The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, August 14, 1968 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Find the solution set of the following system of equations:

$$2x - y = 6$$
$$x - y = 4$$

2 Write an equation of the straight line whose slope is 5 and whose y-intercept is the same as the y-intercept of $y = 2x - 3$.

3 Express $-2i + \frac{3}{2}\sqrt{-16}$ as a monomial in terms of i.

4 Find to the *nearest minute* the positive acute angle for which $\log \tan A = 9.9656 - 10$.

5 Find three binomials whose product is $x^4 - 1$.

6 Express $\log \frac{\sqrt{x}}{y^3}$ in terms of $\log x$ and $\log y$.

7 Express in *simplest* form:

$$\frac{2 + \frac{2b}{a}}{a - \frac{b^2}{a}}$$

8 Find the numerical value of $2 \cos \frac{\pi}{3}$.

9 Express in terms of π the radian measure of an angle of 220°.

10 If s varies directly as the square of t and if $s = 64$ when $t = 2$, what is the value of s when $t = 3$?

11 If the tens digit of a two-digit number is t and the units digit is 8, express the number in terms of t.

12 Express cos 215° as a function of a positive acute angle.

13 Find in degrees a positive value of θ less than 360° which satisfies the equation

$$2 \sin^2 \theta - 3 \sin \theta - 5 = 0.$$

14 The first term of an arithmetic progression is 10 and the 30th term is −77. Find the common difference.

15 In $\triangle ABC$, $a = 8$ inches, $C = 30°$, and the area is 14 square inches. Find the number of inches in the length of b.

16 The bearing of point A from point B is 285° (N 75° W) and the bearing of point C from point B is 160° (S 20° E). Find the number of degrees in the obtuse angle ABC.

17 Express in radical form the positive value of cos (arc sin $\frac{2}{3}$).

Directions (18–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

18 The expression $\frac{(8 \times 10^3)(2 \times 10^{-1})}{4 \times 10^2}$ is equivalent to

(1) 1
(2) 2
(3) 8
(4) 4

19 The expression $\frac{1}{\sqrt{2} - 3}$ is equivalent to

(1) $\frac{3 - \sqrt{2}}{7}$
(2) $\frac{\sqrt{2} - 3}{11}$
(3) $-\frac{\sqrt{2} + 3}{7}$
(4) $-\frac{\sqrt{2} + 3}{11}$

20 If $\cot \theta > 0$ and $\sin \theta > 0$, then angle θ lies in quadrant

(1) I
(2) II
(3) III
(4) IV

21 The sum of any three consecutive integers must be

(1) odd
(2) even
(3) prime
(4) exactly divisible by 3

22 If $\log a + \log \sin B = \log b + \log \sin A$, then a is equal to

(1) $b \sin A \sin B$
(2) $b + \sin A + \sin B$
(3) $\frac{\sin A}{b \sin B}$
(4) $\frac{b \sin A}{\sin B}$

23 One of the roots of the equation $\frac{x - 7}{2} - \frac{1}{x} = -4$ is

(1) -1
(2) -2
(3) -3
(4) -4

24 If $\sin x = a$, then $\cos 2x$ is

(1) $2a$
(2) $1 - 2a$
(3) $1 - 2a^2$
(4) $1 + 2a^2$

25 A quadratic equation, the sum of whose roots is 4 and the product of whose roots is 1, is

(1) $x^2 - x + 4 = 0$
(2) $x^2 - 4x + 1 = 0$
(3) $x^2 + x - 4 = 0$
(4) $x^2 + 4x - 1 = 0$

26 If θ is a positive acute angle, then $\cot \theta$ is equal to

(1) $\frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$
(2) $\frac{\sin \theta}{\sqrt{1 + \sin^2 \theta}}$
(3) $\frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta}$
(4) $\frac{\sqrt{1 + \sin^2 \theta}}{\sin \theta}$

