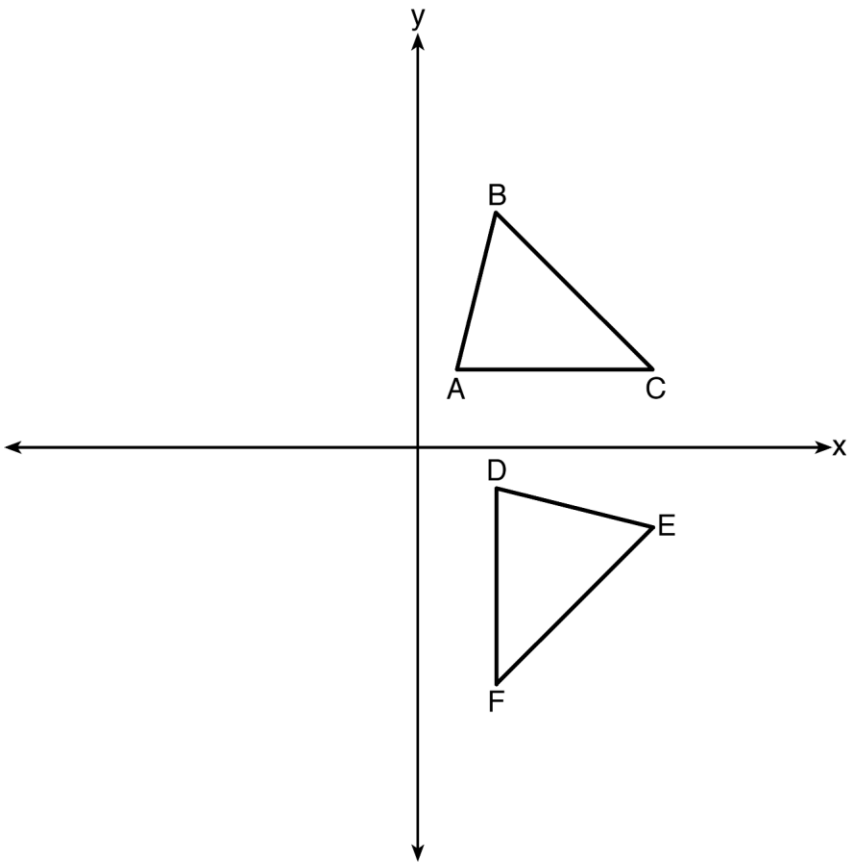
$\tan x = \frac{69}{102}$

$x = \text{arc tan}\left(\frac{69}{102}\right)$

$x = 34.07719528$

Angle of elevation is 34.1°

2 The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$, as shown below.



Which statement is true?

(1) $\overline{BC} \cong \overline{DE}$
(2) $\overline{AB} \cong \overline{DF}$
(3) $\angle C \cong \angle E$
(4) $\angle A \cong \angle D$

Key: 4

Measures CCLS: G-CO.B

Mathematical Practice: 2

Commentary: This question measures G-CO.B because students must reason that the figure resulting from a rigid motion will have angles congruent to the angles of the original figure.

Rationale: Option 4 is correct. The measures of the angles of a triangle remain the same after all rotations because rotations are rigid motions which preserve angle measure.

3 The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

(1) $y = 2x - 4$
(2) $y = 2x - 6$
(3) $y = 3x - 4$
(4) $y = 3x - 6$

Key: 2

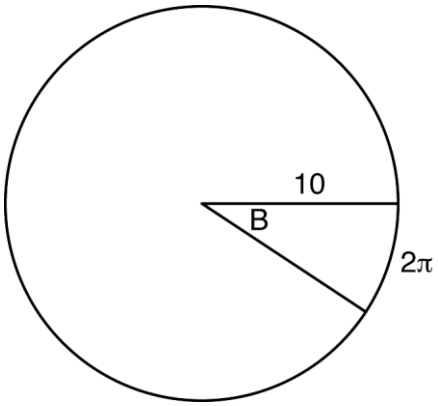
Measures CCLS: G-SRT.A

Mathematical Practice: 1

Commentary: This question measures G-SRT.A because students are required to dilate a line not passing through the center of dilation.

Rationale: Option 2 is correct. The line $y = 2x - 4$ does not pass through the center of dilation, so the dilated line will be distinct from $y = 2x - 4$. Since a dilation preserves parallelism, the line $y = 2x - 4$ and its image will be parallel, with slopes of 2. To obtain the y-intercept of the dilated line, the scale factor of the dilation, $\frac{3}{2}$, can be applied to the y-intercept, $(0,-4)$, therefore, $(0 \cdot \frac{3}{2}, -4 \cdot \frac{3}{2}) \rightarrow (0,-6)$. So the equation of the dilated line is $y = 2x - 6$.

