The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, August 16, 1966 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now complete the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Express $\frac{2}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator.

2 If the number 0.0000004 is expressed as 4×10^n, what is the value of n?

3 If x varies inversely as y and if $x = 9$ when $y = 1$, find the value of x when $y = 3$.

4 If $x = \frac{a-b}{b}$, express $\frac{x+3}{x-2}$ in simplest form in terms of a and b.

5 Express in factored form: $25 \tan^2 x - 16$.

6 If $\log_x 64 = 3$, find the value of x.

7 The graphs of $y = \sin x$ and $y = \frac{1}{2}$ are drawn on the same set of axes from $x = 0$ to $x = 2\pi$ radians. How many points do they have in common?

8 If x is greater than $0°$ but less than $180°$, find in degrees the value of x which satisfies the equation $\sin^2 x + \sin x - 2 = 0$.

9 Write an equation for the line whose slope is -1 and which passes through the point $(0,3)$.

10 Find the value of $(x-2)^{\frac{3}{2}} + 3(x-3)^0$ if $x = 6$.

11 Express in radians the angle through which the hour hand of a clock moves in 4 hours. [Leave answer in terms of π.]

12 If x is an acute angle and $\log \tan x = 0.0723$, find x to the *nearest minute*.

13 Find the fifth term of a geometric progression, the first two terms of which are, respectively, $\sqrt{2}$ and 2.

14 In triangle DEF, $DE = 14$, $DF = 4$, and angle $D = 55°$. Find to the *nearest integer* the area of the triangle.

15 In $\triangle ABC$, if $A = 150°$, $a = 5$, and $\sin B = \frac{2}{5}$, find the value of b.

Directions (16–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

16 The positive acute angle x which satisfies the statement $x = 2 \arccos \frac{\sqrt{3}}{2}$ is

(1) $\frac{\pi}{6}$ (3) $\frac{2\pi}{3}$

(2) $\frac{\pi}{3}$ (4) $\frac{5\pi}{6}$

17 In triangle ABC, if angle $C = 120°$, an expression for c^2 in terms of a and b is

(1) $a^2 + b^2$ (3) $(a-b)^2$

(2) $a^2 + ab + b^2$ (4) $(a+b)^2$

18 If $\cos x = 0.8$ and x is a positive acute angle, the value of $\tan \frac{1}{2} x$ is

(1) $\frac{3}{2}$ (3) 3

(2) $\frac{1}{3}$ (4) $\frac{3}{4}$

19 If $\sin A = \frac{3}{5}$ and A is in the first quadrant, the value of $\sin 2A$ is

(1) $\frac{6}{5}$ (3) $\frac{24}{25}$

(2) $\frac{24}{5}$ (4) $\frac{12}{25}$

20 The expression $\cos(180° - A)$ is equivalent to

(1) $\cos A$ (3) $\sin A$

(2) $-\cos A$ (4) $-\sin A$

21 If $\csc 70° = k$, an expression for $\cos 20°$ in terms of k is

(1) $\frac{1}{k}$ (3) k^2

(2) k (4) $2k$

22 Given the data $A = 130°$, $a = 50$, and $b = 24$, which statement is correct?

1 Triangle ABC can be constructed in which angle B is acute.
2 Triangle ABC can be constructed in which angle B is right.
3 Triangle ABC can be constructed in which angle B is obtuse.
4 No triangle can be constructed with these parts.

23 The expression $\frac{x+2}{x+3} - \frac{x+4}{x-4}$ is meaningless if x has the value of

(1) -4 (3) 3

(2) -2 (4) 4

24 The graph of the equation $2x^2 + y = -3$ is

(1) an ellipse (3) a circle

(2) a parabola (4) a hyperbola

25 If x is a positive acute angle, then $\frac{\sin x}{\sqrt{1 - \sin^2 x}}$ is a ratio for

(1) $\cos x$ (3) $\sec x$

(2) $\tan x$ (4) $\csc x$

26 John can complete a certain job working alone in 3 hours. Henry can complete the same job working alone in 5 hours. The number of hours it will take both of them working together to complete the job is

(1) 8 (3) $\frac{15}{8}$

(2) $\frac{8}{15}$ (4) 4

27 An example of the distributive law is

(1) $a + b = b + a$
(2) $ab + ac = a(b + c)$
(3) $a(bc) = (ab)c$
(4) $a + (b + c) = (a + b) + c$

