ELEVENTH YEAR MATHEMATICS

August 19, 1986

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the answer sheet. [60]

1. Solve for x in terms of a and b:
$$\frac{x}{b} = \frac{1}{a}$$ 1_____

2. For what *positive* acute angle does $\cos x = \sin x$? 2_____

3. Solve the following system of equations for x:
$$\begin{aligned} x + 3y &= -7 \\ 3x - y &= -1 \end{aligned}$$ 3_____

4. Solve for x: $2^{x+1} + 2 = 18$ 4_____

5. The distance d traveled in a given number of hours varies directly as the average rate r. If $d = 300$ when $r = 50$, find d when $r = 40$. 5_____

6. If $f(x) = (x - 3)^2$, find the value of $f(0)$. 6_____

7. Express 200° in radian measure. 7_____

8. In right triangle ABC, $\sin A = \frac{3}{5}$, $m\angle C = 90$, and $c = 10$. What is the area of the triangle? 8_____

9. The reciprocal of a positive number is equal to one-fourth of the number. Find the value of the number. 9_____

10. Express $\dfrac{\sqrt{18x^3}}{\sqrt{2x}}$ in *simplest form.* 10_____

11. Express $\dfrac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a} + \frac{1}{b}}$ as an equivalent fraction in *simplest form.* 11_____

12. Find sin 28° 36′ to four decimal places. 12_____

13. For what value of k will the equation $x^2 - 12x + k = 0$ have equal roots? 13_____

14. Evaluate: $\tan\left(\text{Arc sin } \frac{5}{13}\right)$ 14_____

15. If $f(x) = 3x^0 + x^{\frac{2}{3}}$, find $f(8)$. 15_____

16. In triangle ABC, $\sin A = 0.27$, $a = 4$, and $b = 8$. Find the value of $\sin B$ to the *nearest hundredth*. 16_____

Directions (17-30) : Write in the space provided on the answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

17. If $\frac{12}{6a+12}$ is multiplied by $\frac{a^2-4}{a^2-5a+6}$, the result, in *lowest terms*, is (1) $\frac{-4}{a+6}$ (2) $\frac{-4}{6a(-5a+6)}$ (3) $\frac{2}{a-3}$ (4) $\frac{a+2}{6a(a-3)}$ 17____

18. The solution set of $|x| = -1$ is (1) $\{1\}$ (2) $\{-1\}$ (3) $\{0\}$ (4) $\{\ \}$ 18____

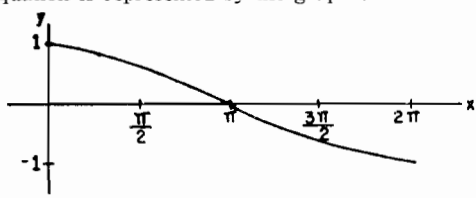
19. If $\sec x < 0$, which statement must be true? (1) $\sin x < 0$ (2) $\tan x < 0$ (3) $\cos x < 0$ (4) $\cot x < 0$ 19____

20. The value of $\sin^2 \pi + \cos^2 \pi$ is (1) 1 (2) 2 (3) -1 (4) 0 20____

21. The graph of which equation passes through the origin and is parellel to the line whose equation is $y = 2x + 1$? (1) $y = -2x$ (2) $y = 2x$ (3) $y = 2x - 1$ (4) $y = x + 2$ 21____

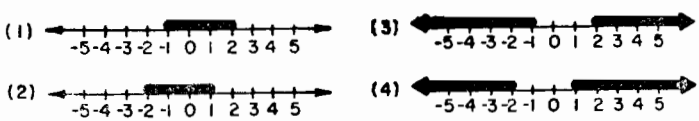
22. For which value of x is $\frac{1}{x^2-4}$ undefined? (1) 0 (2) -2 (3) -4 (4) 4 22____

23. Which equation is represented by the graph shown below?



(1) $y = \sin 2x$ (2) $y = \cos 2x$ (3) $y = \sin \frac{1}{2}x$ (4) $y = \cos \frac{1}{2}x$ 23____

