The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, August 13, 2009—8:30 to 11:30 a.m., only

Student Name: *Steve Sibol*

School Name: *jmap.org*

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [56]
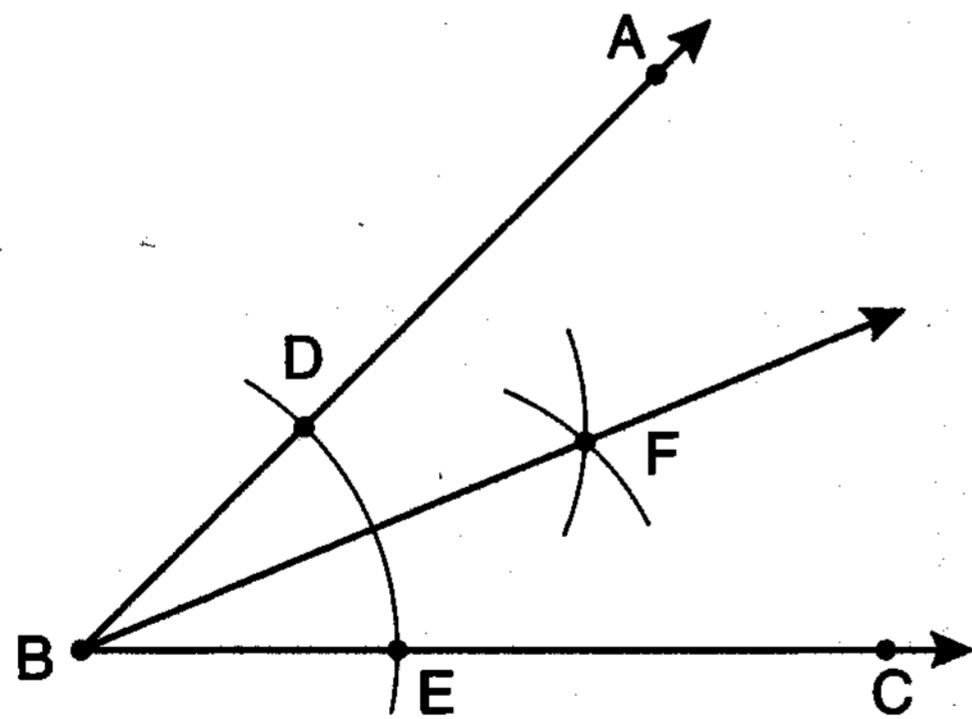
Use this space for computations.

1 Based on the diagram below, which statement is true?
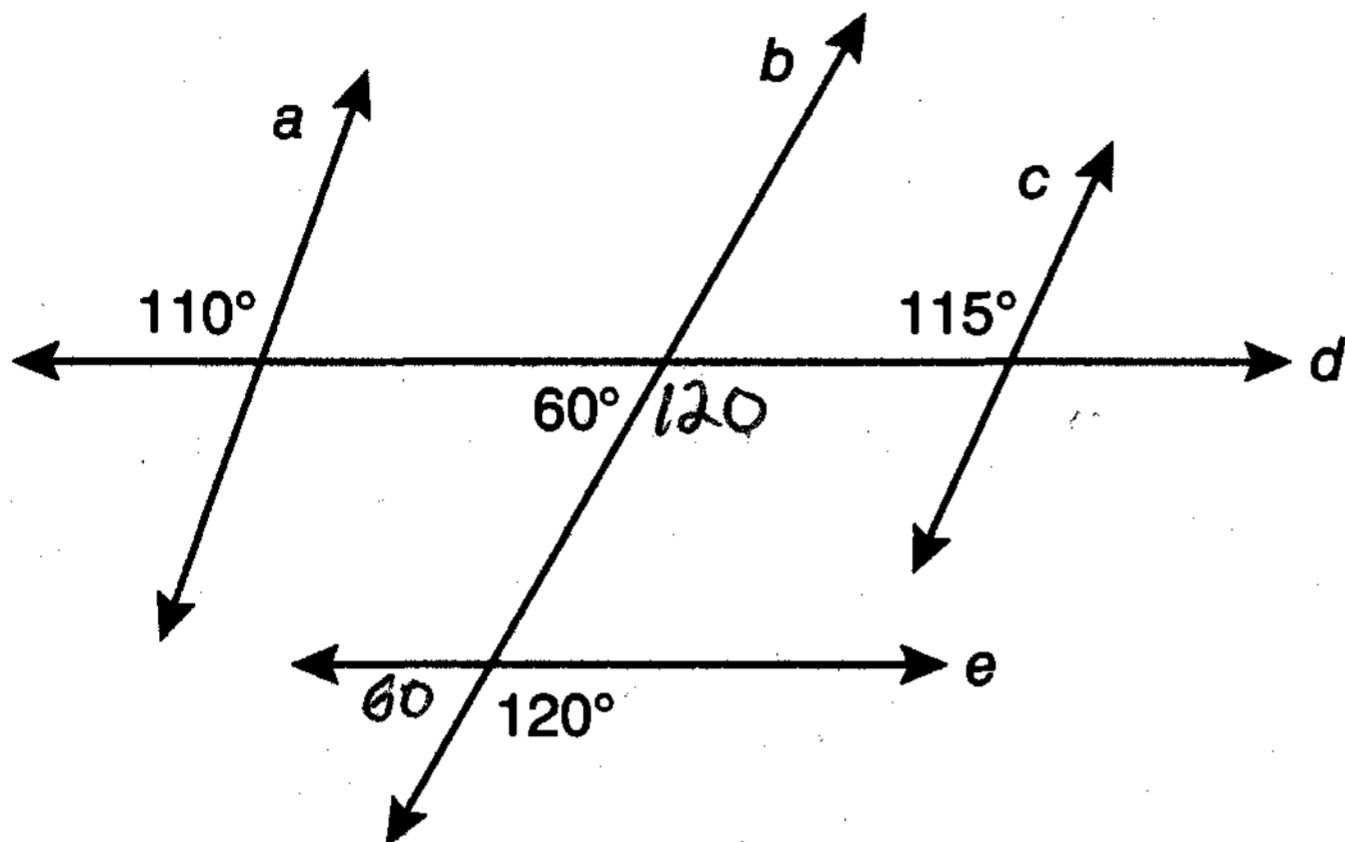
- (1) $a \parallel b$
- (2) $a \parallel c$
- (3) $b \parallel c$
- (4) $d \parallel e$

2 The diagram below shows the construction of the bisector of $\angle ABC$.
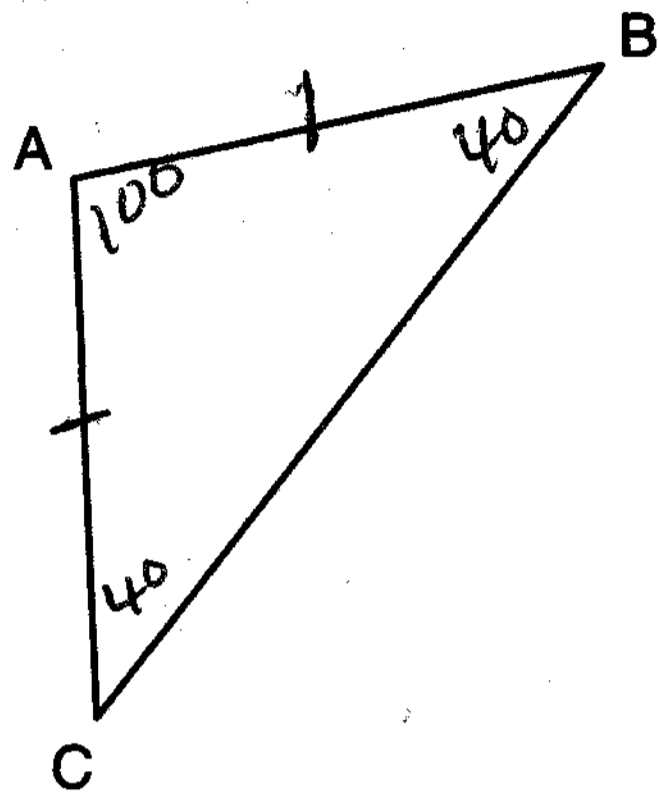

Which statement is *not* true?

