The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Thursday, January 26, 2012 — 9:15 a.m. to 12:15 p.m., only

Student Name: Mr. Sibol

School Name: JMAP

Print your name and the name of your school on the lines above. Then turn to the last page of this booklet, which is the answer sheet for Part I. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

This examination has four parts, with a total of 38 questions. You must answer all questions in this examination. Write your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in Parts II, III, and IV directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will *not* be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice...

A graphing calculator, a straightedge (ruler), and a compass must be available for you to use while taking this examination.

The use of any communications device is strictly prohibited when taking this examination. If you use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

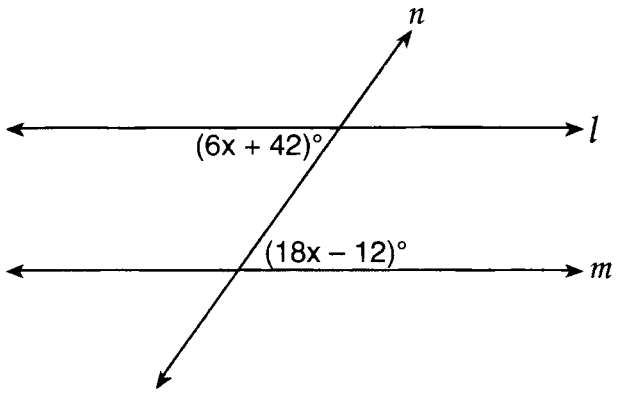
DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 28 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. For each question, write on the separate answer sheet the numeral preceding the word or expression that best completes the statement or answers the question. [56]

Use this space for computations.

1 Line n intersects lines l and m, forming the angles shown in the diagram below.



$6x+42 = 18x-12$
$42 = 12x-12$
$54 = 12x$
$\frac{54}{12} = x$
$\frac{9}{2} = x$

Which value of x would prove $l \parallel m$?

(1) 2.5
(2) 4.5
(3) 6.25
(4) 8.75

2 In a given triangle, the point of intersection of the three medians is the same as the point of intersection of the three altitudes. Which classification of the triangle is correct?

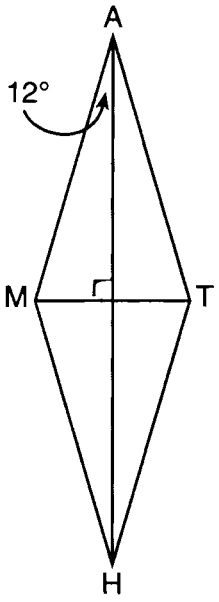
(1) scalene triangle
(2) isosceles triangle
(3) equilateral triangle
(4) right isosceles triangle

3 A circle has the equation $(x - 2)^2 + (y + 3)^2 = 36$. What are the coordinates of its center and the length of its radius?

(1) (−2,3) and 6
(2) (2,−3) and 6
(3) (−2,3) and 36
(4) (2,−3) and 36

Use this space for computations.

4 In the diagram below, $MATH$ is a rhombus with diagonals $\overline{AH}$ and $\overline{MT}$.



The diagonals of a rhombus are perpendicular

$180 - (90 + 12) = 78$

If $m\angle HAM = 12$, what is $m\angle AMT$?

(1) 12
(2) 78
(3) 84
(4) 156

5 A line segment has endpoints (4,7) and (1,11). What is the length of the segment?

(1) 5
(2) 7
(3) 16
(4) 25

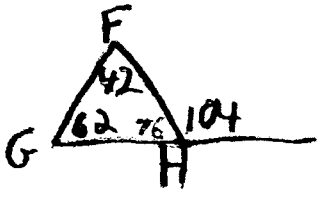
$\sqrt{(4-1)^2 + (7-11)^2}$

$\sqrt{3^2 + (-4)^2}$

$\sqrt{25} = 5$

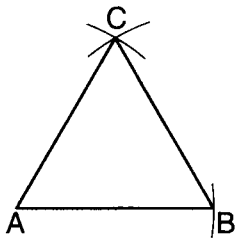
6 In $\triangle FGH$, $m\angle F = 42$ and an exterior angle at vertex H has a measure of 104. What is $m\angle G$?