4 In the diagram below, the circle shown has radius 10. Angle *B* intercepts an arc with a length of 2π.



What is the measure of angle *B*, in radians?

(1) $10 + 2\pi$

(2) 20π

(3) $\frac{\pi}{5}$

(4) $\frac{5}{\pi}$

Key: 3

Measures CCLS: G-C.B

Mathematical Practice: 1

Commentary: This question measures G-C.B because students are required to use the fact that the length of the arc intercepted by an angle is proportional to the radius.

Rationale: Option 3 is correct.

$s = \theta r$ $\qquad$ $C = 2\pi r$

$2\pi = A(10)$ $\qquad$ or $\qquad$ $C = 2\pi(10)$

$\frac{2\pi}{10} = A$ $\qquad$ $C = 20\pi$

$\frac{2\pi}{20\pi} = \frac{x}{360}$ $\qquad$ $\frac{2\pi}{20\pi} = \frac{x}{2\pi}$

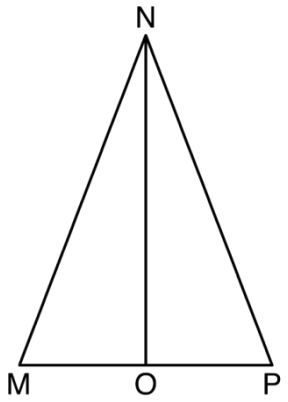
$20x = 720$ $\qquad$ $20x = 4\pi$

$x = 36$ $\qquad$ $x = \frac{2\pi}{10}$

$36 \cdot \frac{\pi}{180}$

$\frac{2\pi}{10}$

5 In isosceles $\triangle MNP$, line segment NO bisects vertex $\angle MNP$, as shown below. If $MP = 16$, find the length of $\overline{MO}$ and explain your answer.



Key: See Rationale below.

Measures CCLS: G-SRT.B

Mathematical Practice: 2,3

Commentary: This question measures G-SRT.B because students are required to solve a problem by applying triangle congruence criteria.

Rationale: Triangle *MNO* is congruent to triangle *PNO* by side-angle-side criteria. Since $\triangle MNO \cong \triangle PNO$, then $\overline{MO} \cong \overline{PO}$ by CPCTC. So $\overline{NO}$ must divide $\overline{MP}$ in half, therefore, $MO = 8$.

Rubric:

[2] 8, and correct work is shown. A complete and correct explanation is written.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 8, and correct work is shown, but the explanation is incomplete or is incorrect.

[0] 8, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

6 A contractor needs to purchase 500 bricks. The dimensions of each brick are 5.1 cm by 10.2 cm by 20.3 cm, and the density of each brick is 1920 kg/m^3.

The maximum capacity of the contractor's trailer is 900 kg. Can the trailer hold the weight of 500 bricks? Justify your answer.

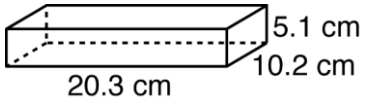
Key: See Rationale below.

Measures CCLS: G-MG.A

Mathematical Practice: 4,6

Commentary: This question measures G-MG.A because students are required to apply modeling, density, and volume to solve a weight-constraint problem.

Rationale:



$V_{brick} = (5.1)(10.2)(20.3)$

$V_{brick} = 1056.006 \text{ cm}^3$

$V_{Total} = 500(1056.006) \text{ cm}^3$

$V_{Total} = 528,003 \text{ cm}^3$

$V_{Total} = \frac{528,003}{100^3} \text{ m}^3$

$V_{Total} = 0.528003 \text{ m}^3$

$$Total\ Weight = (1920)(0.528003)$$

$$Total\ Weight \approx 1013 \text{ kg}$$

The trailer cannot hold the weight of 500 bricks. The total weight of all the bricks is approximately 1013 kg and since the trailer can only hold 900 kg, the weight of 500 bricks is too much for the trailer.

Rubric:

[2] No, and correct work is shown.

[1] Appropriate work is shown, but one computational error is made.

or

[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 0.528003, the total volume of the bricks is found, but no further correct work is shown.

[0] No, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

7 In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.

Key: See Rationale below.