- (1) $m\angle EBF = \frac{1}{2} m\angle ABC$
- (2) $m\angle DBF = \frac{1}{2} m\angle ABC$
- (3) $m\angle EBF = m\angle ABC$
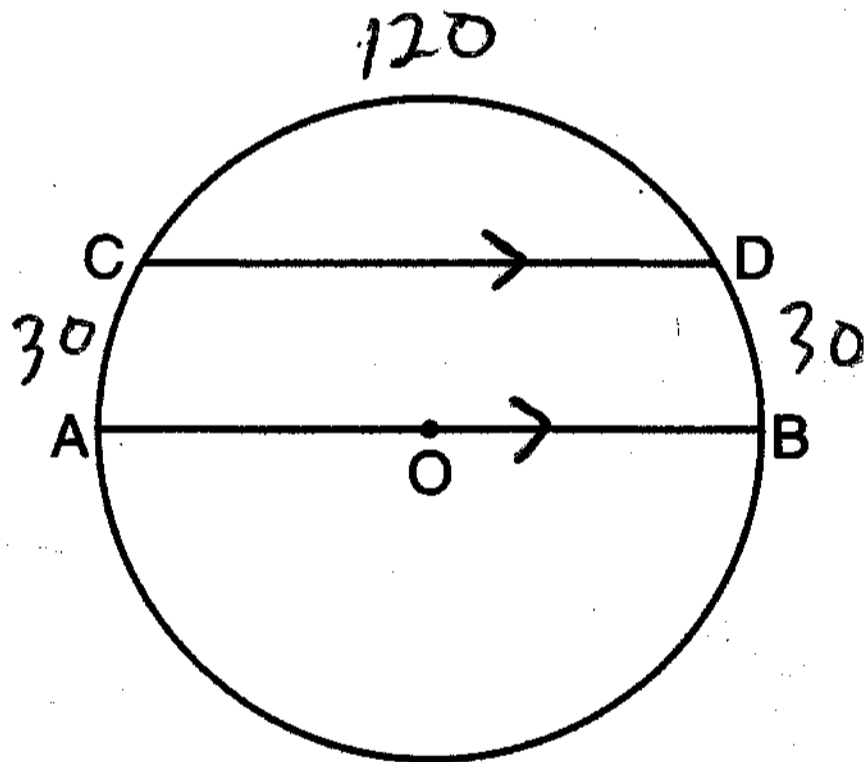- (4) $m\angle DBF = m\angle EBF$

3 In the diagram of $\triangle ABC$ below, $\overline{AB} \cong \overline{AC}$. The measure of $\angle B$ is 40°.

What is the measure of $\angle A$?

(1) 40° (3) 70°

(2) 50° (4) 100°

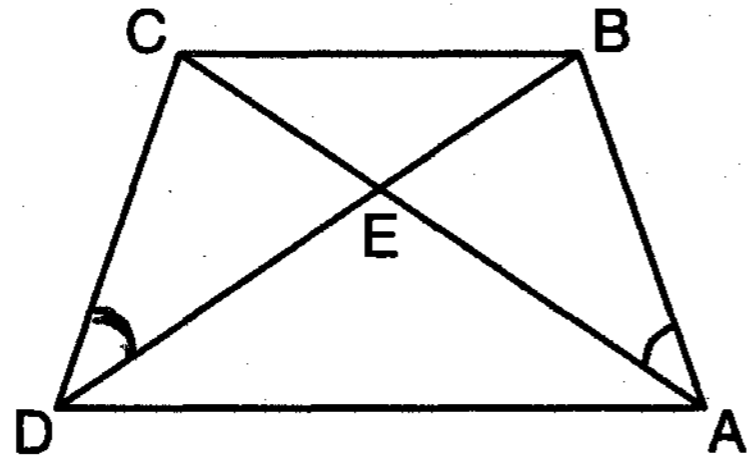**4** In the diagram of circle O below, chord $\overline{CD}$ is parallel to diameter $\overline{AOB}$ and $m\overset{\frown}{AC} = 30$.

What is $m\overset{\frown}{CD}$?

(1) 150 (3) 100

(2) 120 (4) 60

5 In the diagram of trapezoid *ABCD* below, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at *E* and $\triangle ABC \cong \triangle DCB$.
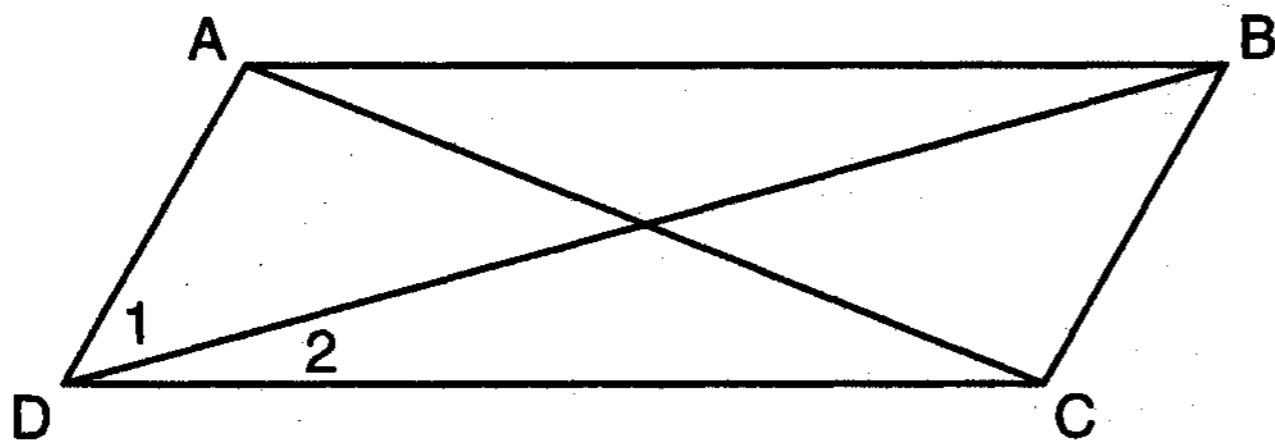
Which statement is true based on the given information?