(1) 34
(2) 62
(3) 76
(4) 146

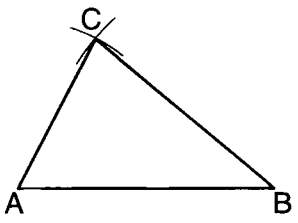


Use this space for computations.

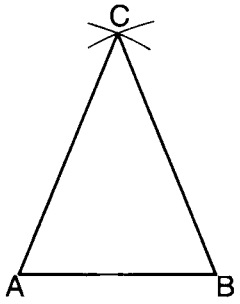
7 Which diagram represents a correct construction of equilateral $\triangle ABC$, given side $\overline{AB}$?



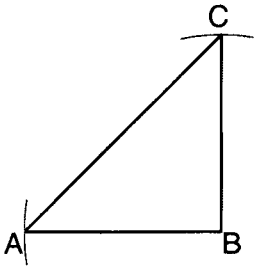
(1)



(3)

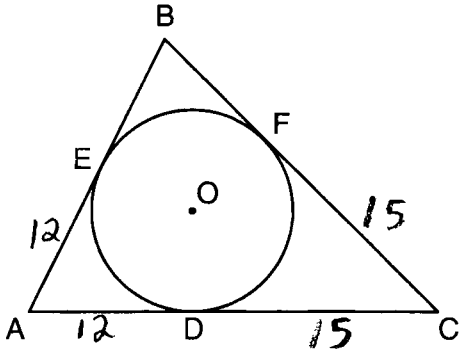


(2)



(4)

8 In the diagram below, $\triangle ABC$ is circumscribed about circle O and the sides of $\triangle ABC$ are tangent to the circle at points D, E, and F.



If $AB = 20$, $AE = 12$, and $CF = 15$, what is the length of $\overline{AC}$?

(1) 8

(2) 15

(3) 23

(4) 27

Use this space for computations.

9 In $\triangle ABC$ and $\triangle DEF$, $\frac{AC}{DF} = \frac{CB}{FE}$. Which additional information would prove $\triangle ABC \sim \triangle DEF$?

(1) $AC = DF$
(2) $CB = FE$
(3) $\angle ACB \cong \angle DFE$
(4) $\angle BAC \cong \angle EDF$

10 The angles of triangle ABC are in the ratio of 8:3:4. What is the measure of the *smallest* angle?

15

(1) 12°
(2) 24°
(3) 36°
(4) 72°

$\frac{3}{15} \cdot 180 = 36$

11 When a quadrilateral is reflected over the line $y = x$, which geometric relationship is *not* preserved?

(1) congruence
(2) orientation
(3) parallelism
(4) perpendicularity

12 Which equation represents circle O with center $(2,-8)$ and radius 9?

(1) $(x + 2)^2 + (y - 8)^2 = 9$
(2) $(x - 2)^2 + (y + 8)^2 = 9$
(3) $(x + 2)^2 + (y - 8)^2 = 81$
(4) $(x - 2)^2 + (y + 8)^2 = 81$

Use this space for computations.

13 Which statement is the negation of "Two is a prime number" and what is the truth value of the negation?

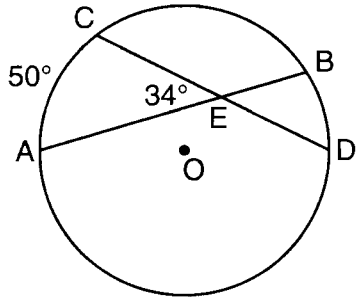
(1) Two is not a prime number; false

(2) Two is not a prime number; true

(3) A prime number is two; false

(4) A prime number is two; true

14 In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E.



$\frac{50 + x}{2} = 34$

$50 + x = 68$

$x = 18$

If $m\angle AEC = 34$ and $m\overset{\frown}{AC} = 50$, what is $m\overset{\frown}{DB}$?

(1) 16

(2) 18

(3) 68

(4) 118

15 The volume of a rectangular prism is 144 cubic inches. The height of the prism is 8 inches. Which measurements, in inches, could be the dimensions of the base?

