The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR
MATHEMATICS

Tuesday, August 17, 1971 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Find the positive value of x which satisfies the equation $x^2 - 6x = 16$.

2 If x is an angle greater than $0°$ and less than $180°$, find in degrees the value of x satisfying the equation $\sqrt{3 + \sin x} = 2$.

3 If y varies directly as x and if $x = \dfrac{9}{2}$ when $y = 3$, find the value of x when $y = 5$.

4 In triangle ABC, $a = 10$, $b = 6$, and $\sin A = 0.5$. Find the value of $\sin B$.

5 Two cars start from the same place at the same time and travel in opposite directions. The average rate of the first car in miles per hour is r and the average rate of the other car is $2r$. How far apart, in terms of r, are they in one-half hour?

6 Find the solution set of $2x + 1 < 10$.

7 Express the fraction $\dfrac{3}{2\sqrt{2} + 1}$ with a rational denominator.

8 Find the numerical value of $3 \cos \dfrac{\pi}{3}$.

9 In right triangle ABC, angle A is acute. One leg of the triangle is numerically equal to $\sin A$ and the other leg is equal to $\cos A$. Find the numerical value of the hypotenuse.

10 If $\log \sin A = 9.9657 - 10$, find the measure of angle A to the *nearest minute*.

11 Find the y-intercept of the line whose equation is $5x - y + 3 = 0$.

12 Express as a product of two binomials: $3(x - 1) + x(x - 1)$

13 Find the numerical value of k if $\log_2 k = 3$.

14 One root of the equation $x^2 - 4x + k = 0$ is -3. What is the other root?

15 Express $\sin(-200°)$ as a function of a positive acute angle.

16 What is the positive value of
$$\sin \left[\text{arc tan} \left(- \frac{1}{\sqrt{3}} \right) \right] ?$$

Directions (17–30): Write in the space provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

17 For what integral values of x is $| 2x + 1 | < 5$?
(1) $\{-1,0,1,2\}$
(3) $\{-3,-2,-1,0,1,2\}$
(2) $\{-2,-1,0,1,2,3\}$
(4) $\{-2,-1,0,1\}$

18 The velocity of light is approximately 186,000 miles per second. In 10 seconds, the distance light travels would be most nearly
(1) 1.86×10^4 miles
(3) 1.86×10^{-6} mile
(2) 1.86×10^6 miles
(4) 1.86×10^7 miles

19 Which equation is an example of the distributive principle for real numbers?
(1) $3(2 + 7) = 6 + 21$
(2) $3 + 5 = 5 + 3$
(3) $8 \times (3 \times 2) = (8 \times 3) \times 2$
(4) $8(5) = 5(8)$

20 If, in a circle of radius 5, an arc subtends a central angle of 2.4 radians, the length of the arc is
(1) 24
(3) .48
(2) 12
(4) 5π

21 The graph of $y = \frac{1}{2} \sin 2x$ has
(1) an amplitude of 2 and a period of 4π
(2) an amplitude of 2 and a period of π
(3) an amplitude of $\frac{1}{2}$ and a period of 4π
(4) an amplitude of $\frac{1}{2}$ and a period of π

22 The sum of $\sqrt{-\dfrac{1}{16}}$ and $\sqrt{-\dfrac{1}{4}}$ expressed as a monomial in terms of i is

(1) $-\dfrac{i}{2}$ (3) $\dfrac{3i}{4}$

(2) $-\dfrac{3i}{4}$ (4) $\dfrac{i}{2}$

23 The relation defined by the set of ordered pairs, $\{(0,1),\ (1,1),\ (0,0),\ (-1,1),\ (-2,-2)\}$, is not a function. Which ordered pair, if omitted from this relation, will make the resulting set a function?

(1) $(1,1)$ (3) $(-1,1)$
(2) $(0,0)$ (4) $(-2,-2)$

24 The solution set of $\sqrt{x+1} + 3 = 0$ is

(1) $\{8\}$ (3) $\{-10\}$
(2) $\{2\}$ (4) $\{\ \ \}$

25 When combined into a single term and simplified, the expression

$$\dfrac{3}{3x-1} - \dfrac{1}{x} \text{ becomes}$$

(1) $\dfrac{6x-1}{x(3x-1)}$ (3) 1

(2) $\dfrac{-1}{x(3x-1)}$ (4) $\dfrac{1}{x(3x-1)}$

26 If, in $\triangle ABC$, $m\angle A = 45$, $a = \sqrt{2}$, and $b = 2$, then $\angle B$

(1) must be acute
(2) must be obtuse
(3) must be a right angle
(4) may be either acute or obtuse

