August 18, 1981

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

Directions (1-17): Write in the space provided on the answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

1. A value of x which satisfies the inequality $x + 3 < 2x + 7$ is
(1) -5 (2) -10 (3) -3 (4) -4 1____

2. If $\cos x = \frac{3}{5}$ and angle x lies in the fourth quadrant, what is the value of $\tan x$? (1) $5/4$ (2) $-5/4$ (3) $3/4$ (4) $-4/3$ 2____

3. The fraction $\frac{(x+2)(x-5)}{(x-1)(x+3)}$ is undefined for which value of x? (1) -1 (2) -2 (3) -3 (4) 5 3____

4. If $\sin A < 0$, in which quadrants may angle A lie?
(1) I, II (2) II, III (3) I, IV (4) III, IV 4____

5. The graph of the equation $\frac{x^2}{4} + \frac{y^2}{16} = 1$ is (1) a hyperbola (2) a parabola (3) a circle (4) an ellipse 5____

6. The numeral value of $\sin \frac{7\pi}{6}$ is (1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{\sqrt{3}}{2}$ 6____

7. The solution set of the equation $2x^2 + 5x - 3 = 0$ is
(1) $\{\frac{1}{2}, -3\}$ (2) $\{-\frac{1}{2}, 3\}$ (3) $\{3\}$ (4) $\{\frac{1}{2}\}$ 7____

8. If triangle ABC is a right triangle and angle C is the right angle, then which is *always* true? (1) $\sin A = \cos B$ (2) $\sin A \cos B = 1$ (3) $\sin A + \cos B = 1$ (4) $\sin A - \cos B = 1$ 8____

9. Which statement about the graphs of the equations $2x + 3y = 5$ and $2x + 3y = -5$ is true? (1) They coincide. (2) They intersect (3) They are parallel. (4) They are perpendicular. 9____

10. The fraction $\frac{\frac{1}{x} + 1}{x - \frac{1}{x}}$ is equivalent to (1) 1 (2) $\frac{1}{x}$ (3) $\frac{1}{x+1}$ (4) $\frac{1}{x-1}$ 10____

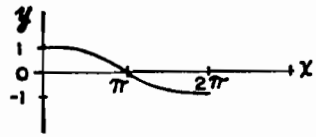
11. The equation $\sqrt{x-2} = x - 4$ is satisfied when x is equal to (1) both 3 and 6 (2) 6, only (3) 3, only (4) neither 3 nor 6 11____

12. If $2^x = 7$, what is the numerical value of 2^{2x}? (1) 49 (2) 14 (3) 5 (4) 4 12____

13. In the interval $0 < A \leq \frac{\pi}{2}$, which value of A satisfies the equation $\tan^2 A - \tan A = 0$? (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$ 13____

14. The range of the function $y = 2 \sin x$ is (1) $y \geq 1$ (2) $x \geq 1$ (3) $-2 \leq y \leq 2$ (4) $-2 \leq x \leq 2$ 14____

15. Which is the equation of the graph shown below?



(1) $y = \sin \frac{1}{2}x$

(2) $y = \cos \frac{1}{2}x$

(3) $y = \frac{1}{2} \sin x$ (4) $y = \frac{1}{2} \cos x$ 15____

16. Which is equivalent to $\sin x \cot x + \sec x \cos x$? (1) 1 (2) $\cos x$ (3) $\sin x + 1$ (4) $\cos x + 1$ 16____

17. If $m\angle B = 30$ and $AB = 10$, it is possible to construct two distinct triangles when AC is (1) 10 (2) 6 (3) 5 (4) 4 17____

Directions (18-30): Write your answers in the spaces provided on the answer sheet. Unless otherwise specified, answers may be left in terms of π or in radical form.

18. What is the solution set of the equation $50(x - \frac{1}{3}) = 45x$? 18____

19. If $y = \text{Arc sin}\left(\frac{\sqrt{2}}{2}\right)$, what is the value of y? 19____

20. If x varies inversely as y and $x = 8$ when $y = 3$, find x when $y = 6$. 20____

21. When the number 56,100,000 is written in the form of 5.61×10^n, what is the value of n? 21____

22. Solve for x: $2x + 3y = 13$
$5x - 2y = 4$ 22____

23. Steve has \$1.40 in nickels and dimes. If he has twice as many nickels as dimes, how many dimes does he have? 23____

24. Solve for the positive value of x: $\frac{2x}{3\sqrt{2}} = \frac{3\sqrt{2}}{x}$ 24____

25. In triangle ABC, $a = 8$, $b = 12$, and $\sin A = \frac{1}{3}$. Find the value of $\sin B$. 25____

26. If $f(x) = x^{\frac{3}{4}} - 4x^0$, evaluate $f(8)$ in *simplest form*. 26_____

27. Express $\frac{1}{3 + \sqrt{2}}$ as a fraction with a rational denominator. 27_____

28. Using logarithms, find the value of $\sqrt[3]{5}$ to the *nearest hundredth*. 28_____

29. In a circle, a central angle of 1 radian intercepts an arc of 2 centimeters. What is the length in centimeters of the radius of this circle? 29_____

30. What is the value of tan 23° 38′ to *four decimal places*? 30_____

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31. *a* Find, to the *nearest tenth*, the value(s) of tan θ which satisfy the equation $\tan^2 \theta - 3 \tan \theta + 1 = 0$. [8]

b Using the answer(s) obtained in part *a*, find the quadrant(s) in which angle θ may lie. [2]

32. A pendulum formula is given by the equation below:

$$t = 6.28 \sqrt{\frac{L}{32.2}}$$

Using logarithms, find t to the *nearest tenth* if $L = 25.5$. [10]

33. If the perimeter of a rectangle is 46 units and its diagonal is 17 units, find its length and width. [*Only an algebraic solution will be accepted.*] [5, 5]

34. *a* On the same set of axes, sketch the graphs of $y = 2 \cos \frac{1}{2}x$ and $y = \frac{1}{2}$ for values of x in the interval $-\pi \leq x \leq \pi$. [Label *each* graph with its equation.] [6, 2]

b From the graphs sketched in part *a*, find the number of values of x in the interval $-\pi \leq x \leq \pi$ that satisfy $2 \cos \frac{1}{2}x = \frac{1}{2}$. [2]

35. Given: obtuse angle x and $\sin x = \frac{24}{25}$.

Find:

a $\sin \frac{x}{2}$ [5]

b $\sin 2x$ [5]

36. The sides of triangle ABC are $a = 10$, $b = 12$, and $c = 18$. Find, to the *nearest degree*, the measure of the largest angle of triangle ABC. [10]

37. *a* On the same set of axes, graph the following system of inequalities: $y \geq x^2 - 2x - 8$ and $x - y + 2 > 0$ [8]

b Find the coordinates of a third quadrant point with integer values which satisfies the system in part *a*. [2]