(1) 3.3 by 5.5 18.15

(2) 2.5 by 7.2 18

(3) 12 by 8 96

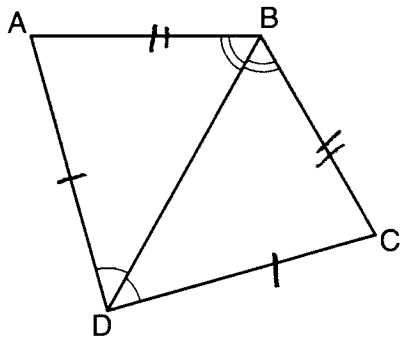
(4) 9 by 9 81

$L \cdot W \cdot 8 = 144$

$LW = 18$

Use this space for computations.

16 The diagram below shows a pair of congruent triangles, with $\angle ADB \cong \angle CDB$ and $\angle ABD \cong \angle CBD$.



Which statement must be true?

(1) $\angle ADB \cong \angle CBD$

(2) $\angle ABC \cong \angle ADC$

(3) $\overline{AB} \cong \overline{CD}$

(4) $\overline{AD} \cong \overline{CD}$

17 What is an equation of the line that is perpendicular to the line whose equation is $y = \frac{3}{5}x - 2$ and that passes through the point $(3,-6)$?

(1) $y = \frac{5}{3}x - 11$

(3) $y = -\frac{5}{3}x - 1$

(2) $y = -\frac{5}{3}x + 11$

(4) $y = \frac{5}{3}x + 1$

$m = \frac{3}{5}$

$m_\perp = -\frac{5}{3}$

$y = mx + b$

$-6 = -\frac{5}{3}(3) + b$

$-6 = -5 + b$

$-1 = b$

18 Point A lies in plane $\mathcal{B}$. How many lines can be drawn perpendicular to plane $\mathcal{B}$ through point A?

(1) one

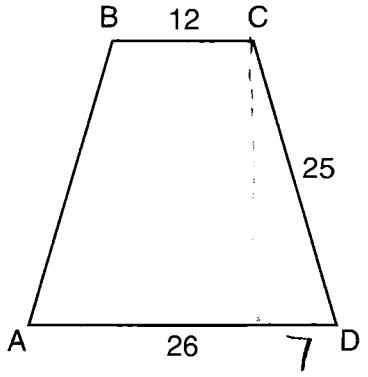
(3) zero

(2) two

(4) infinite

Use this space for computations.

19 In the diagram below of isosceles trapezoid $ABCD$, $AB = CD = 25$, $AD = 26$, and $BC = 12$.

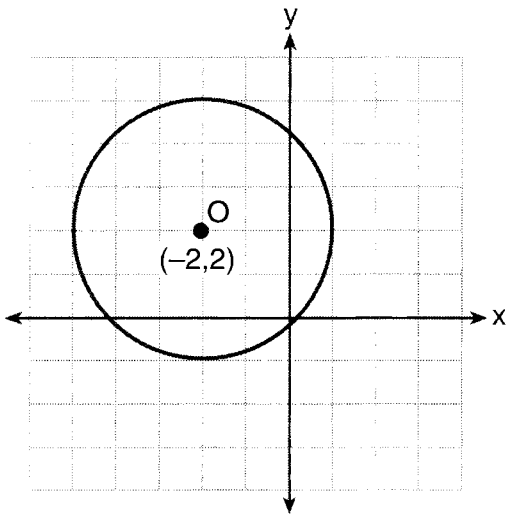


$\sqrt{25^2 - 7^2} = 24$

What is the length of an altitude of the trapezoid?

(1) 7
(2) 14
(3) 19
(4) 24

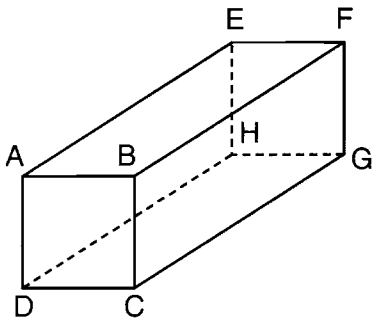
20 What is an equation of circle O shown in the graph below?



