The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, August 17, 1964 — 12:30 to 3:30 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Express $\frac{\sqrt{3}}{\sqrt{3}-1}$ as an equivalent fraction with a rational denominator.

2 Find the integral root of $2x^2 + 3x - 20 = 0$.

3 Express as a monomial in terms of i: $3\sqrt{-4} - \sqrt{-1}$

4 If A is an angle in the second quadrant and if $\tan A = -\frac{12}{5}$, find the value of $\sin A$.

5 The lengths of two sides of a triangle are 15 and 24, and the included angle contains 60°. Find the length of the third side.

6 Express sec (—125°) as a function of a positive acute angle.

7 A central angle of 2.25 radians intercepts an arc of 6.75 inches. Find the number of inches in the radius of the circle.

8 Find the arithmetic mean between the roots of the equation $x^2 - 6x + q = 0$.

9 Write an equation of the line through the point (—1,3) parallel to the line whose equation is $3x - 2y = 6$.

10 Find the sixth term of the geometric progression 4, —12, 36, ...

11 Given the formula $W = \frac{ad}{a+d}$. Express d in terms of W and a.

12 If θ is a positive acute angle, express $\cot \theta$ in terms of $\sin \theta$.

13 Find in degrees the value of y between 0° and 90° which satisfies the following system of equations:

$$2 \sec x - 4 \tan y = 0$$
$$\sec x + \tan y = \sqrt{3}$$

14 In triangle RST, $t = 8$, $r = 5\frac{1}{3}$ and $\sin T = \frac{1}{4}$. Find the number of degrees in the acute angle R.

15 If the logarithm of 64 to the base x is 3, find the numerical value of x.

16 Express $\frac{\frac{1}{a} + \frac{1}{b}}{\frac{2}{ab}}$ as an equivalent fraction in simplest form.

17 Find in degrees the smallest positive value of x if $\sec 3x = \csc 27°$.

18 If $\cos \theta = \frac{31}{50}$, find the positive value of $\cos \frac{\theta}{2}$.

19 Find the positive value of $\sin\left(\text{arc sec } \frac{5}{3}\right)$.

20 Express $\tan (45° + x)$ in terms of $\tan x$.

21 If x varies directly as y and if $x = 4$ when $y = 9$, find the value of y when $x = 6$.

22 Find the logarithm of 0.06386.

23 Find the value of x if $3^x = \frac{1}{81}$.

Directions (24–30) : Indicate the correct completion for *each* of the following by writing the number 1, 2, 3 or 4 in the space provided on the separate answer sheet.

24 A root of the equation $2 \sin x + \sqrt{3} = 0$ is

(1) 60° (2) 120° (3) 210° (4) 300°

25 When drawn on the same set of axes, the graph of $y = 3$ will intersect the graph of

(1) $y = 4 \sin x$
(2) $y = 2 \sin 2x$
(3) $y = \sin 4x$
(4) $y = 2 \sin x$

26 If the graph of $y = x^2 + 4x + C$ is tangent to the x-axis, then the roots of $x^2 + 4x + C = 0$ are

(1) real, unequal and irrational
(2) real, unequal and rational
(3) real, equal and rational
(4) imaginary

27 A rectangular swimming pool with dimensions x and y is bordered by a walk of uniform width a. The area of the walk is

(1) $(x + a)(y + a) - xy$
(2) $(x + 2a)(y + 2a) - xy$
(3) $2a(x + y)$
(4) $4a^2$

28 For all values of x, the expression

$$\cos 6x \cos 4x - \sin 6x \sin 4x$$

is equal to

(1) $\cos 2x$
(2) $\sin 2x$
(3) $\cos 10x$
(4) $\sin 10x$

29 An equation of the axis of symmetry of the graph of $y = 3x^2 - 9x + 8$ is

(1) $x = 3$
(2) $y = 3$
(3) $x = \frac{3}{2}$
(4) $y = \frac{3}{2}$

30 The equation $x + \sqrt{x - 2} = 4$ has

(1) only one real root
(2) two real roots
(3) one real root and one imaginary root
(4) no real roots

Answers to the following questions are to be written on paper furnished by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Solve for x: $(2x + 3)^2 - 17x - 10 = 0$ [Answers should be left in radical form.] [8]