(1) $\overline{AC} \cong \overline{BC}$ (3) $\angle CDE \cong \angle BAD$

(2) $\overline{CD} \cong \overline{AD}$ (4) $\angle CDB \cong \angle BAC$

6 Which transformation produces a figure similar but *not* congruent to the original figure?

(1) $T_{1,3}$ (3) $R_{90°}$

(2) $D_{\frac{1}{2}}$ (4) $r_{y=x}$

7 In the diagram below of parallelogram *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$, m$\angle 1 = 45$ and m$\angle DCB = 120$. } *consecutive angles are supplementary*
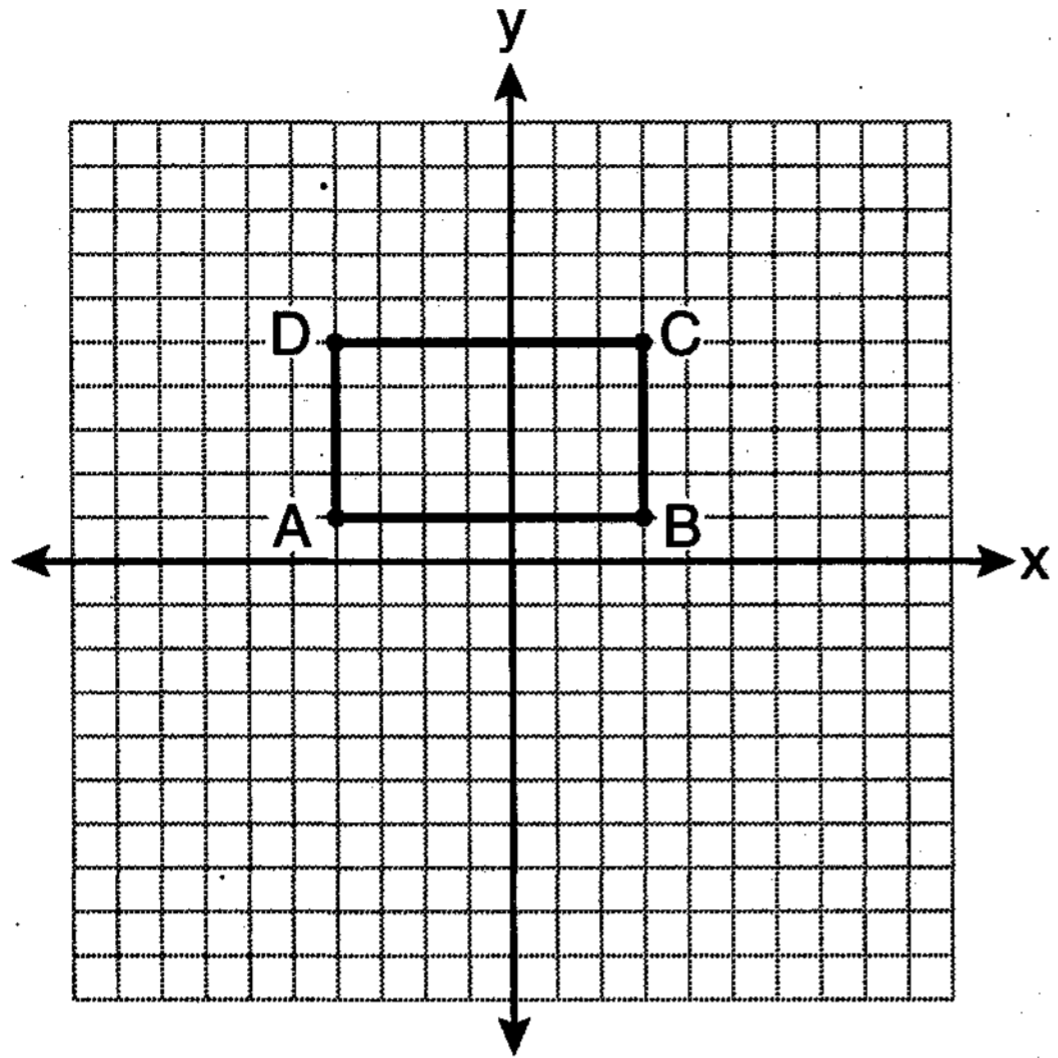
m$\angle CDA = 60$ }

What is the measure of $\angle 2$?

(1) 15°   (3) 45°

(2) 30°   (4) 60°

8 On the set of axes below, Geoff drew rectangle $ABCD$. He will transform the rectangle by using the translation $(x,y) \rightarrow (x + 2, y + 1)$ and then will reflect the translated rectangle over the x-axis.

Distance is preserved in translations and reflections (handwritten)

What will be the area of the rectangle after these transformations?

(1) exactly 28 square units

(2) less than 28 square units

(3) greater than 28 square units

(4) It cannot be determined from the information given.

9 What is the equation of a line that is parallel to the line whose equation is $y = x + 2$?

(1) $x + y = 5$

(3) $y - x = -1$

(2) $2x + y = -2$

(4) $y - 2x = 3$

10 The endpoints of $\overline{CD}$ are $C(-2,-4)$ and $D(6,2)$. What are the coordinates of the midpoint of $\overline{CD}$?

(1) $(2,3)$

(3) $(4,-2)$

(2) $(2,-1)$

(4) $(4,3)$

11 What are the center and the radius of the circle whose equation is $(x - 3)^2 + (y + 3)^2 = 36$?

(1) center = $(3,-3)$; radius = 6

(2) center = $(-3,3)$; radius = 6

(3) center = $(3,-3)$; radius = 36

(4) center = $(-3,3)$; radius = 36

12 Given the equations:

$$y = x^2 - 6x + 10$$

$$y + x = 4$$

$y = -x + 4$

What is the solution to the given system of equations?

(1) $(2,3)$ (3) $(2,2)$ and $(1,3)$

(2) $(3,2)$ (4) $(2,2)$ and $(3,1)$

Handwritten computations:

$x^2 - 6x + 10 = -x + 4$

$x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 3 \quad x = 2$

$y + x = 4$
$y + 3 = 4$
$y = 1$

$y + x = 4$
$y + 2 = 4$
$x = 2$

13 The diagonal $\overline{AC}$ is drawn in parallelogram $ABCD$. Which method can *not* be used to prove that $\triangle ABC \cong \triangle CDA$?

(1) SSS   (3) SSA

(2) SAS   (4) ASA

14 In the diagram below, line k is perpendicular to plane P at point T.
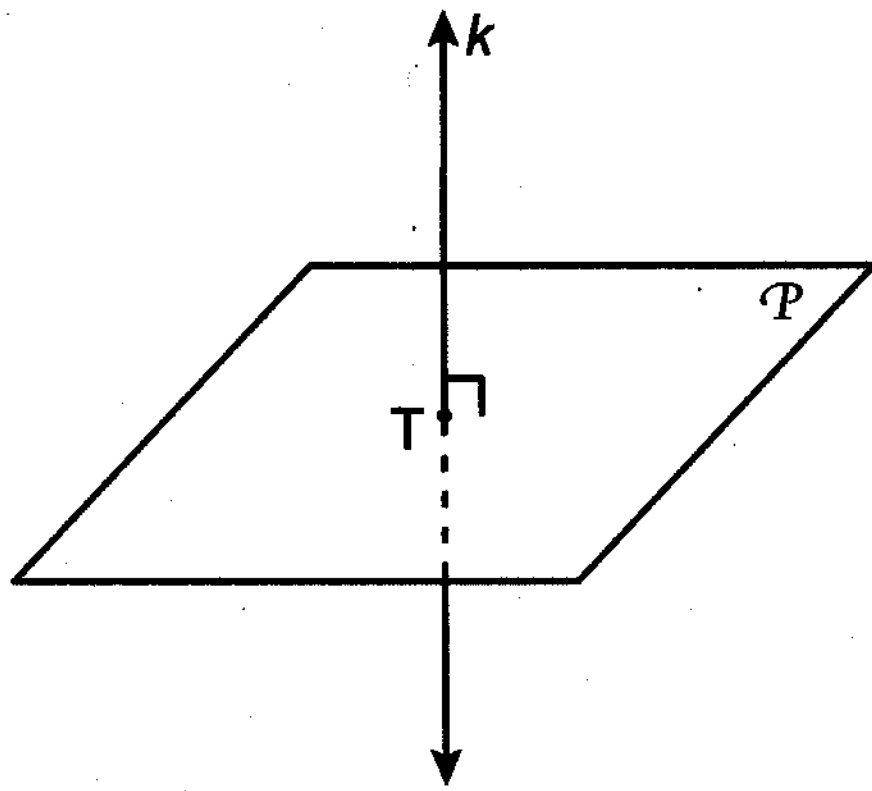
Which statement is true?

(1) Any point in plane P also will be on line k.

(2) Only one line in plane P will intersect line k.

(3) All planes that intersect plane P will pass through T.

(4) Any plane containing line k is perpendicular to plane P.