Measures CCLS: G-SRT.C

Mathematical Practice: 7

Commentary: This question measures G-SRT.C because students are required to apply their knowledge of right triangle trigonometry: that the sine of an acute angle is equal to the cosine of its complement.

Rationale: $4x - 0.7 = 2x + 0.1$
$2x - 0.7 = 0.1$
$2x = 0.8$
$x = 0.4$

Sin A is the ratio of the opposite side and the hypotenuse while cos B is the ratio of the adjacent side and the hypotenuse. The side opposite angle A is the same side as the side adjacent to angle B, therefore, the sin A = cos B.

Rubric:

[2] 0.4, and correct work is shown. A correct explanation is written.

[1] Appropriate work is shown, but one computational error is made.

or

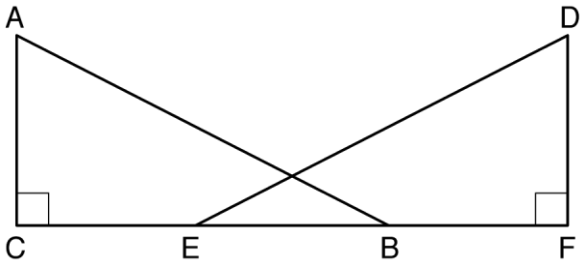
[1] Appropriate work is shown, but one conceptual error is made.

or

[1] 0.4, but the explanation is incomplete or is missing.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

8 Given right triangles ABC and DEF where $\angle C$ and $\angle F$ are right angles, $\overline{AC} \cong \overline{DF}$ and $\overline{CB} \cong \overline{FE}$. Describe a precise sequence of rigid motions which would show $\triangle ABC \cong \triangle DEF$.



Key: See Rationale below.

Measures CCLS: G-CO.A

Mathematical Practice: 3,6

Commentary: This question measures G-CO.B because students are required to describe a sequence of rigid motions that will carry one triangle onto another.

Rationale: Translate triangle *ABC* along line *CF* such that point *C* maps onto point *F*, resulting in image $\triangle A'B'C'$. Then reflect $\triangle A'B'C'$ over the line *DF* such that $\triangle A'B'C'$ maps onto $\triangle DEF$.

or

Reflect $\triangle ABC$ over the perpendicular bisector of *EB* such that $\triangle ABC$ maps onto $\triangle DEF$.

Rubric:

[2] A correct precise sequence of transformations is written.

[1] Appropriate transformations are written, but mapping of corresponding points is not written or is written incorrectly.

or

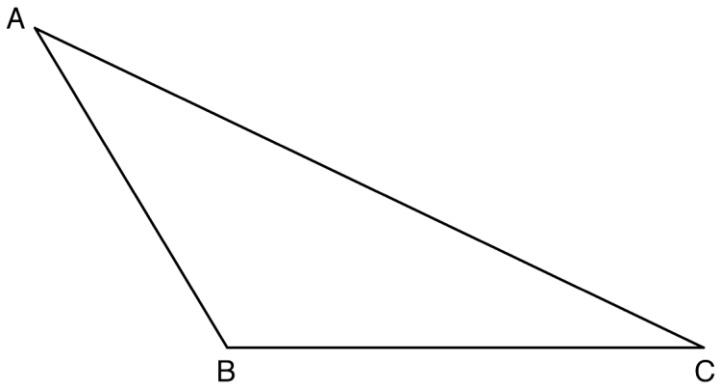
[1] One correct precise transformation is written.

or

[1] Appropriate work is shown, but one conceptual error is made.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