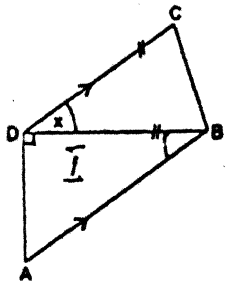
b If in part *a*, $x = \cos \theta$, determine the quadrant(s) in which angle θ lies. [2]

32 *a* Draw the graph defined by the equation $y = -x^2 - 2x + 3$, using all integral values of x from $x = -4$ to $x = 2$, inclusive. [6]

b Find a value of k for which the roots of the equation $-x^2 - 2x + 3 = k$ are imaginary. [2]

c Write an equation of the circle with its center at the origin and passing through the y-intercept of the graph drawn in part *a*. [2]

33 In the accompanying diagram, $AD \perp DB$, $AB \parallel DC$, $DB = DC$ and angle $CDB = x$.



a Show that the area of $\triangle ADB$ is $\frac{1}{2}(DB)^2 \tan x$. [4]

b Show that the area of quadrilateral $ABCD$ is $\frac{1}{2}(DB)^2(\sin x + \tan x)$. [6]

34 A formula for side a of $\triangle ABC$ in terms of the area K and the angles A, B and C is $a = \sqrt{\frac{2K \sin A}{\sin B \sin C}}$.

Using logarithms, find to the *nearest tenth* the length of side a of $\triangle ABC$ if $K = 820$, angle $A = 78^\circ\ 10'$ and angle $C = 43^\circ\ 20'$. [10]

35 When a two-digit number is divided by the sum of its digits, the quotient is 4 and the remainder is 9. If the digits are interchanged, then the new number exceeds the original number by 18. Find the original number. [*Only an algebraic solution will be accepted.*] [5, 5]

36 Answer *either* *a* or *b* but *not* both:

a A pilot flies at 333° 10′ (N 26° 50′ W) from his base B to a landing field C and then flies 319 miles at 42° 40′ (N 42° 40′ E) to an airport A. If the bearing of the airport from the base is 31° 20′ (N 31° 20′ E), find to the *nearest mile* the distance of the airport from the base. [6, 4]

OR

b A body is acted upon by two forces, one of 150 pounds and the other of 220 pounds, respectively. If the resultant is 280 pounds, find to the *nearest ten minutes* the angle between the resultant and the 150-pound force. [10]

*37 *a* Show that the following equality is an identity: [7]

$$\frac{\sin 3x - \sin x}{\cos 3x - \cos x} = \frac{\sin 10x + \sin 6x}{\cos 10x - \cos 6x}$$

b In $\triangle ABC$, $b = 8$, $c = 6$ and angle $A = 60^\circ$. Find in radical form the value of $\tan \frac{1}{2}(B - C)$. [3]

*This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS

Monday, August 17, 1964 — 12:30 to 3:30 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 24–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{3+\sqrt{3}}{2}$

(2) -4

(3) $5i$

(4) $\frac{12}{13}$

(5) 21

(6) $-\sec 55°$ *or* $-\csc 35°$

(7) 3

(8) 3

(9) $3x - 2y = -9$

(10) -972

(11) $\frac{aW}{a-W}$

(12) $\frac{\sqrt{1-\sin^2\theta}}{\sin\theta}$

(13) 30

(14) 30

(15) 4

(16) $\frac{a+b}{2}$

(17) 21

(18) $\frac{9}{16}$

(19) $\frac{4}{5}$

(20) $\frac{1+\tan x}{1-\tan x}$

(21) $13\frac{1}{2}$

(22) 8.8052 — 10

(23) -4

(24) 4

(25) 1

(26) 3

(27) 2

(28) 3

(29) 3

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* $\frac{5 \pm \sqrt{41}}{8}$ [8]

b II, III [2]

(32) *b* Accept any value for k greater than 4. [2]

c $x^2 + y^2 = 9$ [2]

(34) 52.4 [10]

(35) Analysis [5]
57 [5]

(36) *a* Analysis [6]
352 [4]

b 51° 20′ [10]

*(37) *b* $\frac{\sqrt{3}}{7}$ [3]