15 In the diagram below, which transformation was used to map $\triangle ABC$ to $\triangle A'B'C'$?
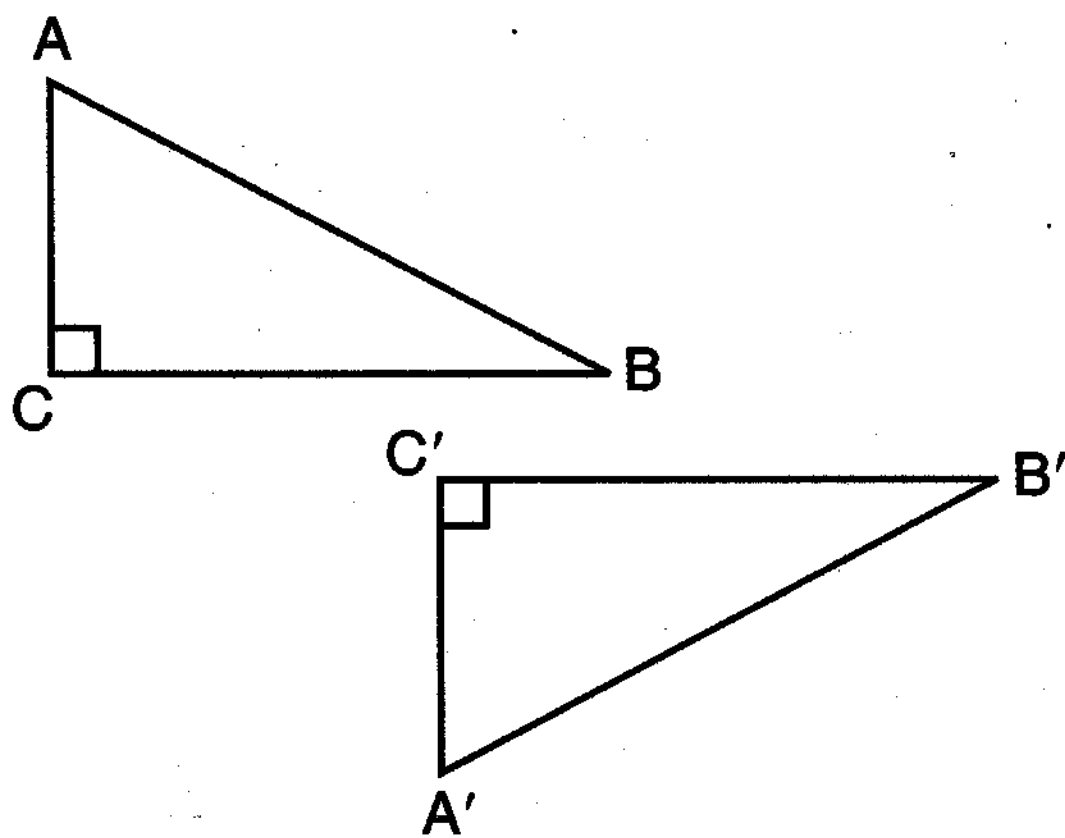
(1) dilation   (3) reflection

(2) rotation   (4) glide reflection

16 Which set of numbers represents the lengths of the sides of a triangle?

(1) {5, 18, 13}   (3) {16, 24, 7}

(2) {6, 17, 22}   (4) {26, 8, 15}

$6 + 17 > 22$

17 What is the slope of a line perpendicular to the line whose equation is $y = -\frac{2}{3}x - 5$?

(1) $-\frac{3}{2}$ (3) $\frac{2}{3}$

(2) $-\frac{2}{3}$ (4) $\frac{3}{2}$

18 A quadrilateral whose diagonals bisect each other and are perpendicular is a

(1) rhombus   (3) trapezoid

(2) rectangle   (4) parallelogram

19 If the endpoints of $\overline{AB}$ are $A(-4,5)$ and $B(2,-5)$, what is the length of $\overline{AB}$?

(1) $2\sqrt{34}$ (3) $\sqrt{61}$

(2) 2   (4) 8

$\sqrt{(-4-2)^2 + (5-(-5))^2} = \sqrt{136}$

$\sqrt{4} \cdot \sqrt{34}$

$2\sqrt{34}$

20 In the diagram below of $\triangle ACT$, D is the midpoint of $\overline{AC}$, O is the midpoint of $\overline{AT}$, and G is the midpoint of $\overline{CT}$.

If $AC = 10$, $AT = 18$, and $CT = 22$, what is the perimeter of parallelogram $CDOG$?

(1) 21
(3) 32
(2) 25
(4) 40

21 Which equation represents circle K shown in the graph below?

(1) $(x + 5)^2 + (y - 1)^2 = 3$ (3) $(x - 5)^2 + (y + 1)^2 = 3$
(2) $(x + 5)^2 + (y - 1)^2 = 9$ (4) $(x - 5)^2 + (y + 1)^2 = 9$

22 In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.
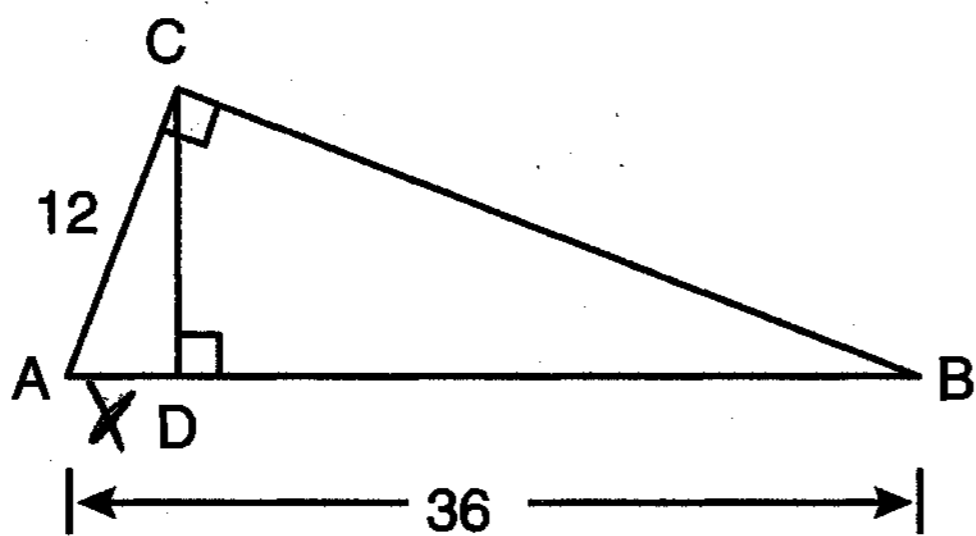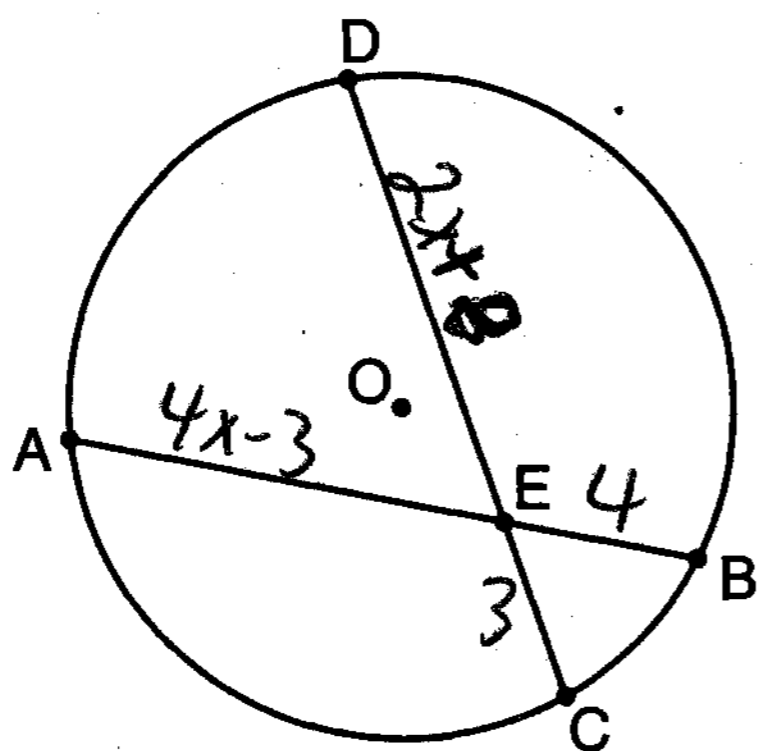
$$36x = 12^2$$
$$x = 4$$

If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?

(1) 32

(3) 3

(2) 6

(4) 4 ⊛

23 In the diagram of circle O below, chord $\overline{AB}$ intersects chord $\overline{CD}$ at E, $DE = 2x + 8$, $EC = 3$, $AE = 4x - 3$, and $EB = 4$.

$$4(4x-3) = 3(2x+8)$$
$$16x - 12 = 6x + 24$$
$$10x = 36$$
$$x = 3.6$$

What is the value of x?