9 Using a compass and straightedge, construct an altitude of triangle *ABC* below.
[Leave all construction marks.]



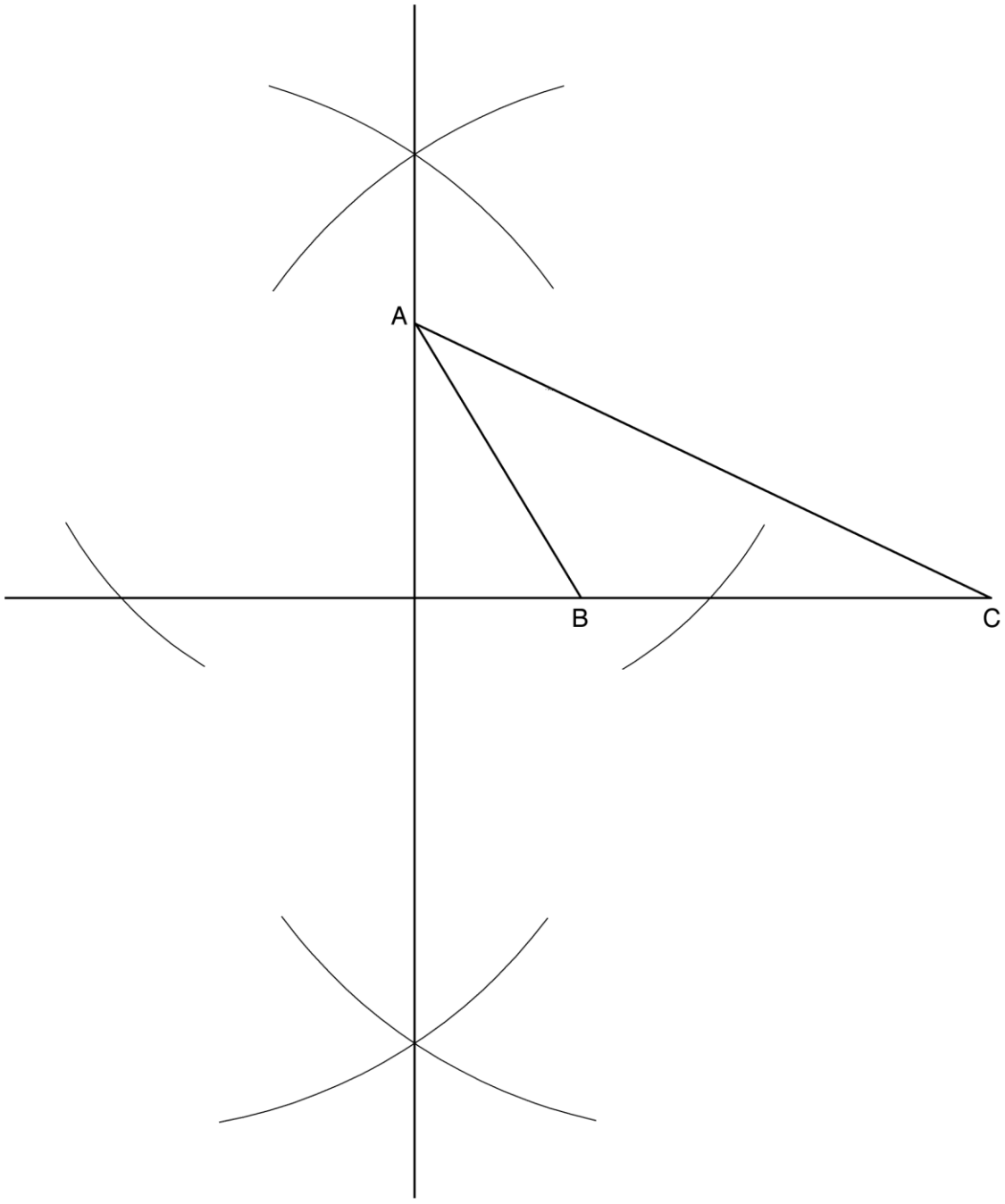
Key: See Rationale below.

Measures CCLS: G-CO.D

Mathematical Practice: 5,6

Commentary: This question measures G-CO.D because students are required to construct an altitude of a triangle.

Rationale:



Rubric:

[2] A correct construction is drawn showing all appropriate arcs, and the altitude is drawn.

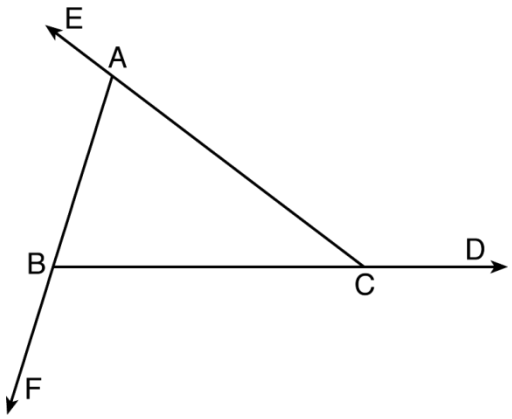
[1] Appropriate construction arcs are drawn, but one construction error is made, such as not drawing in the altitude.

[0] A drawing that is not an appropriate construction is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

10 Prove the sum of the exterior angles of a triangle is 360°.



Key: See Rationale below.

Measures CCLS: G-CO.C

Mathematical Practice: 3

Commentary: This question measures G-CO.C because students are required to prove the theorem: the sum of the exterior angles of a triangle equals 360°.