(1) $(x + 2)^2 + (y - 2)^2 = 9$
(2) $(x + 2)^2 + (y - 2)^2 = 3$
(3) $(x - 2)^2 + (y + 2)^2 = 9$
(4) $(x - 2)^2 + (y + 2)^2 = 3$

Use this space for computations.

21 The diagram below represents a rectangular solid.



Which statement must be true?

(1) $\overline{EH}$ and $\overline{BC}$ are coplanar.

(2) $\overline{FG}$ and $\overline{AB}$ are coplanar.

(3) $\overline{EH}$ and $\overline{AD}$ are skew.

(4) $\overline{FG}$ and $\overline{CG}$ are skew.

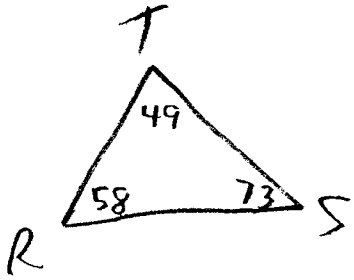
22 In $\triangle RST$, $m\angle R = 58$ and $m\angle S = 73$. Which inequality is true?

(1) $RT < TS < RS$

(2) $RS < RT < TS$

(3) $RT < RS < TS$

(4) $RS < TS < RT$



23 The number of degrees in the sum of the interior angles of a pentagon is

(1) 72

(2) 360

(3) 540

(4) 720

$(5-2)180$
540

24 What is the equation of a line passing through $(2,-1)$ and parallel to the line represented by the equation $y = 2x + 1$?

(1) $y = -\frac{1}{2}x$

(2) $y = -\frac{1}{2}x + 1$

(3) $y = 2x - 5$

(4) $y = 2x - 1$

$m=2$

$y = mx + b$
$-1 = 2(2) + b$
$-1 = 4 + b$
$-5 = b$

vertical line

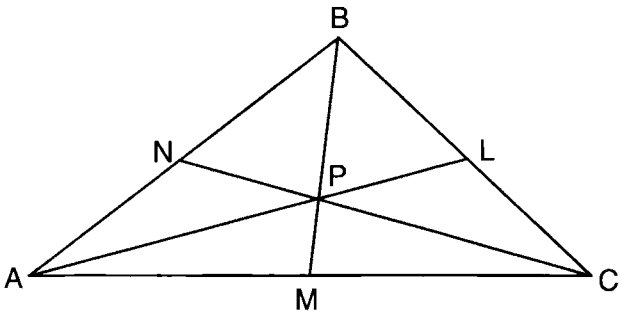
Use this space for computations.

25 The coordinates of the endpoints of $\overline{AB}$ are $A(0,0)$ and $B(0,6)$. The equation of the perpendicular bisector of $\overline{AB}$ is

(1) $x = 0$
(2) $x = 3$
(3) $y = 0$
(4) $y = 3$

a horizontal line through (0,3), the midpoint of $\overline{AB}$

26 In the diagram below, point P is the centroid of $\triangle ABC$.



If $PM = 2x + 5$ and $BP = 7x + 4$, what is the length of $\overline{PM}$?

(1) 9
(2) 2
(3) 18
(4) 27

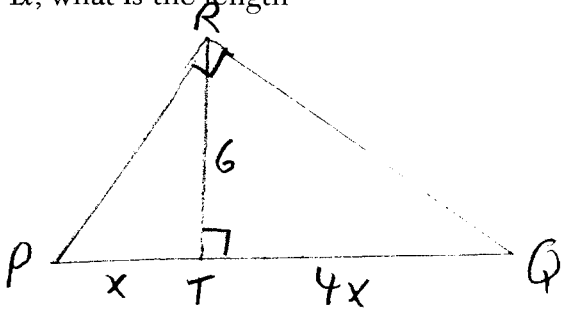
$2(2)+5$
9

$7x+4 = 2(2x+5)$
$7x+4 = 4x+10$
$3x = 6$
$x = 2$

27 In $\triangle PQR$, $\angle PRQ$ is a right angle and $\overline{RT}$ is drawn perpendicular to hypotenuse $\overline{PQ}$. If $PT = x$, $RT = 6$, and $TQ = 4x$, what is the length of $\overline{PQ}$?