(1) 1

(3) 5

(2) 3.6 ⊛

(4) 10.25

24 What is the negation of the statement "Squares are parallelograms"?

(1) Parallelograms are squares.

(2) Parallelograms are not squares.

(3) It is not the case that squares are parallelograms. ⊛

(4) It is not the case that parallelograms are squares.

25 The diagram below shows the construction of the center of the circle circumscribed about △*ABC*.
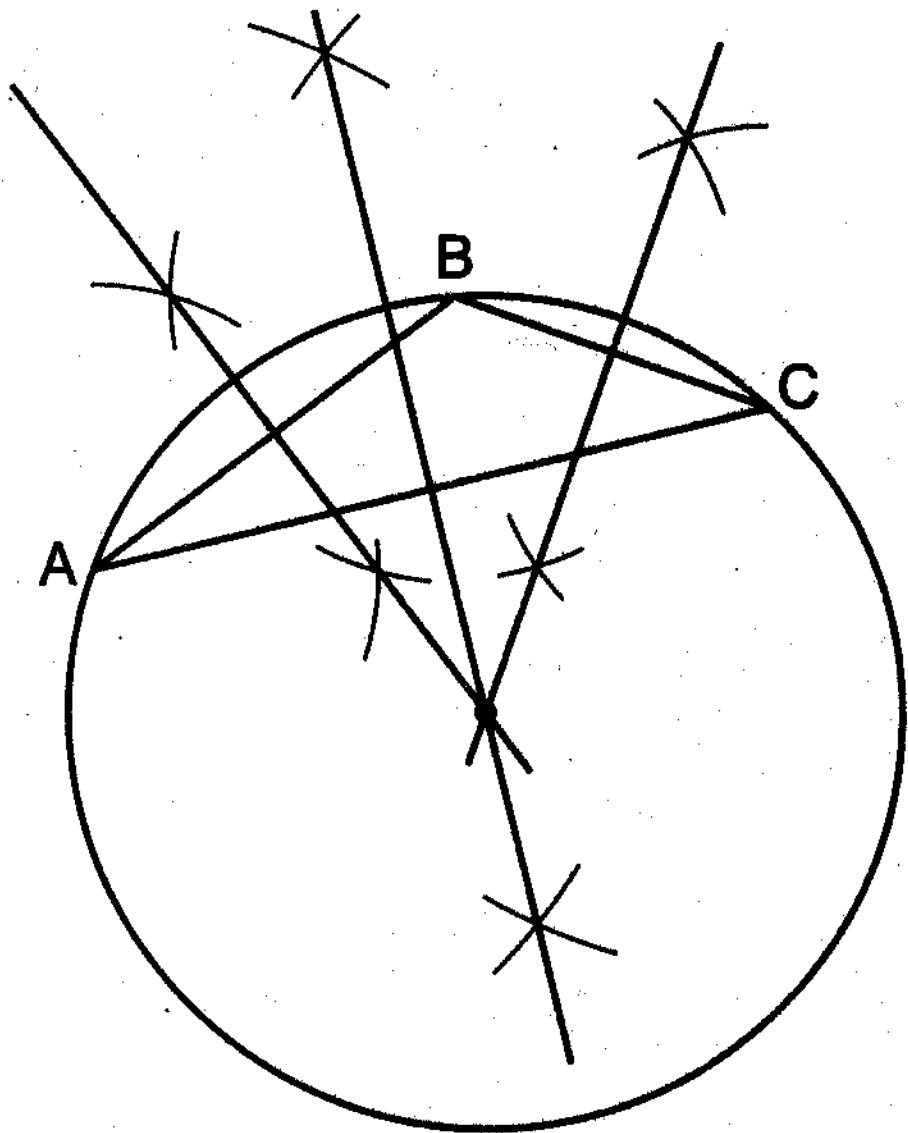
This construction represents how to find the intersection of

(1) the angle bisectors of △*ABC*

(2) the medians to the sides of △*ABC*

(3) the altitudes to the sides of △*ABC*

(4) the perpendicular bisectors of the sides of △*ABC*

26 A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?
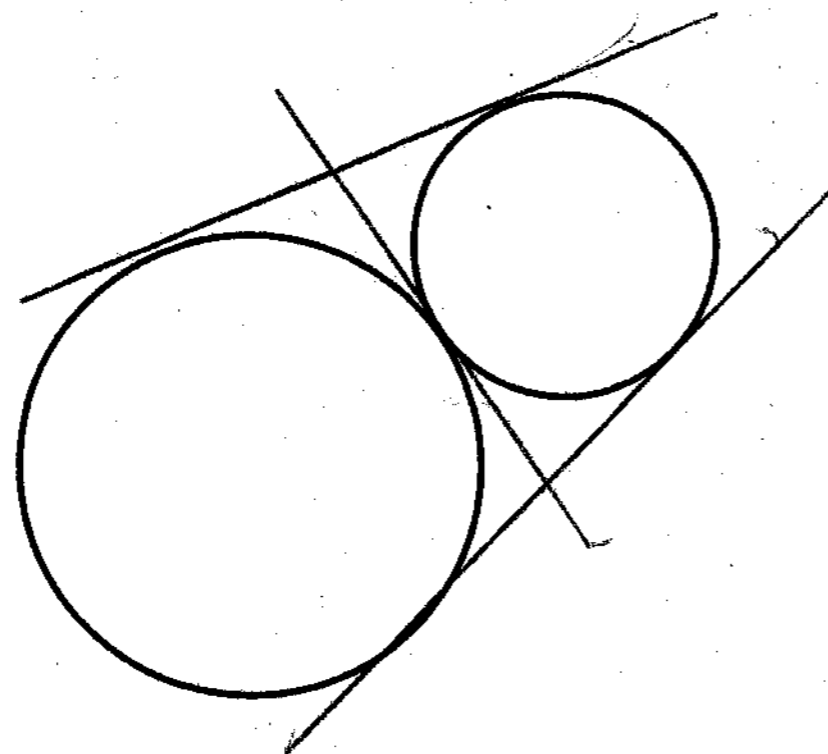
(1) 6.3   (3) 19.8

(2) 11.2   (4) 39.8

$V = \pi r^2 h$

$1000 = \pi r^2 8$

$6.3 \approx r$

27 If two different lines are perpendicular to the same plane, they are

(1) collinear   (3) congruent

(2) coplanar   (4) consecutive

28 How many common tangent lines can be drawn to the two externally tangent circles shown below?

(1) 1   (3) 3
(2) 2   (4) 4

Part II

Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 In the diagram below of isosceles trapezoid $DEFG$, $\overline{DE} \parallel \overline{GF}$, $DE = 4x - 2$, $EF = 3x + 2$, $FG = 5x - 3$, and $GD = 2x + 5$. Find the value of x.

```
        D    4x − 2    E
         /‾‾‾‾‾‾‾‾‾‾\
2x + 5  /            \  3x + 2
       /              \
      G‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾‾F
            5x − 3
```
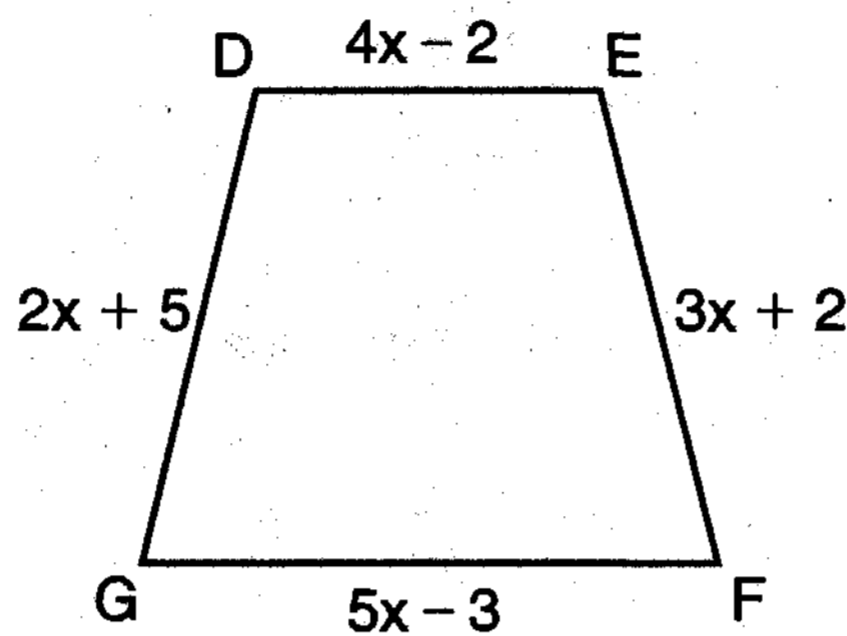
$$2x + 5 = 3x + 2$$
$$3 = x$$