Rationale: As the sum of the measures of the angles of a triangle is 180°, $m\angle ABC + m\angle BCA + m\angle CAB = 180°$. Each interior angle of the triangle and its exterior angle form a linear pair. Linear pairs are supplementary, so $m\angle ABC + m\angle FBC = 180°$, $m\angle BCA + m\angle DCA = 180°$, and $m\angle CAB + m\angle EAB = 180°$. By addition, the sum of these linear pairs is 540°. When the angle measures of the triangle are subtracted from this sum, the result is 360°. So, the sum of the exterior angles of a triangle is 360°.

Rubric:

[4] A complete and correct proof that includes a concluding statement is written.

[3] A proof is written that demonstrates a thorough understanding of the method of proof and contains no conceptual errors, but one statement and/or reason is missing or is incorrect, or the concluding statement is missing.

[2] A proof is written that demonstrates a good understanding of the method of proof and contains no conceptual errors, but two statements and/or reasons are missing or are incorrect.

or

[2] A proof is written that demonstrates a good understanding of the method of proof, but one conceptual error is made.

[1] Only one correct relevant statement and reason are written.

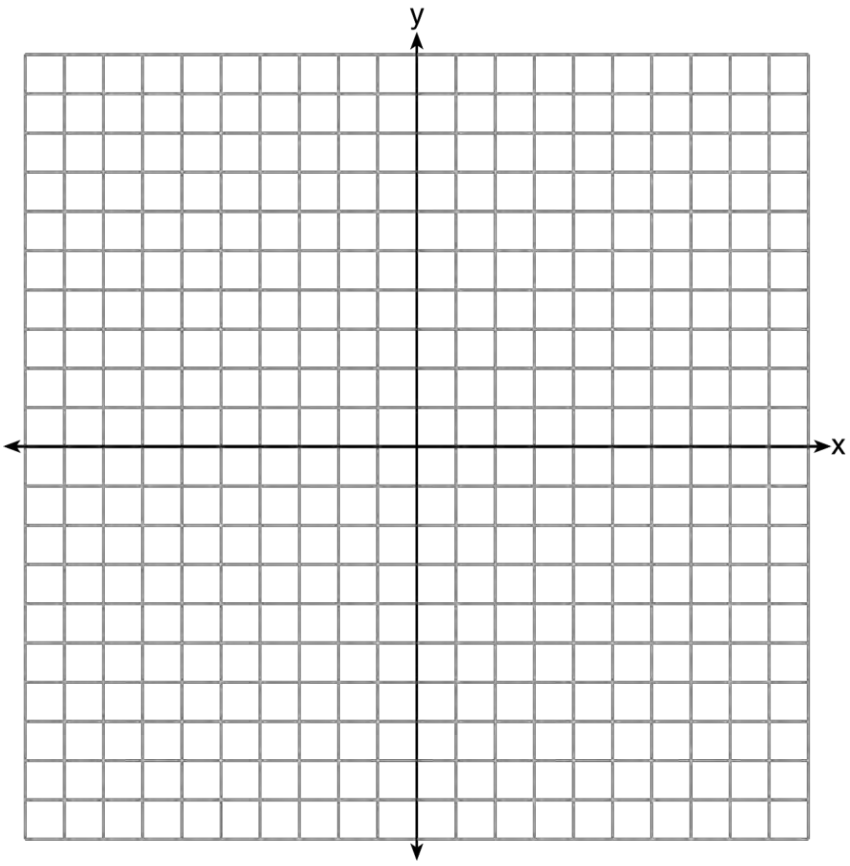
[0] The "given" and/or the "prove" statements are written, but no further relevant statements are written.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

11 In rhombus *MATH*, the coordinates of the endpoints of the diagonal $\overline{MT}$ are $M(0,-1)$ and $T(4,6)$. Write an equation of the line that contains diagonal $\overline{AH}$. [Use of the set of axes below is optional.]

Using the given information, explain how you know that your line contains diagonal $\overline{AH}$.



Key: See Rationale below.

Measures CCLS: G-GPE.B

Mathematical Practice: 1,3

Commentary: This question measures G-GPE.B because students are required to use the slope criteria for perpendicular lines to solve a geometric problem.

Rationale: Midpoint of $\overline{MT}$: $\left(\frac{4+0}{2}, \frac{6-1}{2}\right)$ $\left(2, \frac{5}{2}\right)$

Slope of $\overline{MT}$: $m = \frac{6--1}{4-0}$ $m = \frac{7}{4}$

Perpendicular slope: $\perp m = -\frac{4}{7}$

The diagonals, $\overline{MT}$ and $\overline{AH}$, of rhombus *MATH* are perpendicular bisectors of each other.