24. Which graph represents the solution set of the inequality $x^2 - x - 2 \geq 0$?



24____

25. In the equation $\tan^2 x + \tan x = 0$, angle x may equal
(1) 45° (2) 180° (3) 225° (4) 270° 25____

26. The expression $(3 + 2i)^2$, in $a + bi$ form, is equivalent to
(1) 5 (2) 13 (3) $5 + 12i$ (4) $13 + 12i$ 26____

27. If $\cos \theta = r$ and angle θ is acute, then $\sin \frac{1}{2}\theta$ equals
(1) $\sqrt{1 - r^2}$ (2) $\frac{1}{2}\sqrt{1 - r^2}$ (3) $\frac{\sqrt{1 - r}}{2}$ (4) $\sqrt{\frac{1 - r}{2}}$
27____

28. The graph of which equation is a parabola? (1) $3x^2 + y = 10$
(2) $3x^2 + y^2 = 10$ (3) $3x^2 - y^2 = 10$ (4) $3x + y = 10$ 28____

29. When expressed as a function of a positive acute angle, $\cos(-137°)$ is equivalent to (1) $\cos 43°$ (2) $-\cos 43°$
(3) $\cos 47°$ (4) $-\cos 47°$ 29____

30. If $m\angle A = 30$, $a = 10$, and $b = 20$, then $\triangle ABC$
(1) must be a right triangle (2) may be either an acute or an obtuse triangle (3) must be an obtuse triangle (4) cannot be constructed 30____

Part II

Answer four questions from this part.

Show all work unless otherwise directed. [40]

31. *a* Find, to the *nearest tenth*, the values of x which satisfy the equation

$$3x = \frac{4}{x} - 2. \quad [8]$$

b If, in the equation given in part *a*, x is replaced with $\sin \theta$, determine the quadrant or quadrants in which angle θ may lie. [2]

32. *a* On the same set of axes, sketch the graphs of the equations $y = -\sin x$ and $y = \cos 2x$ over the interval $0 \leq x \leq 2\pi$. Label each graph with its equation. [4, 4]

b From the graphs sketched in part *a*, determine the number of values of x in the interval $0 \leq x \leq 2\pi$ which satisfy the equation $-\sin x = \cos 2x$. [2]

33. Write an equation or a system of equations which can be used to solve *each* of the following problems. In *each* case, state what the variable or variables represent. [*Solution of the equations is not required.*]

a A tank contains 40 liters of a solution which is 25% acid. How many liters of water must be added to make a new solution which is 20% acid? [5]

b The length of a rectangle is 4 units more than the width. The number of square units in the area is 4 less than the number of units in the perimeter. What are the dimensions of the rectangle? [5]

34. *a* Starting with the formula for $\sin(x + y)$, derive the formula for $\sin 2x$. [3]

b If $\cos A = -\frac{3}{5}$ and A lies in the second quadrant, find the value of $\sin 2A$. [3]

c For all values of θ for which the expressions are defined, show that the following is an identity:

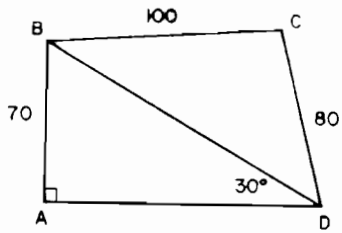
$$\tan\theta + \cot\theta = \frac{1}{\cos\theta \sin\theta} \quad [4]$$

35. *a* Using logarithms, find the value of $\sqrt[3]{0.0297}$ to the *nearest hundredth*. [4]

b Using logarithms, find, to the *nearest tenth*, the value of x if $3^x = 7$. [4]

c Find $\log_3 81$. [2]

36. In the accompanying diagram, $m\angle A = 90$, $m\angle BDA = 30$, $AB = 70$, $BC = 100$, and $DC = 80$. Find $m\angle C$ to the *nearest degree*. [10]



*37. *a* On the same set of axes, graph the following system of inequalities.

$\{(x,y) \mid x^2 + y^2 \leq 25\}$
$\{(x,y) \mid x - y < 0\}$ [4, 3]

b Label the solution set of the system with an S. [1]

c Give the coordinates of a point in the solution set of $\{(x,y) \mid x^2 + y^2 \leq 25\}$ that is *not* in the solution set of $\{(x,y) \mid x - y < 0\}$. [2]

* This question is based on an optional topic in the syllabus.