30 A regular pyramid with a square base is shown in the diagram below.
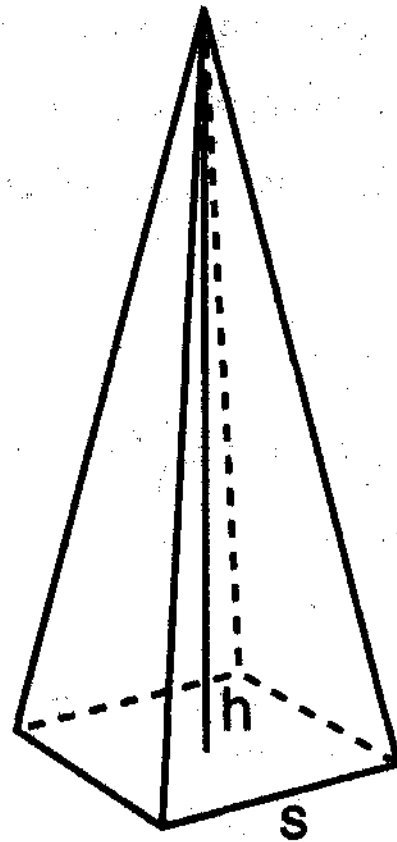
A side, s, of the base of the pyramid is 12 meters, and the height, h, is 42 meters. What is the volume of the pyramid in cubic meters?

$$V = \frac{1}{3} Bh$$

$$= \frac{1}{3} \cdot 12^2 \cdot 42$$

$$= 2016$$

31 Write an equation of the line that passes through the point $(6, -5)$ and is parallel to the line whose equation is $2x - 3y = 11$.

$$m = \frac{-A}{B} = \frac{-2}{-3} = \frac{2}{3}$$

Slope/Intercept

$$y = mx + b$$

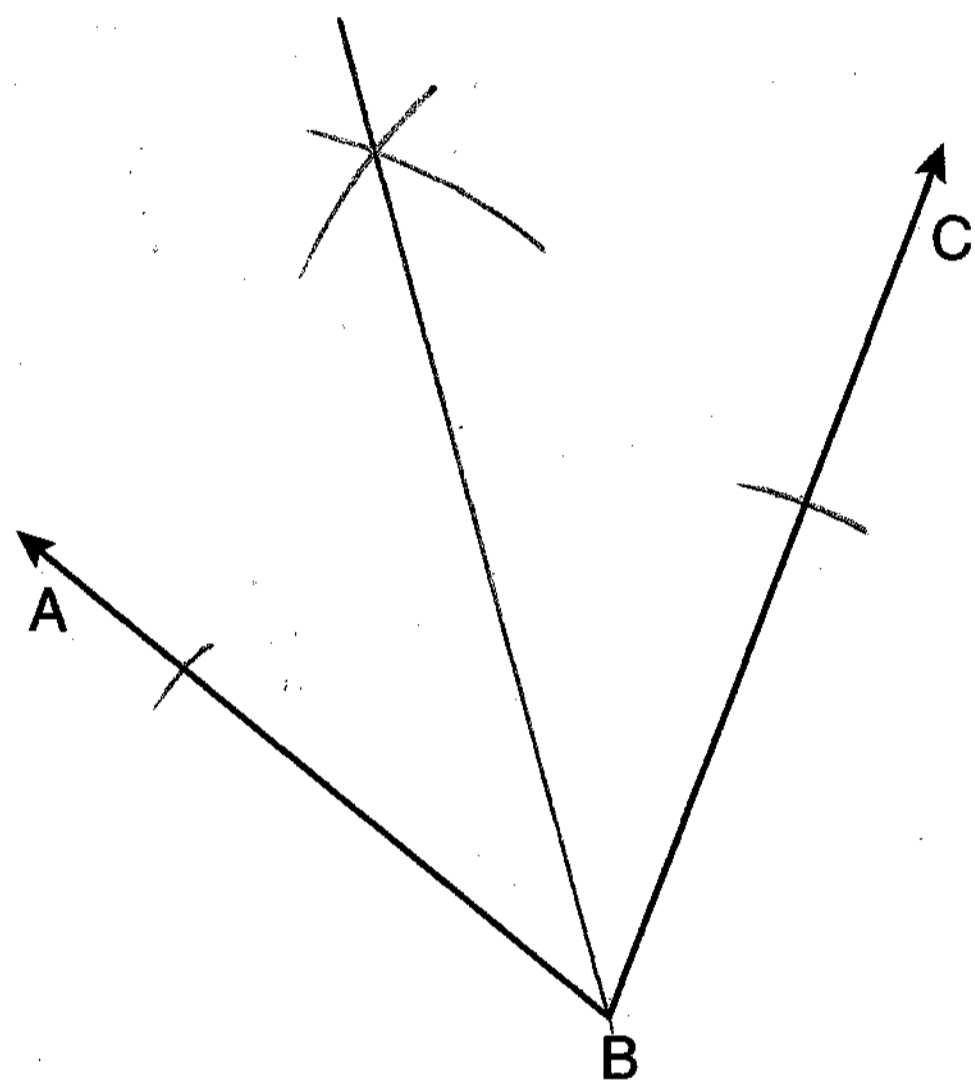$$-5 = \frac{2}{3}(6) + b$$

$$-5 = 4 + b$$

$$-9 = b$$

$$y = \frac{2}{3}x - 9$$

Point/Slope

$$m(x - x_1) = y - y_2$$

$$\frac{2}{3}(x - 6) = y + 5$$

32 Using a compass and straightedge, construct the angle bisector of $\angle ABC$ shown below. [Leave all construction marks.]
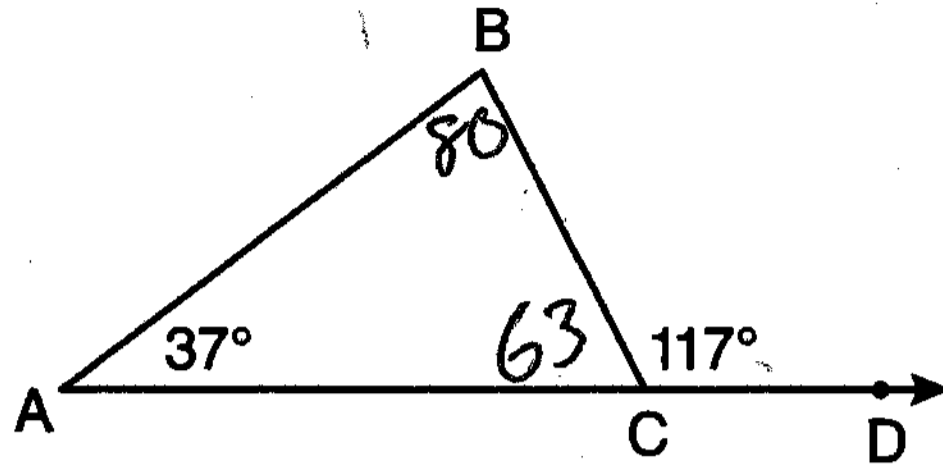
33 The degree measures of the angles of $\triangle ABC$ are represented by x, $3x$, and $5x - 54$. Find the value of x.

$$x + 3x + 5x - 54 = 180$$
$$9x = 234$$
$$x = 26$$

34 In the diagram below of $\triangle ABC$ with side $\overline{AC}$ extended through D, $m\angle A = 37$ and $m\angle BCD = 117$. Which side of $\triangle ABC$ is the longest side? Justify your answer.