Rubric:

[4] $y - 2.5 = -\frac{4}{7}(x - 2)$ or equivalent equation, and a correct explanation is written.

[3] Appropriate work is shown, but one computational or graphing error is made.

or

[3] $y - 2.5 = -\frac{4}{7}(x - 2)$ is written, but the explanation is incorrect or missing.

[2] Appropriate work is shown, but one conceptual error is made.

or

[2] $-\frac{4}{7}$, the slope of $\overline{AH}$, and (2,2.5), the midpoint of the diagonals, are found, but no further correct work is shown.

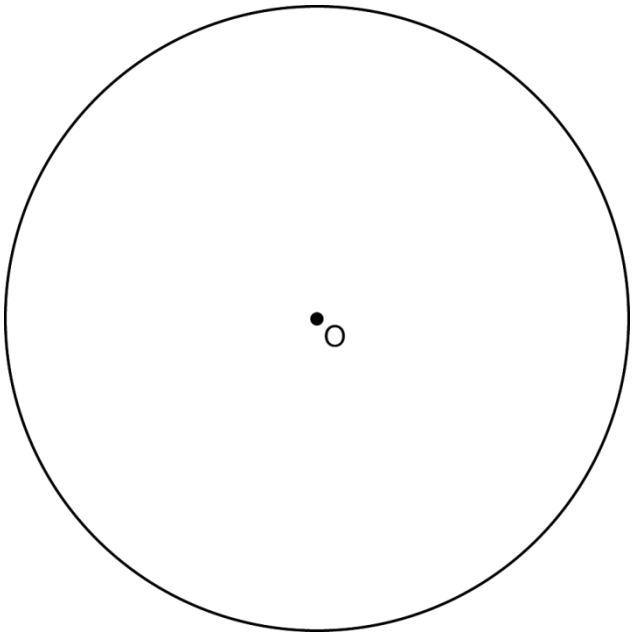
[1] Appropriate work is shown, but one computational or graphing error and one conceptual error are made.

or

[1] $-\frac{4}{7}$, slope of $\overline{AH}$, or (2, 2.5), the midpoint of the diagonals is found, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

12 Using a straightedge and compass, construct a square inscribed in circle *O* below.
[Leave all construction marks.]



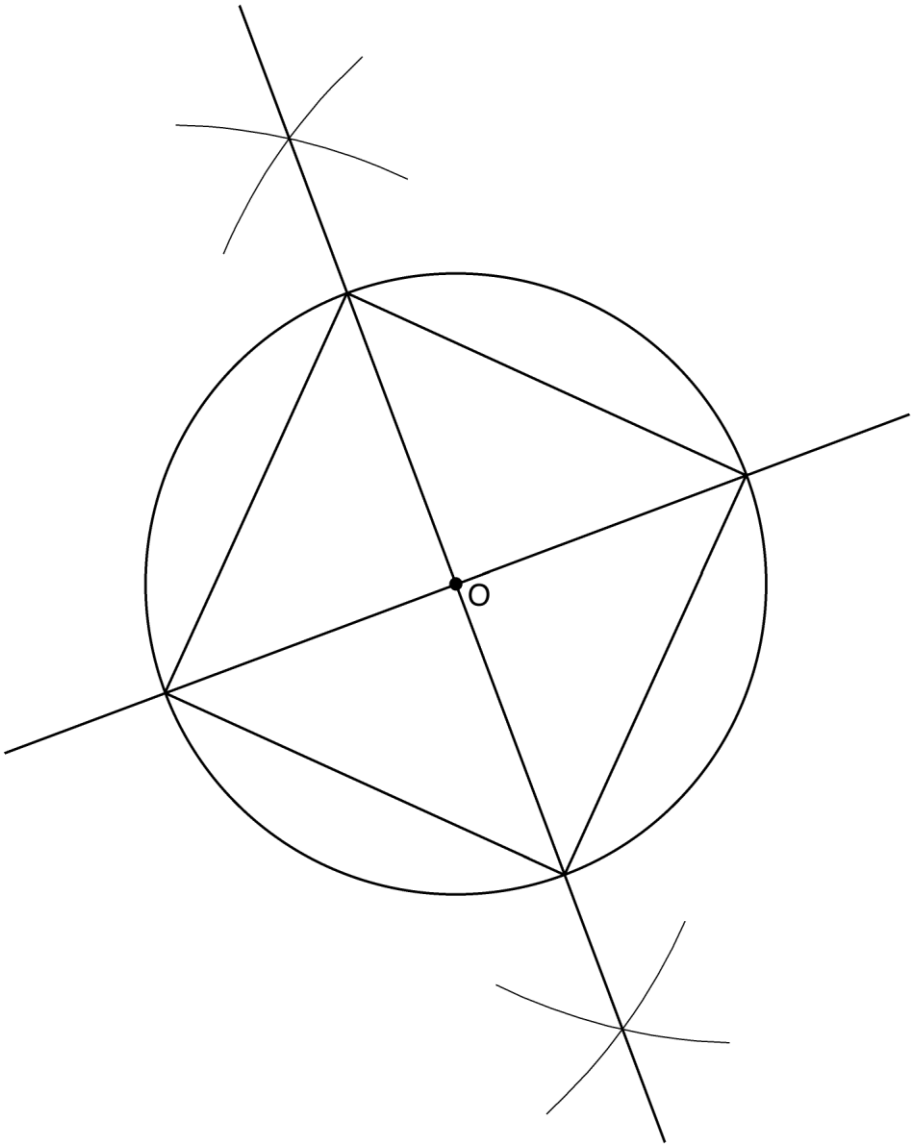
Determine the measure of the arc intercepted by two adjacent sides of the constructed square.
Explain your reasoning.

