The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, August 20, 1962 — 12:30 to 3:30 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\frac{3}{3+\sqrt{5}}$ as an equivalent fraction with a rational denominator. 1.....

2 If $x = 64$, what is the value of $3x^0 - 3x^{-\frac{1}{3}}$? 2.....

3 Express the sum of $-4i$ and $\sqrt{-16}$ as a single term. 3.....

4 Find log 0.8924. 4.....

5 Find the smallest positive value of θ to the *nearest minute* if $\theta = \arccos 0.8560$. 5.....

6 Solve for x: $2^{x+3} = 4^x$ 6.....

7 The sum of the first 17 terms of an arithmetic progression is 85. If the first term is 2, find the 17th term. 7.....

8 Express $\cos(270° - \theta)$ as a function of θ. 8.....

9 Find the numerical value of $\tan \frac{3\pi}{4} + \sin \frac{\pi}{6}$. 9.....

10 Find the number of degrees in the smallest positive value of x which satisfies the equation $\sqrt{\tan^2 x + 1} = 2$. 10.....

11 If 0.00123 is written in the form 1.23×10^n, what is the value of n? 11.....

12 Write an equation of the line which passes through the point $(0, 0)$ and is parallel to the line $4x - 2y = 7$. 12.....

13 In triangle ABC, $b = 3$, $B = 30°$ and $C = 120°$. Find c. 13.....

14 What is the minimum value of $2 \cos \frac{1}{2}x$? 14.....

15 In triangle ABC, $a = 3$, $b = 5$ and $c = 7$. Find the value of the cosine of the largest angle. 15.....

16 If θ is an obtuse angle and $\cos \theta = -\frac{20}{29}$, find the value of $\tan \frac{1}{2}\theta$. 16.....

17 Find the coordinates of the minimum point of the graph of $y = x^2 - 6x + 5$. 17.....

18 Express the sum of $\sqrt{75}$ and $6\sqrt{\frac{1}{3}}$ as a single term. 18.....

19 Find the number of inches in the radius of a circle in which an arc of 4 inches subtends a central angle of 2 radians. 19.....

20 The area of triangle ABC is 10. If $a = 10$ and $b = 18$, find the value of $\sin C$. 20.....

Directions (21–27) : Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

21 The expression $\frac{1 - \sin^2 x}{\cot^2 x}$ is equivalent to

(1) 1 (2) $\sin^2 x$ (3) $\frac{\cos^4 x}{\sin^2 x}$ (4) $\tan^2 x$ 21.....

22 The graph of $4x^2 - 4y^2 = 9$ is

(1) a hyperbola (2) a parabola (3) an ellipse (4) a circle 22.....

23 The period of the curve $y = 2 \cos \frac{1}{2}x$ is

(1) $\frac{\pi}{2}$ (2) π (3) 2π (4) 4π 23.....

24 L varies inversely as the square of d. If d is multiplied by 3, then L is

(1) multiplied by 3
(2) divided by 3
(3) multiplied by 9
(4) divided by 9 24.....

25 Which of the following is a value of x which satisfies the equation $\sin^2 2x + \sin 2x - 2 = 0$?

(1) 30° (2) 45° (3) 60° (4) 90° 25.....

26 The expression $(a + b)\log x$ is equivalent to

(1) $\log ax + \log bx$
(2) $\log (a + x) + \log (b + x)$
(3) $a \log x + b \log x$
(4) $\log abx$ 26.....

27 In the equation $x^2 + bx + c = 0$, the sum of the roots is p and the product of the roots is q. The discriminant of the equation is

(1) $-p^2 - 4q$ (2) $q^2 - 4p$ (3) $p^2 - 4q$ (4) $p^2 + 4q$ 27.....

Directions (28–30): Indicate whether *each* of the following statements is true for

(1) all real values of x,
(2) some, but not all, real values of x,
(3) no real value of x,

by writing on the line at the right the number 1, 2 or 3.

28 $\dfrac{3}{x^2 + 4} = -1$ 28.....

29 $\sin\left(\dfrac{\pi}{2} + x\right) = \cos(-x)$ 29.....

30 $\cos 4x = 1 - 2\sin^2 2x$ 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find the values of θ between $0°$ and $360°$ which satisfy the equation $2\cos 2\theta - 11\cos\theta - 1 = 0$. [Express approximate answers to the *nearest degree*.] [10]

32 The diameter d of a sphere whose volume is V is given by the formula $d = \sqrt[3]{\frac{6V}{\pi}}$.

By means of logarithms, find to the *nearest thousandth of an inch* the diameter of a spherical ball bearing whose volume is 0.254 cubic inch. [Use the approximation $\pi = 3.14$.] [10]

33 *a* On the same set of axes, *sketch* the graphs of $y = 2\sin 2x$ and $y = \cos x$ in the interval from 0 to 2π, inclusive. [Label each curve with its equation.] [5, 3]

b From the graphs made in answer to part *a*, determine the number of values of x in the interval from 0 to 2π for which $2\sin 2x = \cos x$. [2]

34 *a* The three positive numbers $\frac{1}{x}$, $2 + \frac{1}{x}$ and $4 + \frac{5}{x}$ form a geometric progression in that order. Find x. [6]

b Find *both* number pairs which satisfy the following system of equations: [4]

$$xy + x = 0$$
$$x + y = 0$$

35 Ordinarily two pumps working together can fill a tank in 12 hours. However, after the two pumps had been operating together for 4 hours, one broke down. It then required 12 more hours for the second pump to complete filling the tank. Find the number of hours required for *each* pump to fill the tank alone. [5, 5]

36 *a* Prove that the following equality is an identity: [7]

$$\tan\theta + \frac{2 - 4\sin^2\theta}{\sin 2\theta} = \cot\theta$$

b Starting with the formula for $\tan(x + y)$, derive the formula for $\tan(x - y)$. [3]

*37 Town B is 100 miles due south of town A on a coastline. A yachtsman has sailed 45.7 miles from B in the direction N 37° 40′ E to a point C. Using the law of tangents, find to the *nearest ten minutes* the new course to sail directly from C to A. [4, 6]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING ELEVENTH YEAR MATHEMATICS

Monday, August 20, 1962 — 12:30 to 3:30 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{9 - 3\sqrt{5}}{4}$

(2) $2\frac{1}{4}$

(3) 0

(4) 9.9506 — 10

(5) 31° 8′

(6) 3

(7) 8

(8) $-\sin\theta$

(9) $-\frac{1}{2}$

(10) 60

(11) —3

(12) $y = 2x$

(13) $3\sqrt{3}$ *or* 5.2

(14) —2

(15) $-\frac{1}{2}$

(16) $\frac{7}{3}$

(17) (3, —4)

(18) $7\sqrt{3}$

(19) 2

(20) $\frac{1}{9}$

(21) 2

(22) 1

(23) 4

(24) 4

(25) 2

(26) 3

(27) 3

(28) 3

(29) 1

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 104° and 256° [10]

(32) 0.786 [10]

(33) *b* 4 [2]

(34) *a* 1 [6]
b (0, 0) and (1, —1) [4]

(35) Analysis [5]
18 and 36 [5]

*(37) Analysis [4]
N 23° 40′ W *or* 336° 20′ [6]