27 For the function defined by the equation $y = 2 \sin \frac{1}{2}x$, the values of y are in the interval defined by

(1) $-2 < y < 2$
(2) $-2 \leqq y \leqq 2$
(3) $-\frac{1}{2} < y < \frac{1}{2}$
(4) $-\frac{1}{2} \leqq y \leqq \frac{1}{2}$

28 The sixth term of the geometric progression 4^5, 4^4, 4^3, . . . is

(1) 1
(2) 0
(3) $\frac{1}{4}$
(4) 4

29 Which is an equation of a parabola?

(1) $x^2 = 9 - y^2$
(2) $x^2 = 9 + y^2$
(3) $x^2 = 9 - y$
(4) $x^2 = 9y^2$

30 In triangle ABC, if $a = \sqrt{3}$, $b = \sqrt{3}$, and $c = 1$, then $\cos C$ is equal to

(1) 1
(2) -1
(3) $\frac{5}{6}$
(4) $-\frac{5}{6}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find to the *nearest tenth* the roots of the equation $3x^2 - 2x = 4$. [10]

32 *a* On the same set of axes, sketch the graphs of $y = \tan x$ and $y = \sin 2x$ for values of x from 0 to π radians, inclusive. [4,4]

b How many values in the interval $0 \leq x \leq \pi$ satisfy the equation $\tan x = \sin 2x$? [2]

33 A formula for finding the area of a triangle when given the length of one side and any two angles is

$$K = \frac{a^2 \sin B \sin C}{2 \sin A}.$$

Using logarithms, find the area of triangle ABC to the *nearest integer* if $a = 18$, $A = 42°$, and $C = 63°$. [10]

34 The area of a rectangle is 180 square inches. If its length is increased by 5 inches and its width is diminished by 3 inches, then a second rectangle is formed whose area is the same as that of the given rectangle. Find, in inches, the dimensions of the given rectangle. [5,5]

35 Answer *both* *a* and *b*:

a Verify the identity $\frac{1 - \cos 2x}{\sec^2 x - \tan^2 x} = 2 \sin^2 x$. [5]

b Starting with formulas for $\sin (x + y)$ and $\cos (x + y)$, derive a formula for $\tan (x + y)$ in terms of $\tan x$ and $\tan y$. [5]

36 Answer *either* *a* or *b* but *not* both:

a Line segments OP and OQ, drawn from the origin O, make angles of 68° 30′ and 125° 40′, respectively, with the positive direction of the x-axis. If OP is 30 units and OQ is 40 units, find the distance from P to Q, to the *nearest integer*. [5,5]

OR

b Two points, A and B, both at sea level and 2,000 feet apart, are due west of a mountain peak C. The angles of elevation at A and B of the mountain peak are 30° 00′ and 40° 20′, respectively. Find, to the *nearest 100 feet*, the height of the mountain above sea level. [5,5]

37 Solve the following system of equations for x, y, and z and check: [8,2]

$$3x + 2y - z = 7$$
$$x + y + 2z = 3$$
$$4x - y + z = 2$$

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 18–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) (2,—2) *or* $x = 2$, $y = -2$

(2) $y = 5x - 3$

(3) $4i$

(4) 42° 44′

(5) $(x - 1)$, $(x + 1)$, $(x^2 + 1)$

(6) $\frac{1}{2} \log x - 3 \log y$

(7) $\frac{2}{a - b}$

(8) 1

(9) $\frac{11}{9}\pi$

(10) 144

(11) $10t + 8$

(12) —cos 35° *or* —sin 55°

(13) 270

(14) —3

(15) 7

(16) 125

(17) $\frac{\sqrt{5}}{3}$

(18) 4

(19) 3

(20) 1

(21) 4

(22) 4

(23) 2

(24) 3

(25) 2

(26) 3

(27) 2

(28) 1

(29) 3

(30) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 1.5, —0.9 [10]

(32) *b* 4 [2]

(33) 208 [10]

(34) Analysis [5]
15 by 12 [5]

(36) *a* Analysis [5]
35 [5]
b Analysis [5]
3,600 [5]

(37) $x = 1$, $y = 2$, $z = 0$ [8]
Check [2]