The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, August 16, 1967 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

Directions (1–16): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

1 If the product of 0.002 and 0.0003 is written in the form 6×10^n, what is the value of n?

(1) 7 (3) -7
(2) 9 (4) -9

2 If $11x - 7y = 28$ and $2x = 7$, then y is equal to

(1) 7 (3) 3
(2) $\frac{7}{2}$ (4) $\frac{3}{2}$

3 An equivalent expression for the imaginary number $3i$ is

(1) -9 (3) $\sqrt{-3}$
(2) $-\sqrt{3}$ (4) $3\sqrt{-1}$

4 The expression $\frac{14}{3 - \sqrt{2}}$ is equivalent to

(1) $14\sqrt{2}$ (3) $6 + \frac{\sqrt{2}}{7}$
(2) $6 + 2\sqrt{2}$ (4) $\frac{14(3 + \sqrt{2})}{5}$

5 A car traveled a certain distance in 8 hours at an average speed of r miles per hour. At what rate in miles per hour would the car have to travel in order to cover the same distance in h hours?

(1) $\frac{8r}{h}$ (3) $8rh$
(2) $\frac{h}{8r}$ (4) $8r$

6 What is the product of the roots of the equation $3x^2 - 2x - 5 = 0$?

(1) $\frac{2}{3}$ (3) $-\frac{5}{3}$
(2) $\frac{5}{3}$ (4) -5

7 The graph of the equation $x^2 + y = 9$ is

(1) an ellipse (3) a circle
(2) a parabola (4) a straight line

8 What is the positive value of tan [arc cos $(-\frac{3}{5})$]?

(1) $\frac{4}{5}$ (3) $\frac{3}{4}$
(2) $\frac{5}{4}$ (4) $\frac{4}{3}$

9 If A and B are acute angles and tan $A = \frac{1}{4}$ and tan $B = \frac{3}{5}$, what is the value of tan $(A + B)$?

(1) 1 (3) $\frac{3}{17}$
(2) 2 (4) $\frac{3}{20}$

10 If x and y are the first two terms, respectively, of a geometric progression, the third term of the progression, expressed in terms of x and y, is

(1) $\frac{x + y}{2}$ (3) $\frac{x}{y^2}$
(2) $\frac{xy}{2}$ (4) $\frac{y^2}{x}$

11 If a central angle of $\frac{4}{3}$ radians intercepts an arc of 12 inches, what is the number of inches in the radius of the circle?

(1) 9 (3) 3
(2) 16 (4) 4

12 A tower 100 feet high stands on level ground. From point A, 200 feet from the base of the tower, the angle of elevation of the top of the tower, to the *nearest degree,* is

(1) 27 (3) 60
(2) 30 (4) 63

13 The replacement of $2a + 2b$ by $2(a + b)$ is justified by the

(1) commutative law of multiplication
(2) associative law of multiplication
(3) associative law of addition
(4) distributive law

14 As angle x increases from 0° to 360°, cos x and sin x both decrease in quadrant

(1) I (3) III
(2) II (4) IV

15 In triangle ABC, if $a = 4$, $c = 2\sqrt{3}$, and $A = 60°$, what is the value of sin C?

(1) $\frac{\sqrt{3}}{2}$ (3) $\frac{3}{2}$

(2) $\frac{\sqrt{3}}{4}$ (4) $\frac{3}{4}$

16 What is the value of $\sin \frac{\pi}{6} + \cos \pi$?

(1) $-\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$

(2) $\frac{1}{2}$ (4) $\frac{\sqrt{3} - 2}{2}$

17 Solve for m: $T = mg - mf$

18 The first row in an auditorium has 32 seats, and each succeeding row has 2 seats more than the row in front of it. How many seats are there in the 17th row?

