The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, August 21, 1961 — 12 m. to 3 p.m., only

Name of pupil.....Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\dfrac{1}{\sqrt{3}+1}$ as an equivalent fraction with a rational denominator. 1.....

2 Find the value of $(2x)^0 + 8x^{\frac{2}{3}}$ if $x = 8$. 2.....

3 If s varies directly as the square of t and if $s = 45$ when $t = 3$, find the constant of variation. 3.....

4 Find the slope of the graph whose equation is $2x + 3y = 6$. 4.....

5 The sum of the roots of the equation $ax^2 + 6x - 8 = 0$ is 12. Find the value of a. 5.....

6 Solve for x: $\sqrt{x^2 - 4} = x - 2$ 6.....

7 Solve for x: $\dfrac{1}{x} - \dfrac{1}{a} = 1$ 7.....

8 If the numbers 3, x, y and -24 form a geometric progression in this order, find the value of x. 8.....

9 In triangle ABC, $a = 6$, $b = 3$ and $A = 150°$. Find the value of $\sin B$. 9.....

10 Find the number whose logarithm is 9.8783–10. 10.....

[OVER]

11 Find cot $32° 54'$. 11.....

12 Find the numerical value of $\cos \frac{2\pi}{3}$. 12.....

13 If x is an obtuse angle, express $\cos x$ in terms of $\sin x$. 13.....

14 If θ is an acute angle and $\cos \theta = m$, express $\cos \frac{\theta}{2}$ in terms of m. 14.....

15 Each of the equal sides of an isosceles triangle is a and a base angle is $15°$. Express the area of the triangle in terms of a. 15.....

16 Find the number of degrees in the positive acute angle x if $\sin^2 x - \cos^2 x = 0$. 16.....

17 If θ is an angle in quadrant IV and $\theta = \text{arc cos } \frac{1}{2}$, find the value of $\sin \theta$. 17.....

18 Express $\tan \left(\frac{\pi}{4} + x \right)$ in terms of $\tan x$. 18.....

19 In triangle ABC, $a = 3$, $b = 3$ and $c = 2$. Find the value of $\cos C$. 19.....

Directions (20–27): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement.

20 The value of $\frac{1}{\frac{1}{2} + \frac{1}{2}}$ is

(1) 1
(2) 2
(3) 3
(4) 4

20.....

21 If a, b and c are *positive unequal* numbers, the graph of $ax^2 + by^2 = c$ is

(1) a circle
(2) a parabola
(3) an ellipse
(4) a hyperbola

21.....

22 If the discriminant of a quadratic equation with integral coefficients is 1, the roots are

(1) rational and equal
(2) rational and unequal
(3) irrational and unequal
(4) imaginary

22.....

23 Log $\frac{x^3}{100}$ equals

(1) $3 \log x - 2$
(2) $\frac{3 \log x}{2}$
(3) $3x - 100$
(4) $\frac{3x}{100}$

23.....

24 The tens digit of a two-digit number is 3 times the units digit. If x represents the units digit, then the number can be represented by

(1) $4x$
(2) $11x$
(3) $13x$
(4) $31x$

24.....

25 In triangle ABC, $a = 12$, $b = 9$ and $A = 42°$. These data determine for B

(1) two values, one less than 90° and one greater than 90°
(2) two values, both less than 90°
(3) exactly one value, that value being less than 90°
(4) exactly one value, that value being greater than 90°

25.....

26 An equivalent expression for $\sqrt{-8}$ is

(1) $8i$
(2) $-\sqrt{8}$
(3) $2i\sqrt{2}$
(4) $-2\sqrt{2}$

26.....

27 The minimum value of $\frac{1}{2} \sin 2x$ is

(1) -1
(2) -2
(3) $-\frac{1}{2}$
(4) $\frac{1}{2}$

27.....

Directions (28–30): If the blank space in each statement below is replaced by the word *always, sometimes* (but not always) or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

28 The expression $\cos (A - B)$ is ... equal to $\cos (B - A)$. 28.....

29 If $A + B = 90°$, $\sin^2 A + \sin^2 B$ is ... equal to 1. 29.....

30 If a, b and c represent *positive unequal* numbers,

$\frac{a + b}{c + b}$ is ... equal to $\frac{a}{c}$. 30.....

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Solve for x, expressing the answer in *inverse trigonometric form*: [8]

$\sin^2 x - 2 \sin x + \frac{1}{4} = 0$

b Using the result obtained in part *a*, determine the quadrant(s) in which angle x lies. [2]

32 *a* On the same set of axes, draw the graphs of $y = \frac{1}{2}x^2$ and $y = -x + 3$. [5, 3]

b From your graph, estimate to *tenths* the roots of the equation $\frac{1}{2}x^2 = -x + 3$. [2]

33 *a* Starting with a formula for $\sin(x + y)$, develop a formula for $\sin(x - y)$. [3]

b Prove that the following equality is an identity: [7]

$\sin(x + y) \sin(x - y) = \sin^2 x - \sin^2 y$

34 The area of a regular pentagon inscribed in a circle of radius R is given by the formula $A = \frac{5R^2}{2} \sin 72°$. By means of logarithms, find R to the *nearest integer* if $A = 340$. [10]

35 Three positive numbers a, b and c form an arithmetic progression in this order.

a Express b in terms of a and c. [4]

b If the ratio of a to b is $\frac{5}{12}$, find the ratio of a to c. [6]

36 *a* Two forces of 8 pounds and 6 pounds, respectively, act on a body so that the magnitude of the resultant is 9 pounds. Find to the *nearest degree* the angle between the two forces. [8]

b If the angle between the 8-pound force and the 6-pound force changes, the magnitude of the resultant changes. When the angle between the two forces is 90°, what is the value of the magnitude of the resultant? [2]

*37 In triangle ABC, angle C is 120° and side a is twice side b.

a Using the law of tangents, show that $\tan \frac{1}{2}(A - B) = \frac{\sqrt{3}}{9}$. [5]

b Using the result in part *a*, find angle A to the *nearest degree*. [5]

*This question is based upon an optional topic in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 20–27, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{\sqrt{3}-1}{2}$

(2) 33

(3) 5

(4) $-\frac{2}{3}$

(5) $-\frac{1}{2}$

(6) 2

(7) $\frac{a}{a+1}$

(8) -6

(9) $\frac{1}{4}$

(10) 0.7557

(11) 1.5458

(12) $-\frac{1}{2}$

(13) $-\sqrt{1-\sin^2 x}$

(14) $\sqrt{\frac{1+m}{2}}$ *or* $\frac{1}{2}\sqrt{2+2m}$

(15) $\frac{1}{4}a^2$

(16) 45

(17) $-\frac{\sqrt{3}}{2}$

(18) $\frac{1+\tan x}{1-\tan x}$

(19) $\frac{1}{9}$

(20) 1

(21) 3

(22) 2

(23) 1

(24) 4

(25) 3

(26) 3

(27) 3

(28) always

(29) always

(30) never

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(31) *a* $x = \text{arc sin } \frac{2-\sqrt{3}}{2}$ [8]

b I, II [2]

(32) *b* 1.6 and —3.6 [2]

(34) 12 [10]

(35) *a* $\frac{a+c}{2}$ [4]

b $\frac{5}{19}$ [6]

(36) *a* 101 [8]

b 10 [2]

(37) *b* 41 [5]