The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, August 19, 1963 — 12:30 to 3:30 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express the sum of $2i$ and $2\sqrt{-9}$ as a monomial in terms of i. 1.....

2 If $x = 2$, find the numerical value of $(3x + 2)^{-\frac{1}{3}} + 2x^0$. 2.....

3 Express $\dfrac{3}{2+\sqrt{2}}$ as an equivalent fraction having a rational denominator. 3.....

4 If an arc 15 feet long on a wheel subtends an angle of 2.5 radians at the center of the wheel, find the number of feet in the radius of the wheel. 4.....

5 Write the value of cot 29° 44′. 5.....

6 If $\log n = 1.8278$, find the value of n. 6.....

7 Express cos (—75°) as a function of a positive acute angle. 7.....

8 Write an equation of the line which passes through the point (0,4) and is parallel to the line whose equation is $2x + 3y = 5$. 8.....

9 Express in radical form the value of $2 \sin \dfrac{\pi}{4} - \sin \dfrac{\pi}{2}$. 9.....

10 A number of poles of uniform size are stacked in layers. There are 21 poles in the bottom layer and one less pole in each succeeding layer. There is only one pole in the top layer. What is the total number of poles in the stack? 10.....

[OVER]

11 Solve $\sqrt{5 \tan^2 x + 4} - 3 = 0$ for the number of degrees in the smallest positive value of x. 11.....

12 Find the value of cot (arc tan $\frac{3}{4}$). 12.....

13 If 0.00066 is written in the form 6.6×10^n, what is the value of n? 13.....

14 If one root of $x^2 - x - k = 0$ is -2, what is the value of k? 14.....

15 In triangle ABC, $a = 6$, $\sin A = 0.3$ and $\sin B = 0.6$. Find the value of b. 15.....

16 In triangle ABC, $b = 6$, $c = 4$ and $\cos A = -\frac{1}{4}$. Find the value of a. 16.....

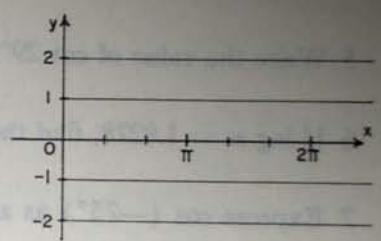
17 Express in simplest form: $\dfrac{2 + \dfrac{1}{x}}{4 - \dfrac{1}{x^2}}$ 17.....

18 In an isosceles triangle, the equal sides are each 20 and the base angles are each 65°. Find to the *nearest integer* the area of the triangle. 18.....

19 Solve the equation $\frac{1}{r} + \frac{1}{s} = \frac{1}{t}$ for s in terms of r and t. 19.....

20 Solve for x: $3^{x-1} = 27^x$ 20.....

21 On the coordinate axes at the right, *sketch* the graph of $y = \sin x$ for values of x from 0 to 2π radians.



Directions (22–28): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

22 If $\cos \theta = \frac{7}{25}$, the positive value of $\tan \frac{\theta}{2}$ is 22.....

(1) $\frac{3}{4}$

(2) $\frac{4}{3}$

(3) $\frac{12}{7}$

(4) $\frac{12}{25}$

23 The graph of $2x^2 - 9 = 2y^2$ is

(1) a circle
(2) an ellipse
(3) a parabola
(4) a hyperbola

23.....

24 The numbers $\sqrt{2}$, 2, $\sqrt{8}$ taken in the given order form

(1) a geometric but not an arithmetic progression
(2) an arithmetic but not a geometric progression
(3) both a geometric and an arithmetic progression
(4) neither a geometric nor an arithmetic progression

24.....

25 The graph of $y = x^2 + kx + 9$ will be tangent to the x-axis if k equals

(1) $+18$ or -18
(2) $+6$ or -6
(3) $+3$ or -3
(4) 0

25.....

26 The period of the function defined by the equation $y = 2 \sin 3x$ is

(1) π
(2) $\frac{\pi}{3}$
(3) $\frac{2\pi}{3}$
(4) $\frac{3\pi}{2}$

26.....