(1) 9
(2) 12
(3) 3
(4) 15

$x \cdot 4x = 6^2$
$4x^2 = 36$
$x^2 = 9$
$x = 3$
$PQ = 4x + x = 5x = 15$



28 In $\triangle ABC$, $AB = 5$ feet and $BC = 3$ feet. Which inequality represents all possible values for the length of $\overline{AC}$, in feet?

(1) $2 \leq AC \leq 8$
(2) $2 < AC < 8$
(3) $3 \leq AC \leq 7$
(4) $3 < AC < 7$

$5 - 3 = 2$
$5 + 3 = 8$

Part II

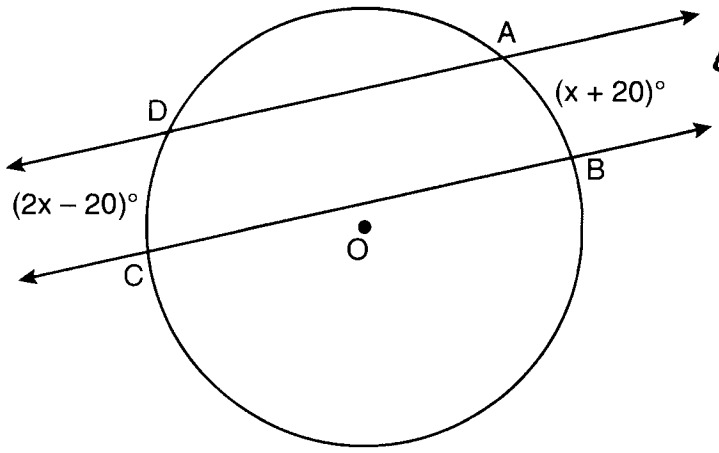
Answer all 6 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

29 In the diagram below, two parallel lines intersect circle O at points A, B, C, and D, with $m\overset{\frown}{AB} = x + 20$ and $m\overset{\frown}{DC} = 2x - 20$.

Find $m\overset{\frown}{AB}$.

$2x - 20 = x + 20$

$x = 40$

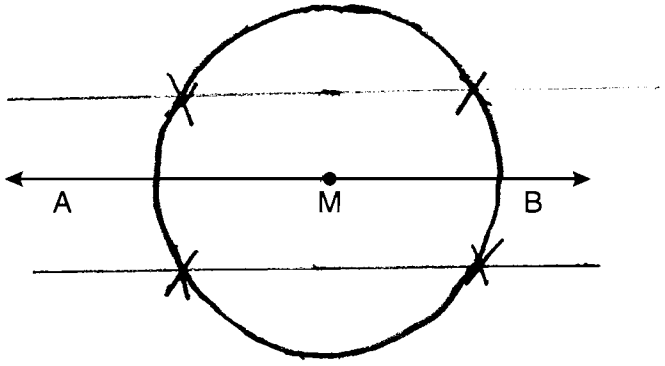


$40 + 20 = 60$

30 In the diagram below, point M is located on $\overleftrightarrow{AB}$.

Sketch the locus of points that are 1 unit from $\overleftrightarrow{AB}$ and the locus of points 2 units from point M.

Label with an **X** all points that satisfy both conditions.



31 Determine whether the two lines represented by the equations $y = 2x + 3$ and $2y + x = 6$ are parallel, perpendicular, or neither.