Key: See Rationale below.

Measures CCLS: G-CO.D

Commentary: This question measures G-CO.D because students are required to construct an inscribed square in a circle.

Rationale:



Since the square is inscribed, each vertex of the square is on the circle and the diagonals of the square are diameters of the circle, therefore, each angle of the square is an inscribed angle in the circle that intercepts the circle at the endpoints of the diameters. Each angle of the square, which is an inscribed angle, measures 90 degrees. Therefore, the measure of the arc intercepted by two adjacent sides of the square is 180 degrees because it's twice the measure of its inscribed angle.

Rubric:

[4] A correct construction is drawn showing all appropriate arcs, and 180°, and correct explanation is shown.

[3] 180° with correct explanation, but one construction error is shown.

or

[3] A correct construction is drawn showing all appropriate arcs, but one computational error is made.

[2] A correct construction is drawn showing all appropriate arcs, but no further correct work is shown.

or

[2] 180° with a correct explanation, but no further correct work is shown.

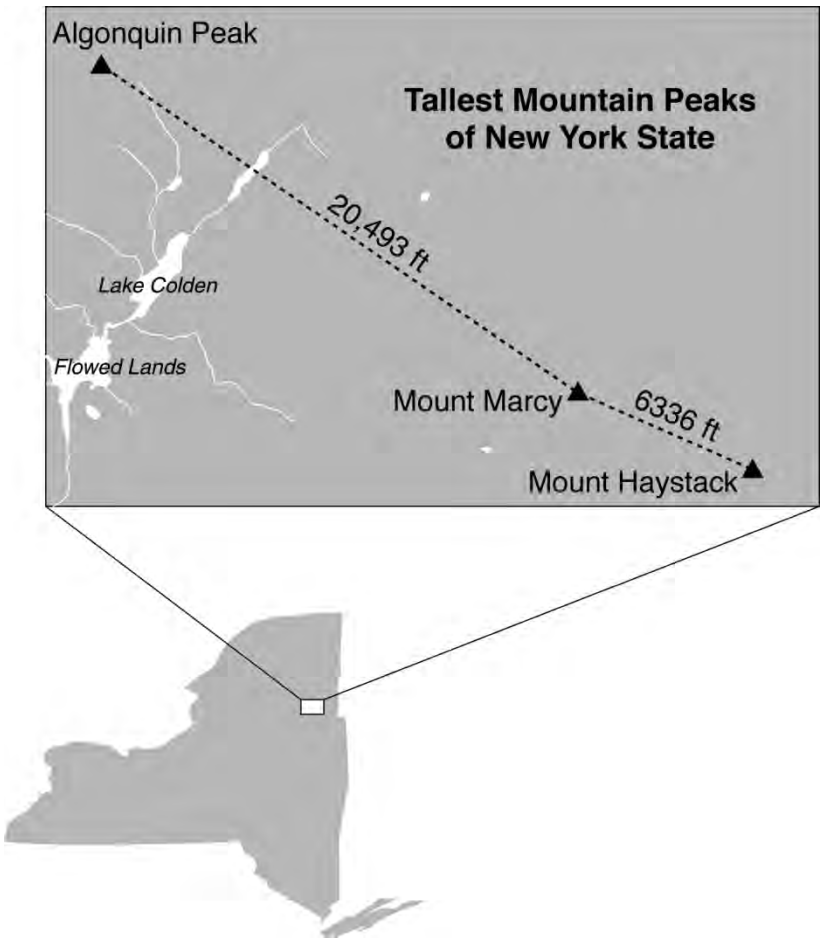
[1] 180°, but no further correct work is shown.

