ELEVENTH YEAR MATHEMATICS

Wednesday, August 13, 1969 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 Find the numerical value of a number x, such that, if the product of b and x is increased by $4b$ (where $b \neq 0$), the result is $8b$.

2 Solve for x: $(x - 2)^2 + 3x = x^2 - 6x - 1$

3 Factor completely: $a \tan^2 x - a$

4 Solve the equation $\sqrt{2 \sin x + 8} = 3$ for the value of x in the interval $90° < x < 180°$.

5 Express $75°$ in radian measure.

6 Find the value of log 96.14.

7 In triangle ABC, $a = x$, $b = y$, and $\sin A = m$. Express $\sin B$ in terms of x, y, and m.

8 In triangle ABC, if $a = 5$, $b = 6$, and $c = 9$, find $\cos C$.

9 What is the additive inverse of the complex number $3 - 2i$?

10 Find the solution set for the inequality $\{x \mid 3x + 7 > 13\}$.

11 What is the slope of any line which is perpendicular to a line whose slope is 3?

Directions (12–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

12 If the expressions $\dfrac{x}{2}$, $\dfrac{x}{3}$, and $\dfrac{x}{5}$ all represent integers, then x may be any multiple of
(1) 6   (3) 15
(2) 10   (4) 30

13 Given $\sqrt{3} = 1.73$ and $\sqrt{2} = 1.41$. What is the value of $\dfrac{1}{\sqrt{3} - \sqrt{2}}$ to the *nearest tenth*?
(1) 3.0   (3) 3.2
(2) 3.1   (4) 3.3

14 What is the value of x which satisfies the equation $4^{-x} = 8$?
(1) -2 (3) $-\frac{3}{2}$
(2) 2   (4) $\frac{3}{2}$

15 As a body falls from rest, its velocity varies directly as the time of the fall. If at the end of 2 seconds the velocity of a body is 64.4 feet per second, what is the velocity of the body in feet per second at the end of 5 seconds?
(1) 161 ft./sec.   (3) 322 ft./sec.
(2) 193 ft./sec.   (4) 644 ft./sec.

16 The sum of $\dfrac{1}{x + 1}$ and $\dfrac{3}{x^2 - 1}$, where the expressions are defined, expressed as a single fraction is
(1) $\dfrac{4}{(x + 1)(x - 1)}$ (3) $\dfrac{2}{x - 1}$
(2) $\dfrac{x + 2}{(x + 1)(x - 1)}$ (4) $\dfrac{2}{x + 1}$

17 The function $y = \dfrac{x}{x-1}$ is undefined when the value of x is
(1) 1
(2) 2
(3) 0
(4) —1

18 If $\cos \theta = 0.5380$, then θ may be equal to
(1) 32° 33′
(2) 32° 37′
(3) 57° 23′
(4) 57° 27′

19 The expression $\sin 54° \cos 36° + \cos 54° \sin 36°$ is equivalent to
(1) 1
(2) 0
(3) $\sin 18°$
(4) $\cos 18°$

20 The largest possible value of the fraction
$$\dfrac{3}{2 - \sin A} \text{ is}$$
(1) 1
(2) $\frac{3}{2}$
(3) 3
(4) infinite

21 If $\sin A = b$, then the value of $\sin A \cos A \tan A$ is equal to
(1) 1
(2) $\dfrac{1}{b}$
(3) b
(4) b^2

22 The graph of the equation $x^2 + 4y = 16$ is
(1) a parabola
(2) a hyperbola
(3) a circle
(4) an ellipse

23 The graph of $x^2 + y^2 = 16$ and the graph of $y = b$ are drawn on the same set of axes. The graphs will *not* intersect if
(1) $b > 4$
(2) $b = 4$
(3) $2 < b < 4$
(4) $0 < b < 2$

24 If $A = \text{arc sec } 2$, then a value of A is
(1) 30°
(2) 45°
(3) 60°
(4) 90°

25 The roots of a quadratic equation are real and irrational. If n is a positive integer and a perfect square, then which one of the following could be the value of the discriminant?
(1) n
(2) n^2
(3) $(n+1)^2$
(4) $n+1$

26 As angle A increases from 270° to 360°, the value of $\cos A$
(1) decreases from 0 to —1
(2) decreases from 1 to 0
(3) increases from —1 to 0
(4) increases from 0 to 1

27 The expression $\tan (180° - x)$ is equivalent to
(1) $\tan x$
(2) $-\tan x$
(3) $-\cot x$
(4) $\cot x$

28 The expression $(-\sin A - \cos A)^2$ is equivalent to
(1) 1
(2) —1
(3) $1 - \sin 2A$
(4) $1 + \sin 2A$

29 In $\triangle ABC$, $B = 35°$ and side $c = 10$. Which value of side b will make it possible for two different triangles to be formed?
(1) $b = 5$
(2) $b = 8$
(3) $b = 10$
(4) $b = 13$