$m = 2$ $m = \frac{-A}{B} = \frac{-1}{2}$

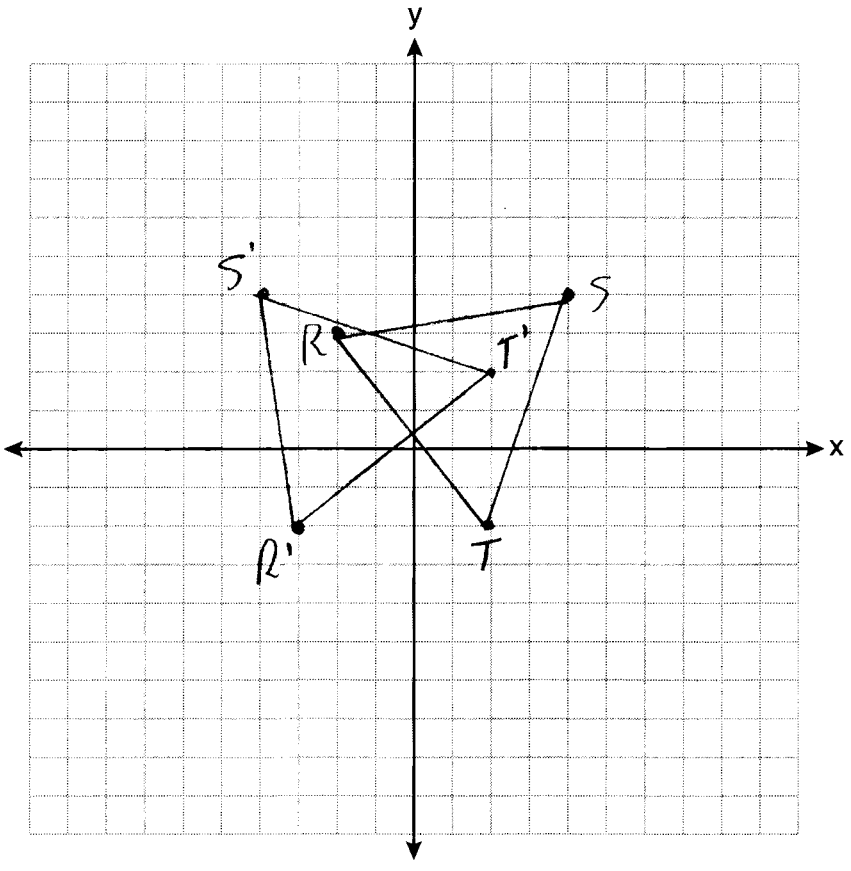
Justify your response.

Since the slopes are opposite reciprocals, the lines are perpendicular.

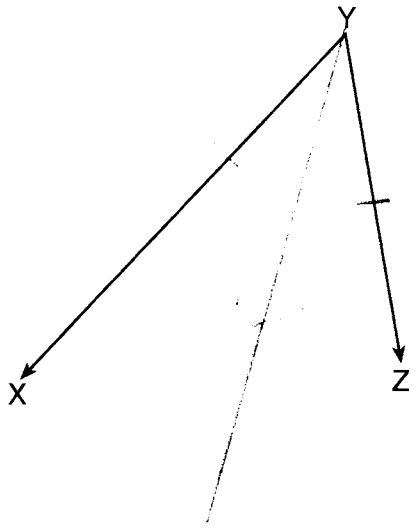
32 The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Triangle $R'S'T'$ is the image of $\triangle RST$ after a rotation of 90° about the origin.

State the coordinates of the vertices of $\triangle R'S'T'$.

[The use of the set of axes below is optional.] R'(3,-2) S'(-4,4) T'(2,2)

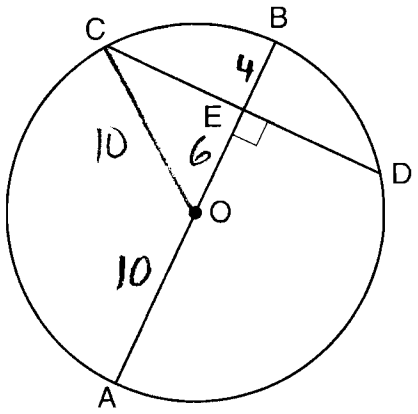


33 On the diagram below, use a compass and straightedge to construct the bisector of $\angle XYZ$. [Leave all construction marks.]



34 In the diagram below of circle O, diameter $\overline{AB}$ is perpendicular to chord $\overline{CD}$ at E.

If $AO = 10$ and $BE = 4$, find the length of $\overline{CE}$.