B

80

37°   63   117°

A   C   D

(Not drawn to scale)

$\overline{AC}$ because it is
opposite the largest angle

Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Write an equation of the perpendicular bisector of the line segment whose endpoints are $(-1,1)$ and $(7,-5)$. [The use of the grid below is optional.]

$$\text{midpoint } \left(\frac{-1+7}{2}, \frac{1+-5}{2}\right) = (3, -2)$$

$$\text{slope } \frac{1--5}{-1-7} = \frac{6}{-8} = -\frac{3}{4}$$

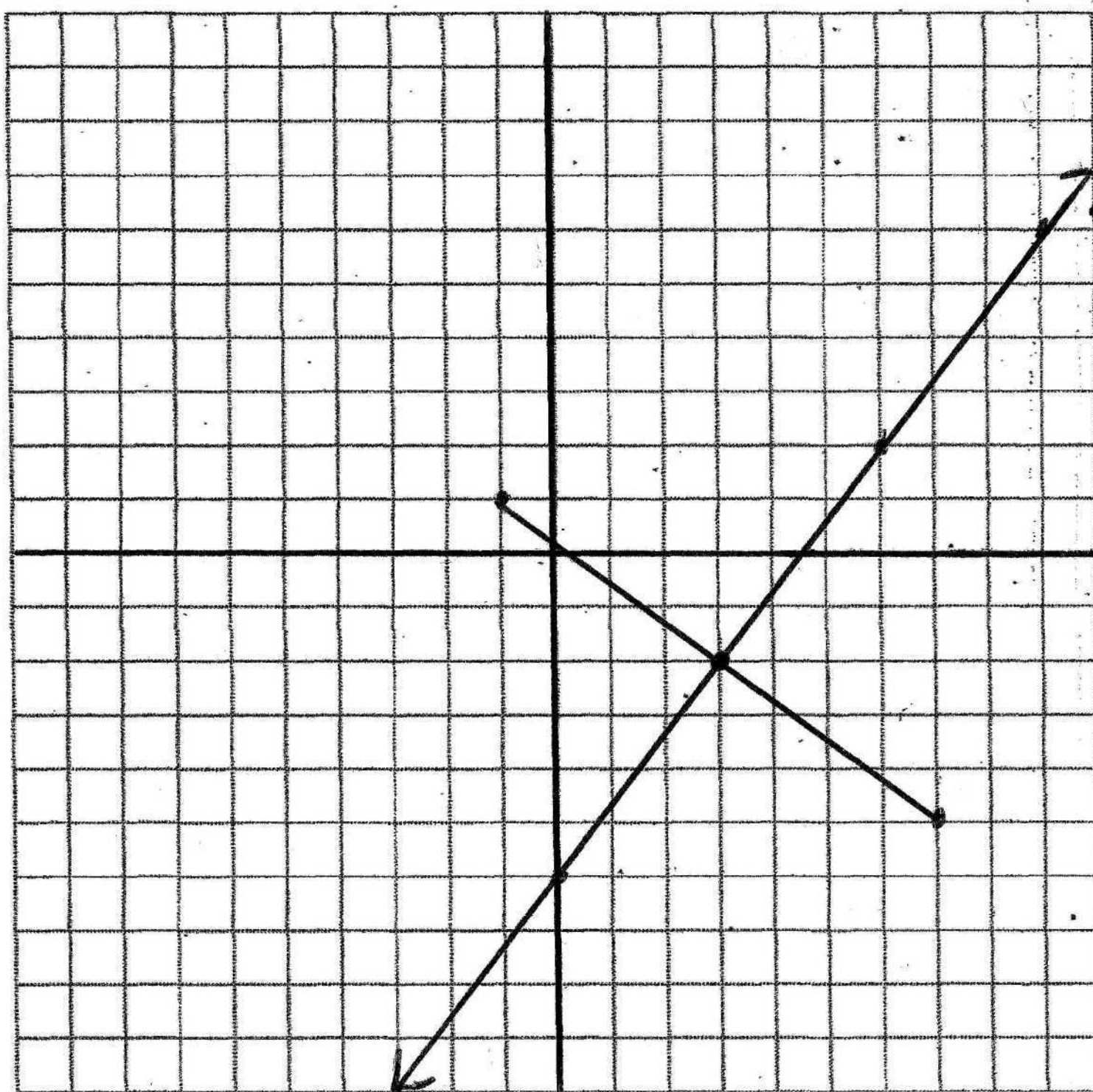$$\text{slope}_\perp = \frac{4}{3}$$

Slope/Intercept
$$y = mx + b$$
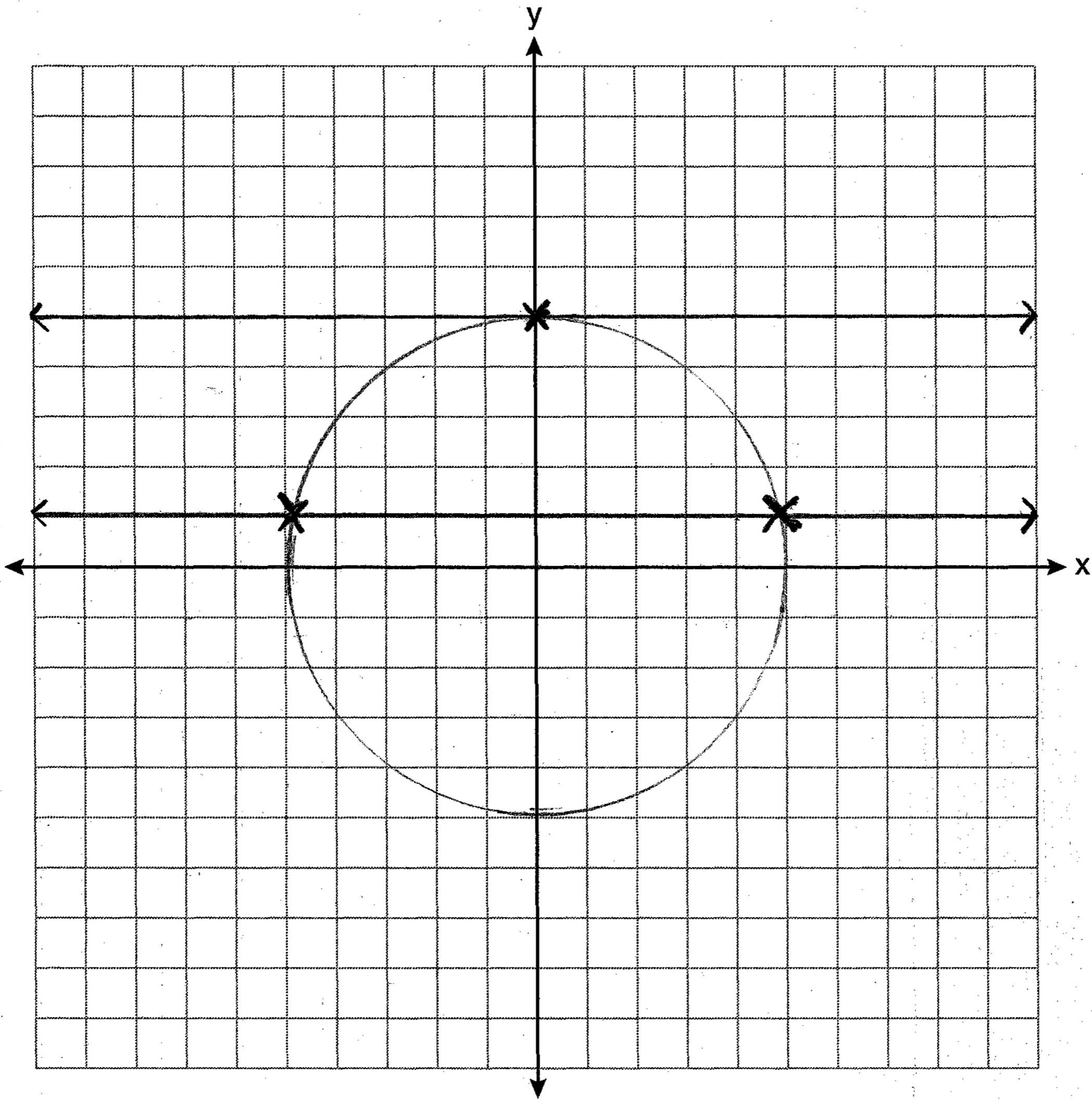$$-2 = \left(\frac{4}{3}\right)3 + b$$
$$-6 = b \qquad y = \frac{4}{3}x - 6$$

Point/Slope
$$\frac{4}{3}(x-3) = y + 2$$

36 On the set of axes below, sketch the points that are 5 units from the origin and sketch the points that are 2 units from the line $y = 3$. Label with an **X** all points that satisfy *both* conditions.