or

[1] One construction error is made, and no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

13 The map below shows the three tallest mountain peaks in New York State: Mount Marcy, Algonquin Peak, and Mount Haystack. Mount Haystack, the shortest peak, is 4960 feet tall. Surveyors have determined the horizontal distance between Mount Haystack and Mount Marcy is 6336 feet and the horizontal distance between Mount Marcy and Algonquin Peak is 20,493 feet.



The angle of depression from the peak of Mount Marcy to the peak of Mount Haystack is 3.47 degrees. The angle of elevation from the peak of Algonquin Peak to the peak of Mount Marcy is 0.64 degrees. What are the heights, to the *nearest foot*, of Mount Marcy and Algonquin Peak? Justify your answer.

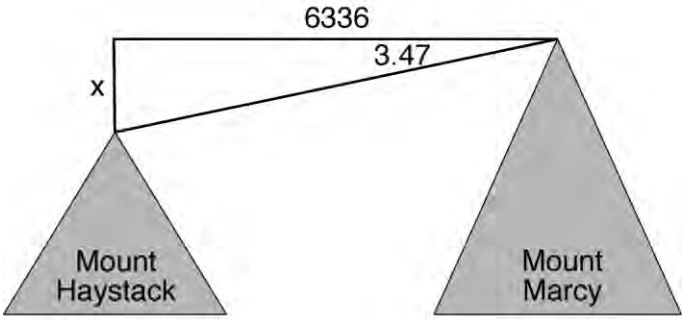
Key: See Rationale below.

Measures CCLS: G-SRT.C

Mathematical Practice: 4

Commentary: This question measures G-SRT.C because students are required to use trigonometric ratios to solve applied problems.

Rationale:

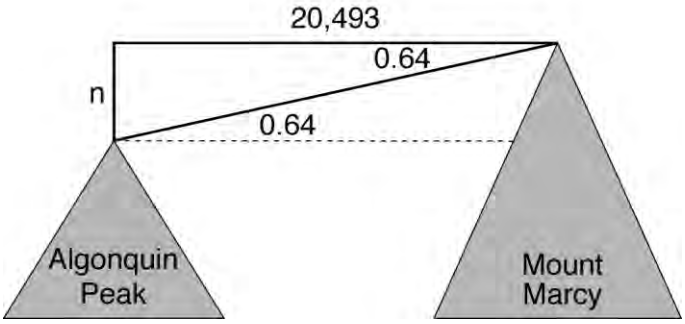


$\tan 3.47 = \frac{x}{6336}$

$x = 6336(\tan 3.47)$

Mount Marcy = 4960 + 6336(tan 3.47)

Mount Marcy = 5344



$\tan 0.64 = \frac{n}{20,493}$

$n = 20,493(\tan 0.64)$

Algonquin Peak = 5344 – 20,493(tan 0.64)

Algonquin Peak = 5115

Rubric:

[6] 5344, the height of Mount Marcy, and 5115, the height of Algonquin, and correct work is shown.

[5] Appropriate work is shown, but one computational or rounding error is made.

[4] Appropriate work is shown, but two computational or rounding errors are made.

or

[4] Appropriate work is shown, but one conceptual error is made.

[3] Appropriate work is shown, but three or more computational or rounding errors are made.

or

[3] Appropriate work is shown, but one conceptual error and one computational or rounding error are made.

or

[3] Appropriate work is shown to find 5344, but no further correct work is shown.

[2] Appropriate work is shown, but two conceptual errors are made.

or

[2] Appropriate work is shown to find 384 or 229, the height differences between Mount Marcy and other two mountains, but no further correct work is shown.

[1] $\tan 3.47 = \frac{x}{6336}$ or $\tan 0.64 = \frac{n}{20,493}$ is written, but no further correct work is shown.

or

[1] Correct diagrams are drawn and labeled showing the height relationships between Mount Marcy and Mount Haystack and Algonquin Peak, but no further correct work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.