$\sqrt{10^2 - 6^2}$

$\sqrt{64}$

8

Part III

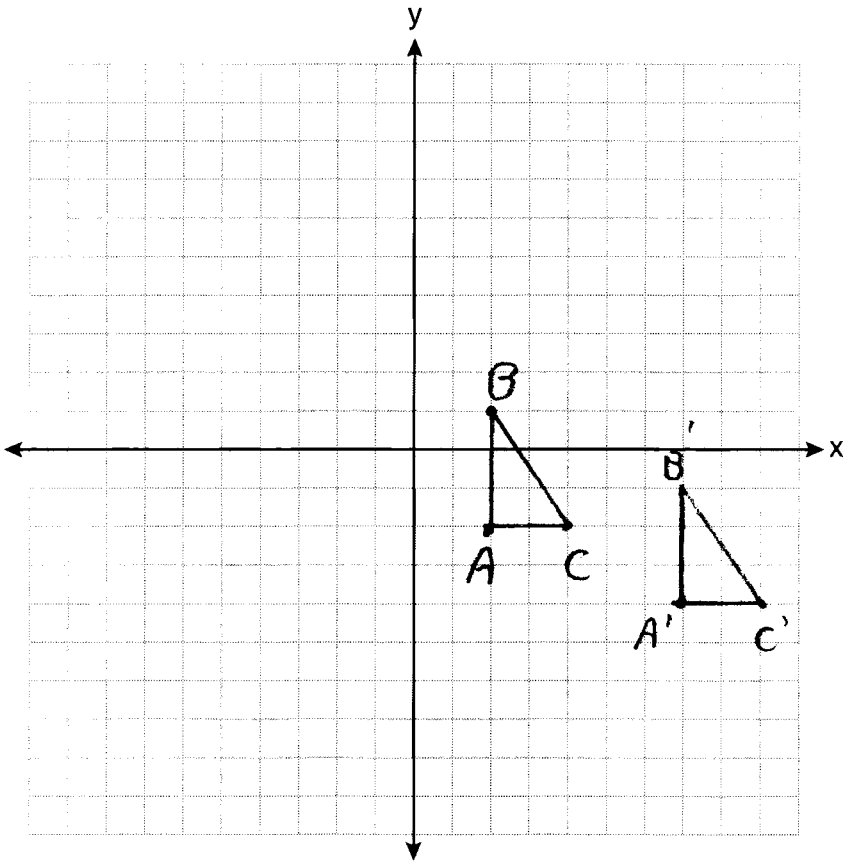
Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

35 Triangle ABC has coordinates $A(2,-2)$, $B(2,1)$, and $C(4,-2)$. Triangle $A'B'C'$ is the image of $\triangle ABC$ under $T_{5,-2}$. A'(7,-4) B'(7,-1) C'(9,-4)

On the set of axes below, graph and label $\triangle ABC$ and its image, $\triangle A'B'C'$.

Determine the relationship between the area of $\triangle ABC$ and the area of $\triangle A'B'C'$.

Justify your response. The areas are equal because translations preserve distance



36 A paint can is in the shape of a right circular cylinder. The volume of the paint can is 600π cubic inches and its altitude is 12 inches.

Find the radius, in inches, of the base of the paint can. Express the answer in simplest radical form.

$V = \pi r^2 h$

$600\pi = \pi r^2 \cdot 12$

$50 = r^2$

$\sqrt{50} = r$

$\sqrt{25}\sqrt{2} = r$

$5\sqrt{2}$

Find, to the *nearest tenth of a square inch,* the lateral area of the paint can.

$L = 2\pi rh$

$= 2\pi \cdot 5\sqrt{2} \cdot 12$

≈ 533.1

37 Triangle HKL has vertices $H(-7,2)$, $K(3,-4)$, and $L(5,4)$. The midpoint of $\overline{HL}$ is M and the midpoint of $\overline{LK}$ is N.

Determine and state the coordinates of points M and N.

Justify the statement: $\overline{MN}$ is parallel to $\overline{HK}$.

[The use of the set of axes below is optional.]