28 If the discriminant of a quadratic equation with rational coefficients is 4, the roots are

1 rational and unequal
2 rational and equal
3 imaginary
4 irrational and unequal

29 The equation $\sqrt{x^2 - 6x} + 4 = 0$ has

(1) 8 and -2 as its roots
(2) 8 as its only root
(3) -2 as its only root
(4) no roots

30 An equation of the axis of symmetry of the graph of $y = x^2 - \frac{1}{2}$ is

(1) $x = -\frac{1}{2}$ (3) $x = 0$

(2) $y = -\frac{1}{2}$ (4) $y = 0$

Answers to the following questions are to be written on paper supplied by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the roots of the equation $3x^2 - 4x - 2 = 0$. [8]

b If, in part *a*, $x = \cos \theta$, determine the quadrant(s) in which angle θ lies. [2]

32 Solve the following system of equations algebraically and check in both equations: [8, 2]

$$x^2 + y^2 - 2x = 3$$
$$3x + y = 1$$

33 *a* On the same set of axes sketch the graphs of $y = 2 \sin x$ and $y = \cos 2x$ as x varies from 0 to 2π radians. [4, 4]

b From the graphs drawn in answer to part *a*, determine the value(s) of x between 0 and 2π which will satisfy the equation $2 \sin x - \cos 2x = 3$. [2]

34 *a* Starting with the formula for the $\tan (x + y)$, *derive* a formula for $\tan 2x$ in terms of $\tan x$. [3]

b Determine a positive value of x less than 180° for which $\tan 2x$ is undefined. [2]

c Show that the following equality is an identity: [5]

$$\frac{\sec x \sec y}{\tan x - \tan y} = \frac{1}{\sin (x - y)}$$

35 Write an equation or a system of equations which can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a A commuter ordinarily travels the distance from his office to his home at the rate of 30 miles per hour in 50 minutes. Because of construction, he must now take another route which is 10 miles longer. At what rate in miles per hour must he now travel in order still to make the trip in 50 minutes? [5]

b When a certain 2-digit number is divided by the sum of its digits, the quotient is 7. However, if the digits are reversed and the new number is added to twice the original number, the sum is 216. Find the original number. [5]

36 Answer *either a* or *b* but *not* both:

a Given $\triangle ABC$ with $AB = 235$, $\angle A = 52° 10'$, and $\angle B = 73° 30'$. If CD, the altitude to AB, is drawn, find to the *nearest integer* the length of CD. [10]

OR

b A plot of ground has the form of a triangle whose sides are 60 yards, 80 yards, and 110 yards in length.

(1) Find to the *nearest degree* the largest angle of the triangle. [6]

(2) Find to the *nearest ten square yards* the area of the triangle. [4]

*37 A ship in distress is anchored 12.5 miles N 57° 20′ E from a lighthouse, and a freighter is 7.5 miles due south of the lighthouse. Using the law of tangents, find to the *nearest ten minutes* the course the freighter must take in order to reach the distressed ship. [6, 4]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 16–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) $\frac{6 + 2\sqrt{2}}{7}$

(2) -7

(3) 3

(4) $\frac{a + 2b}{a - 3b}$

(5) $(5 \tan x + 4)(5 \tan x - 4)$

(6) 4

(7) 2

(8) 90

(9) $y = -x + 3$

(10) 11

(11) $\frac{2}{3}\pi$

(12) 49° 45′

(13) $4\sqrt{2}$

(14) 23

(15) 4

(16) 2

(17) 2

(18) 2

(19) 3

(20) 2

(21) 1

(22) 1

(23) 4

(24) 2

(25) 2

(26) 3

(27) 2

(28) 1

(29) 4

(30) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 1.7 and —0.4 [8]
b II and III [2]

(32) $x = 1, y = -2$
$x = -\frac{1}{5}, y = \frac{8}{5}$ [8]
Check [2]

(33) *b* $\frac{\pi}{2}$ [2]

(34) *b* 45° [2]

(35) *a* r = rate of travel because of construction
$\frac{5}{6}r = 35$ [5]

b t = tens digit
u = units digit
$$\frac{10t + u}{t + u} = 7$$
$10u + t + 20t + 2u = 216$ [5]

(36) *a* 219 [10]
b (1) 103 [6]
(2) 2340 [4]

*(37) Analysis [6]
N. 36° 30′ E. [4]