19 What is the abscissa of the turning point of the graph whose equation is $y = x^2 - 4x$?

20 If 2 is the amplitude of $y = k \sin x$, what is the amplitude of $y = 3k \sin 2x$?

21 Find the factors of $2 \cot^2 x - 5 \cot x - 3$.

22 What is the slope of the line whose equation is $2x - 3y = -6$?

23 Express the sum of $\frac{a}{a - 2}$ and $\frac{2}{2 - a}$ in *simplest* form.

24 If sin $A = 0.8621$, find the value of acute angle A to the *nearest minute.*

25 If log $n = 2.3125$, find n to the *nearest tenth.*

26 If x is an acute angle and cos $x = a$, express cos $\frac{1}{2}x$ in terms of a.

27 If the value of the discriminant of $ax^2 - 2x - 6 = 0$ is 76, find the value of a.

28 What is the numerical value of tan (−135°)?

29 In triangle ABC, $a = 4$, $b = 5$, and cos $C = -0.6$. Find in radical form the length of side c.

30 If A is an acute angle, express tan A in terms of sin A.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the roots of the equation

$$2x - \frac{4}{x} = 5. \quad [8]$$

b If in part *a*, $x = \cos \theta$, determine the quadrant(s) in which angle θ lies. [2]

32 Solve the following set of equations for x, y, and z and check in all three equations: [8,2]

$$x + y - z = 1$$
$$8x + 3y - 6z = 1$$
$$4x + y - 3z = -1$$

33 *a* On the same set of axes sketch the graphs of $y = \frac{1}{2} \sin x$ and $y = \cos \frac{1}{2}x$ as x varies from 0 to 2π radians. [Label each curve with its equation.] [4,4]

b If a graph of a cosine function has an amplitude of 1 and a period of π radians, write its equation. [2]

34 Write an equation or a system of equations which can be used to solve *each* of the following problems. In each case state what the variable or variables represent. [*Solution of the equations is not required.*]

a Three numbers are in the ratio of 1:2:4. If 1 is subtracted from the first number, the second is left unchanged, and the third number is multiplied by 2, the resulting numbers taken in the same order will form a different geometric progression. Find the original numbers. [5]

b The perimeter of a right triangle is 24 inches. If the hypotenuse is 10 inches, find the length of each leg. [5]

35 *a* Starting with the formulas for $\sin \frac{1}{2}x$ and $\cos \frac{1}{2}x$, *derive* a formula for $\tan \frac{1}{2}x$. [You may assume that x is an angle in the first quadrant.] [4]

b Prove that the following equality is an identity: [6]

$$\frac{\cos 2\theta}{\tan \theta} = \cot \theta - \sin 2\theta$$

36 Answer *either* *a* or *b* but *not* both: [10]

a Two forces act on an object. The first force has a magnitude of 76 pounds and makes an angle of 36° with the resultant. The magnitude of the resultant is 118 pounds. Find to the *nearest pound* the magnitude of the second force.

OR

b Ranger Station *B* is 27.4 miles due east of Ranger Station *A*. A fire is located N 21° 40′ E (21° 40′) from Station *A* and N 33° W (327°) from Station *B*. Find to the *nearest tenth of a mile* the distance from Station *A* to the fire.

*37 In $\triangle ABC$, side $a = 15$, side $b = 19$, and side $c = 13$. Using $\tan \frac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$, find the value of angle A to the *nearest degree*. [10]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 1–16, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 3

(2) 4

(3) 4

(4) 2

(5) 1

(6) 3

(7) 2

(8) 4

(9) 1

(10) 4

(11) 1

(12) 1

(13) 4

(14) 2

(15) 4

(16) 1

(17) $\frac{T}{g - f}$

(18) 64

(19) 2

(20) 6

(21) $(\cot x - 3)(2 \cot x + 1)$

(22) $\frac{2}{3}$

(23) 1

(24) 59° 33′

(25) 205.3

(26) $\sqrt{\frac{1 + a}{2}}$

(27) 3

(28) 1

(29) $\sqrt{65}$

(30) $\frac{\sin A}{\sqrt{1 - \sin^2 A}}$

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 3.1 and — 0.6 [8]
b II and III [2]

(32) $x = 2,\ y = 3,\ z = 4$ [8]
Check [2]

(33) *b* $y = \cos 2x$ [2]

(34) *a* $x =$ first number

$$\frac{2x}{x-1} = \frac{8x}{2x} \quad or \quad \frac{2x}{x-1} = 4 \qquad [5]$$

b $x =$ length of one leg

$x^2 + (14 - x)^2 = 100$ [5]

(36) *a* 72 [10]
b 28.2 [10]

*(37) 52 [10]