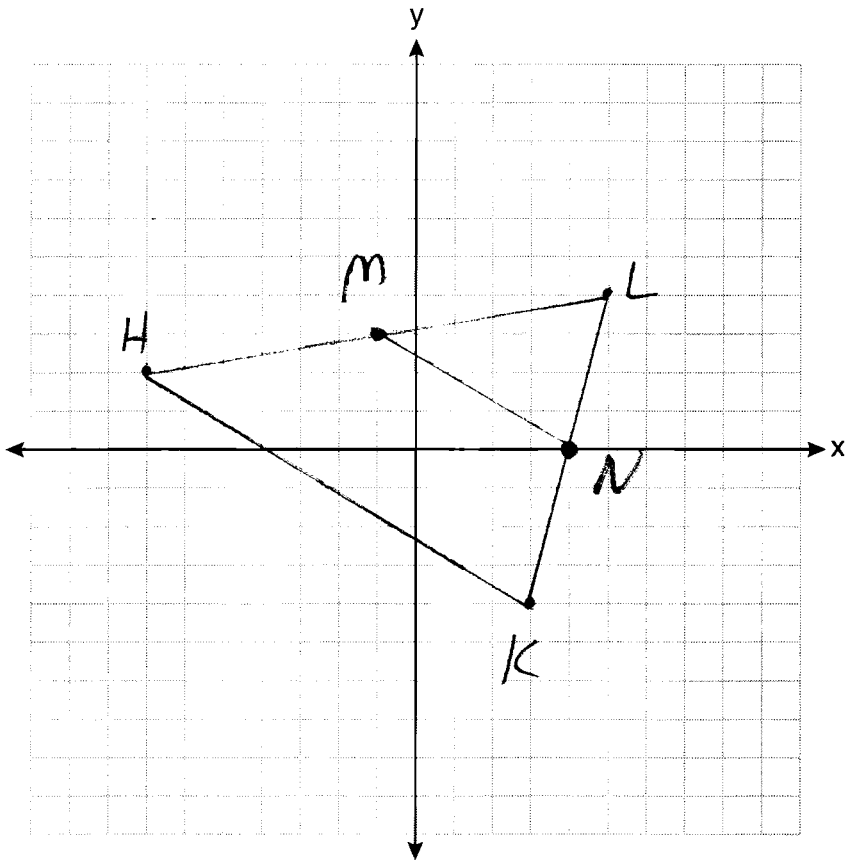
$M\left(\frac{-7+5}{2}, \frac{2+4}{2}\right)$

$M(-1,3)$

$N\left(\frac{3+5}{2}, \frac{-4+4}{2}\right)$

$N(4,0)$

$\overline{MN}$ is a midsegment



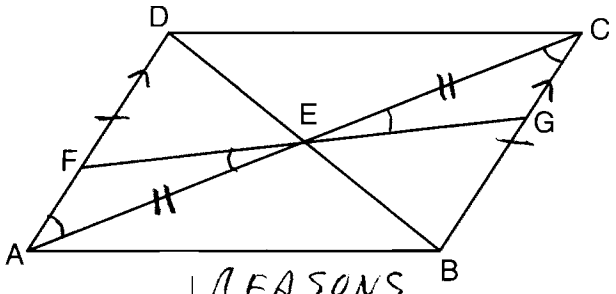
Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

38 In the diagram below of quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$ and $\angle DAE \cong \angle BCE$.

Line segments AC, DB, and FG intersect at E.

Prove: $\triangle AEF \cong \triangle CEG$



STATEMENT	REASONS
(1) Quadrilateral $ABCD$, $\overline{AD} \cong \overline{BC}$, $\angle DAE \cong \angle BCE$	(1) Given
(2) $\overline{AD} \parallel \overline{BC}$	(2) If two lines are cut by a transversal so that a pair of alternate interior angles are congruent, the lines are parallel.
(3) $ABCD$ is a parallelogram	(3) If one pair of opposite sides of a quadrilateral are both congruent & parallel, the quadrilateral is a parallelogram
(4) $\overline{AE} \cong \overline{CE}$	(4) The diagonals of a parallelogram bisect each other
(5) $\angle FEA \cong \angle GEC$	(5) Vertical angles
(6) $\triangle AEF \cong \triangle CEG$	(6) ASA