37 Triangle *DEG* has the coordinates *D*(1,1), *E*(5,1), and *G*(5,4). Triangle *DEG* is rotated 90° about the origin to form △*D'E'G'*. On the grid below, graph and label △*DEG* and △*D'E'G'*. State the coordinates of the vertices *D'*, *E'*, and *G'*. Justify that this transformation preserves distance.

(-1,1) (-1,5) (-4,5)
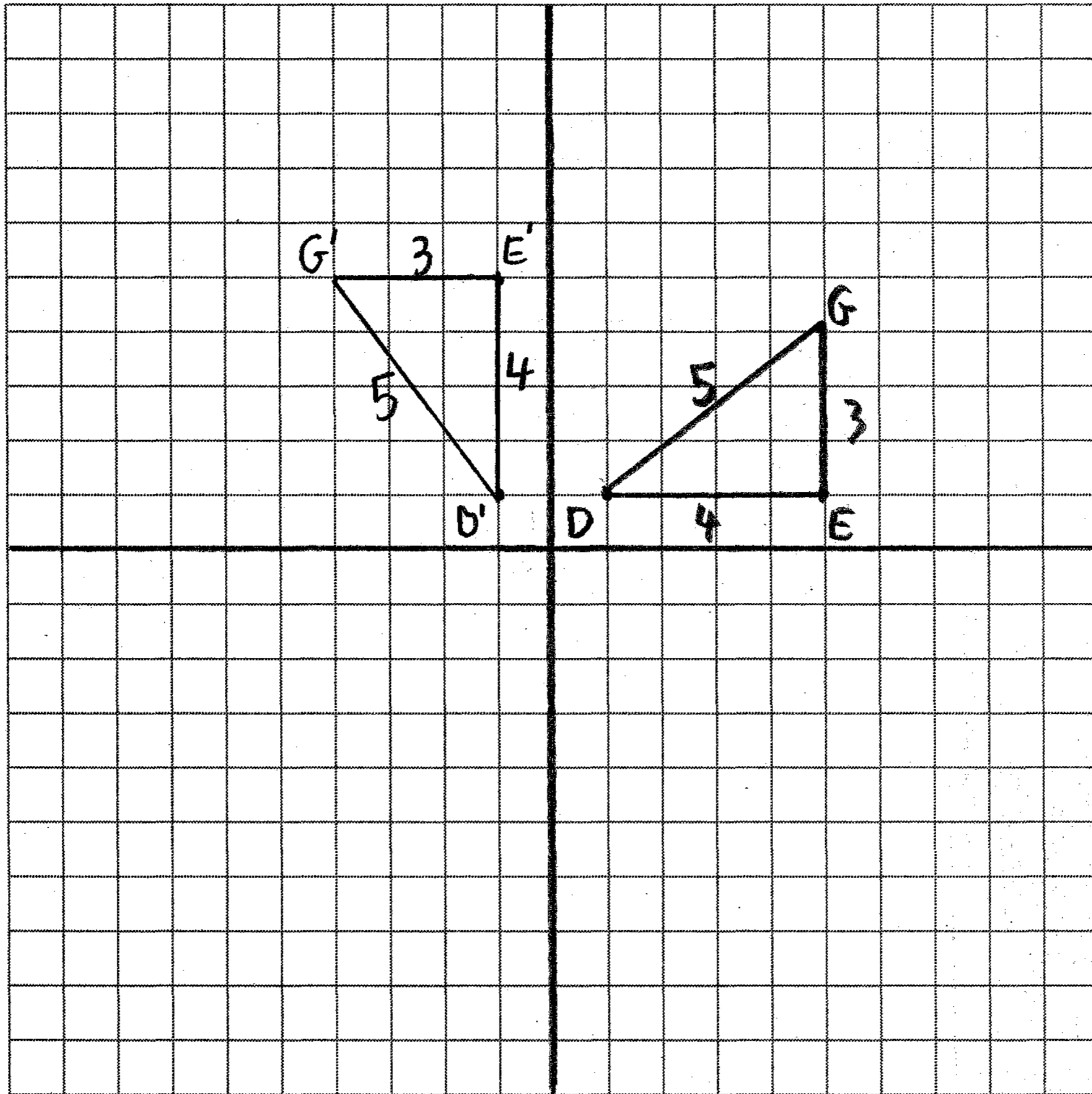
Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

38 Given: Quadrilateral $ABCD$, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$

Prove: $ABCD$ is a parallelogram.
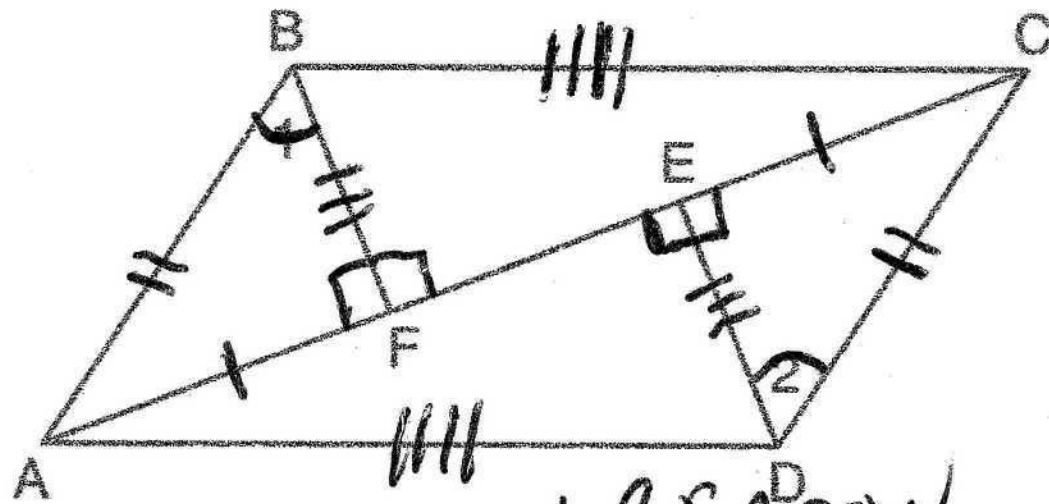
STATEMENT	REASON
① Quadrilateral $ABCD$, diagonal $\overline{AFEC}$, $\overline{AE} \cong \overline{FC}$, $\overline{BF} \perp \overline{AC}$, $\overline{DE} \perp \overline{AC}$, $\angle 1 \cong \angle 2$	① Given
② $\overline{FE} \cong \overline{EF}$	② Reflexive Property
③ $\overline{AE} - \overline{FE} = \overline{CF} - \overline{EF}$	③ Line Segment Subtraction Theorem
④ $\overline{AF} \cong \overline{CE}$	④ Substitution
⑤ $\angle BFA \cong \angle DEC$	⑤ All right angles are congruent
⑥ $\triangle BFA \cong \triangle DEC$	⑥ AAS
⑦ $\overline{AB} \cong \overline{CD}$	⑦ CPCTC
⑧ $\overline{BF} \cong \overline{DE}$	⑧ CPCTC
⑨ $\angle BFC \cong \angle DEA$	⑨ All right angles are congruent
⑩ $\triangle BFC \cong \triangle DEA$	⑩ SAS
⑪ $\overline{AD} \cong \overline{CB}$	⑪ CPCTC
⑫ $ABCD$ is a parallelogram	⑫ Opposite sides of the quadrilateral are congruent