27 If y varies inversely as x and a value of x is multiplied by 2, then the corresponding value of y is multiplied by

(1) $\frac{1}{2}$
(2) 2
(3) $\frac{1}{4}$
(4) 4

27.....

28 The expression $\log \sqrt[3]{\frac{10}{x}}$ is equal to

(1) $3(1 - \log x)$
(2) $3(10 - \log x)$
(3) $\frac{1 - \log x}{3}$
(4) $\frac{10 - \log x}{3}$

28.....

Directions (29–30): Indicate whether each of the following statements is true for

(1) all real values of x,
(2) some but not all real values of x,
(3) no real values of x,

by writing on the line at the right the number 1, 2 or 3.

29 $1 + \cos 2x = 2 \cos^2 x$ 29.....

30 $-\sqrt{x} + 5 = 3$ 30.....

[OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find the values of θ between 0° and 360° which satisfy the equation $\sin^2 \theta - 3 \sin \theta + 1 = 0$. [Express $\sin \theta$ to the *nearest hundredth* and θ to the *nearest degree*.] [10]

32 *a* Draw the graph defined by the equation $y = x^2 + 2x - 4$, using all integral values of x from $x = -4$ to $x = 2$, inclusive. [6]

b On the same set of axes used in part *a*, draw the graph defined by the equation $y = 2$. [2]

c Using the graphs made in answer to part *a* and part *b*, find to the *nearest tenth* the roots of the equation $x^2 + 2x - 4 = 2$. [2]

33 Given the formula $C = 0.0408LD^2$. If $L = 525$ and $C = 12$, use logarithms to find the positive value of D to the *nearest hundredth*. [10]

34 To find the height of a vertical tower standing on a level plain, two points, A and B, are located on the plain in line with the foot of the tower. At A the angle of elevation of the top of the tower is 21° 10′; at B the angle is 36° 40′. The distance from A to B is 570 feet. Find to the *nearest foot* the number of feet in the height of the tower. [10]

35 A rectangular plot is 120 feet by 90 feet. If a strip of uniform width is cut from two adjacent sides of the plot, the area is reduced by 2,000 square feet. Find the number of feet in the width of the strip. [5,5]

36 *a* Starting with a formula for $\cos (x + y)$, *derive* a formula for $\cos 2x$ in terms of $\sin x$. [5]

b Prove that the following equality is true for all values of A for which both members are defined (that is, prove that the equality is an identity): [5]

$$\tfrac{1}{2} \sin 2A = \frac{\tan A}{1 + \tan^2 A}$$

*37 Two forces of 57 pounds and 39 pounds, respectively, act on a body at an angle of 69° with each other. Using the law of tangents, find to the *nearest ten minutes* the angle formed by the resultant and the greater force. [10]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS

Monday, August 19, 1963 — 12:30 to 3:30 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $8i$

(2) $\frac{5}{2}$

(3) $\frac{6-3\sqrt{2}}{2}$

(4) 6

(5) 1.7508

(6) 67.27

(7) cos 75° *or* sin 15°

(8) $2x + 3y = 12$

(9) $\sqrt{2}-1$

(10) 231

(11) 45

(12) $\frac{5}{2}$

(13) -4

(14) 6

(15) 12

(16) 8

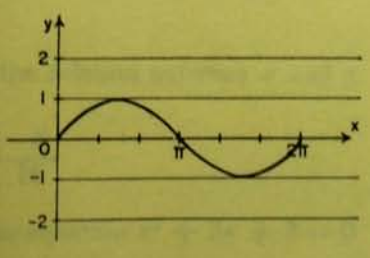
(17) $\frac{x}{2x-1}$

(18) 153

(19) $\frac{rt}{r-t}$

(20) $-\frac{1}{2}$

(21)



(22) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 22°, 158° [10]

(32) *c* 1.6 and —3.6 [Allow 1.5 *or* 1.7 and —3.5 *or* —3.7] [2]

(33) 0.75 [10]

(34) 460 [10]

(35) Analysis [5]
10 [5]

*(37) 27° 10′ [10]