30 The solution set of $|x + 2| = 7$ is
(1) {5}
(2) {—5}
(3) {5, —9}
(4) {—9}

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 a Solve the following equation for cot x to the *nearest tenth*: [7]

$$2 \cot^2 x - 3 \cot x = 1$$

b Using the results obtained in part a, find to the *nearest degree* the value of x in the third quadrant for which $2 \cot^2 x - 3 \cot x = 1$. [2]

c Is there a value of x in each of the four quadrants which satisfies the equation $2 \cot^2 x - 3 \cot x = 1$? [1]

32 Write an equation or a system of equations which can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a One pipe can fill a tank in 3 hours and a drainpipe can empty the tank in 4 hours. By error, both pipes were opened when the tank was empty. One hour later the drainpipe was closed. How many hours, after the drainpipe was closed, did it take to fill the tank? [5]

b A man invested a total of $5,150 in two enterprises. At the end of a year he had gained 6% on one of these investments but had lost 4% on the other. If his net profit for the year on the two investments was $141, find the amount invested at each rate. [5]

33 Answer *either a or b, but not both*:

a A triangular lot ABC has side $AB = 20.8$ yards, side $BC = 41.5$ yards, and angle $B = 83°$ 50'.
  (1) Find to the *nearest square yard* the area of the lot. [7]
 (2) Find to the *nearest yard* the perpendicular distance from C to AB. [3]

OR

b The sides of a triangular piece of land are 220 miles, 150 miles, and 290 miles. Find to the *nearest degree* the angle which is opposite the longest side. [10]

34 If $R = \dfrac{23.8 \sqrt[3]{0.0642}}{\sin 38°}$, using logarithms, find the value of R to the *nearest tenth*. [10]

35 a Given acute $\triangle ABC$ as shown in the accompanying figure. *Derive* a formula for the area of $\triangle ABC$ in terms of a, b, and a function of angle C. [4]
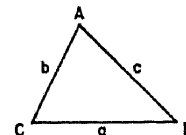
b For all values of A for which the expression is defined, show that the following is an identity: [6]

$$\frac{\cos 2A + \cos A + 1}{\sin 2A + \sin A} = \cot A$$

36 a Find the coordinates of the turning point of the graph of $y = x^2 - 2x - 3$. [2]

b Using all integral values of x in the interval $-2 \leq x \leq 4$, draw the graph of $y = x^2 - 2x - 3$. [6]

c On a number line, indicate the solution set of $\{x \mid x^2 - 2x - 3 \leq 0\}$. [2]

*37 Solve the following system of equations and check: [8, 2]

$$4x + 2y + z = 7$$
$$x - y + 6z = -1$$
$$2x + 3y - 5z = 5$$

  * This question is based on an optional topic in the syllabus.

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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, August 13, 1969 — 12:30 to 3:30 p.m., only

ANSWER SHEET

Pupil...Teacher...

School...

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1...............................   11...............................   21...............................

2...............................   12...............................   22...............................

3...............................   13...............................   23...............................

4...............................   14...............................   24...............................

5...............................   15...............................   25...............................

6...............................   16...............................   26...............................

7...............................   17...............................   27...............................

8...............................   18...............................   28...............................

9...............................   19...............................   29...............................

10...............................   20...............................   30...............................

Your answers for Part II should be placed on paper provided by the school.

[7]

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS
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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 12–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 4

(2) —1

(3) $a(\tan x - 1)(\tan x + 1)$

(4) 150°

(5) $\dfrac{5\pi}{12}$

(6) 1.9829

(7) $\dfrac{my}{x}$

(8) $-\frac{1}{3}$

(9) $-3 + 2i$

(10) $\{x \mid x > 2\}$ *or* $x > 2$

(11) $-\frac{1}{3}$

(12) 4

(13) 2

(14) 3

(15) 1

(16) 2

(17) 1

(18) 4

(19) 1

(20) 3

(21) 4

(22) 1

(23) 1

(24) 3

(25) 4

(26) 4

(27) 2

(28) 4

(29) 2

(30) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 1.8, —0.3   [7]
     b 209   [2]
     c yes   [1]

(32) *a* x = number of hours it took to fill the tank

$$\frac{x}{3} - \frac{x}{4} = 1 \quad [5]$$

     b x = amount invested at 6%
       y = amount invested at 4%

$$x + y = 5{,}150$$
$$.06x - .04y = 141 \quad [5]$$

(33) *a* (1) 429   [7]
       (2) 41   [3]
     b 102   [10]

(34) 15.5   [10]

(36) *a* (1,—4)   [2]

c [2]

*(37) $x = 3, y = -2, z = -1$ [8]

DO YOU KNOW . . .

  . . . that 400 classroom teachers were involved in preparing Regents examinations last year?

- Teachers wrote the questions.

- Other teachers assembled the examinations.

- Still other teachers reviewed the finished product.

And a committee of principals approved all of the examinations before they went to the printer.