27 The expression $\sin 2A + \sin A$ is equivalent to

(1) $3 \sin A$
(2) $\sin A\ (2 \cos A + 1)$
(3) $2 \sin A \cos A + 1$
(4) $\sin A\ (2 \sin A + 1)$

28 If $\cos A = t$, then $\sin^2 \dfrac{A}{2}$ is equal to

(1) $\dfrac{1-t}{2}$ (3) $\dfrac{1-t}{1+t}$

(2) $\dfrac{1+t}{2}$ (4) $\dfrac{1-t}{4}$

29 If $a \neq 0$, $b \neq 0$, and $c \neq 0$, the graph of $ax^2 + by^2 = c$ can not be

(1) an ellipse
(2) a circle
(3) a parabola
(4) a hyperbola

30 In triangle ABC, $a = 6$, $b = 6$, and $c = 9$. The value of $\cos B$ is

(1) $\frac{3}{4}$ (3) $\frac{1}{8}$
(2) $-\frac{3}{4}$ (4) $-\frac{1}{8}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for sin x to the *nearest tenth:* [10]

$2 \sin^2 x - \cos^2 x + \sin x = 0$

32 Determine the solution set of the following system of equations and check solutions in both equations: [8,2]

$(x - 2)^2 + (y - 3)^2 = 25$

$x - y + 2 = 0$

33 *a* On the same set of axes sketch the graphs of $y = \cos x$ and $y = \sin \frac{1}{2}x$ for all values of x which are in the interval $0 \leqq x \leqq 2\pi$. [4,4]

b From the sketches in (*a*) determine the value(s) of x for which $\sin \frac{1}{2}x - \cos x$ is maximum in the interval $0 \leqq x \leqq 2\pi$. [2]

34 Using logarithms, find to the *nearest hundredth* the

value of $\sqrt[3]{\dfrac{17.5 \tan 57° 20'}{896}}$. [10]

35 Write an equation or a system of equations which can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. (*Solution of the equations is not required.*)

a A man invested part of his \$20,000 at $7\frac{1}{2}\%$ and the rest at $6\frac{1}{2}\%$. How much should he invest at each rate in order to realize an annual income of \$1,440? [5]

b Mr. *A* can do a certain job by himself in 5 hours, and his assistant can do the same job by himself in 8 hours. After the two men worked on the job together for 2 hours, Mr. *A* was forced to stop working, and to let his assistant complete the job by himself. How many hours will it take the assistant to complete the job by himself? [5]

36 Answer *a* or *b* but *not* both:

a From two points on level ground in line with the foot of a tower and on the same side of the tower, the angles of elevation of the top of the tower are 41° and 47°. If the distance between the two points is 50 feet, find, to the *nearest ten feet,* the height of the tower. [10]

OR

b Two sides of a triangle are 10 and 12, respectively, and the angle included between these two sides is 130°. Find the length of the third side of the triangle to the *nearest integer.* [10]

37 Answer *both a* and *b*:

a Show that the following is an identity for all values of A for which the expressions are defined. [5]

$\dfrac{\cot A - \tan A}{\sec A + \csc A} = \cos A - \sin A$

b Starting with the formula $\cos (x + y)$, *derive* the formula for $\cos 2x$. [3]

Express the answer:
(1) in terms of $\cos x$ [1]
(2) in terms of $\sin x$ [1]

FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 8	(11) 3 *or* (0,3)	(21) 4
(2) 90	(12) $(x-1)(3+x)$	(22) 3
(3) 7½	(13) 8	(23) 2
(4) ³⁄₁₀	(14) 7	(24) 4
(5) $\dfrac{3r}{2}$	(15) sin 20° *or* cos 70°	(25) 4
(6) {$x < $ ⁹⁄₂}	(16) ½	(26) 3
(7) $\dfrac{6\sqrt{2}-3}{7}$	(17) 4	(27) 2
(8) ³⁄₂	(18) 2	(28) 1
(9) 1	(19) 1	(29) 3
(10) 67° 32′	(20) 2	(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 0.4, —0.8   [10]

(32) $\{(-2,0) \text{ and } (5,7)\}$ [8]

(33) *b* π *or* 180°   [2]

(34) 0.31   [10]

(35) *a* $x =$ amount invested at $7\frac{1}{2}\%$
$y =$ amount invested at $6\frac{1}{2}\%$
$x + y = 20,000$
$.075x + .065y = 1,440$ [5]

b $x =$ number of hours it will take the assistant to complete the job
$\frac{2}{5} + \frac{2}{8} + x/8 = 1$ [5]

(36) *a* 230   [10]
b 20   [10]