The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, August 23, 1960 — 12 m. to 3 p.m., only

Name of pupil..... Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $\frac{3}{\sqrt{5}-1}$ as an equivalent fraction with a rational denominator. 1.....

2 Write in *simplest form* the value of $3x^0 + x^{\frac{3}{3}}$ if $x = 8$. 2.....

3 Find cos 41° 12′. 3.....

4 Find the number whose logarithm is 9.8472—10. 4.....

5 Find the 20th term in the arithmetic progression 2, 5, 8, 11, ... 5.....

6 If angle A is in quadrant I, express tan A in terms of sin A. 6.....

7 Solve the equation $\sqrt{2 \cos x + 10} = 3$ for the smallest positive value of x. 7.....

8 In triangle ABC, $a = 5$, $b = 7$ and cos $C = \frac{1}{7}$. Find the length of side c. 8.....

9 Find the positive value of cos $\frac{1}{2}\theta$ if cos $\theta = \frac{1}{8}$. 9.....

10 In a geometric progression whose terms are all positive, the fifth term is 6 and the seventh term is 12. Find the sixth term of this progression. 10.....

11 Express cos 260° as a function of a positive acute angle. 11.....

12 Find the slope of the line whose equation is $x + 2y = 4$. 12.....

13 Factor: $15\cos^2 x + 7\cos x - 4$. 13.....

14 If the number 0.0068 is expressed in the form 6.8×10^n, find the value of n. 14.....

15 Using the formula $C = \frac{5}{9}(F - 32)$, find F if $C = 80$. 15.....

16 Solve for x: $\frac{1}{a} - \frac{1}{x} = \frac{1}{b}$ 16.....

17 What is the abscissa of the turning point of the graph whose equation is $y = x^2 + 6x + 8$? 17.....

18 Two sides of a triangle are 6 and 10 and the included angle is 120°. Find the area of the triangle. 18.....

19 In a circle whose radius is 20 inches, a central angle intercepts an arc of 15 inches. Find the number of radians in the central angle. 19.....

20 Find the sum of the roots of the equation $x^2 + 2x + 5 = 0$. 20.....

Directions (21–22): Indicate whether the following statements are true for

a all real values of x,
b some but not all real values of x,
c no real value of x,

by writing on the line at the right the letter *a*, *b* or *c*.

21 $\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$ 21.....

22 $x^2 + 4 = 0$ 22.....

Directions (23–30): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

23 Each of the following is an equivalent form of the expression $\tan A + \sin A$. Which one is an illustration of the commutative principle?

(1) $\frac{1}{\cot A} + \frac{1}{\csc A}$ (2) $\frac{\sin A}{\cos A} + \sin A$ (3) $\sin A + \tan A$

(4) $\cot (90° - A) + \cos (90° - A)$ 23.....

24 The period of the function $4 \sin \frac{1}{2}x$ is

(1) $\frac{1}{2}\pi$ (2) π (3) 2π (4) 4π 24.....

25 The positive value of $\sin \left(\text{arc cos } \frac{\sqrt{3}}{2} \right)$ is equal to

(1) 30° (2) 60° (3) $\frac{1}{4}$ (4) $\frac{1}{2}$ 25.....

26 When expressed in terms of the imaginary unit i, $\sqrt{-3}$ is

(1) $3i$ (2) $-3i$ (3) $i\sqrt{3}$ (4) $-i\sqrt{3}$ 26.....

27 The graph of $2x^2 + 3y^2 = 6$ is

(1) a circle (2) an ellipse (3) a hyperbola (4) a parabola 27.....

28 A value for y which satisfies the equation $\tan^2 y = 1$ is

(1) π (2) 2π (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$ 28.....

29 If $A = 90°$ and $B = 30°$, then $\sin A - \sin B$ equals

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $1 - \frac{\sqrt{3}}{2}$ (4) $\frac{\sqrt{3}}{2}$ 29.....

30 As x increases from 0 to π radians, the graphs of $y = \cos x$ and $y = \frac{1}{2}$, when drawn on the same axes,

(1) are tangent (2) intersect in two points (3) intersect in only one point (4) do not intersect 30.....

Part II

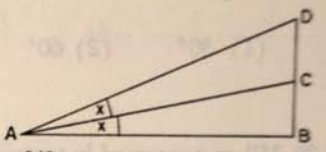
Answer four questions from this part. Show all work unless otherwise directed. Only an algebraic solution will be accepted in 33.

31 Find to the *nearest tenth* the roots of the equation $2x^2 - 9x = 1$. [10]

32 An observer in a boat finds the angle of elevation of a beacon light on a mountain top to be 46° 40′. The boat is due east of the beacon light. After the boat moves 1,000 feet farther east, the new angle of elevation of the beacon light is 42° 10′. Find to the *nearest ten feet* the height of the beacon light above the eye of the observer. [5, 5]

33 How many pounds of water must be evaporated from 84 pounds of a 20% salt solution to raise it to a 35% salt solution? [6, 4]

34 *a* In the accompanying diagram, angle B is a right angle and AC bisects angle DAB in triangle ABD. Using this diagram, *derive* the formula $AC = \dfrac{AD \cos 2x}{\cos x}$. [6]



b If $AD = 210$ feet and $x = 34°$, find AC to the *nearest foot*. [4]

35 *a* Starting with a formula for $\cos 2x$, *derive* the formula for $\cos \frac{1}{2}\theta$ in terms of $\cos \theta$. [5]

b Show that the following equality is an identity: [5]

$$\frac{\cos (x - y)}{\sin x \sin y} = 1 + \cot x \cot y$$

36 *a* Draw the graph of $y = x^2 + 3x - 2$, using all integral values of x from $x = -5$ to $x = 2$, inclusive. [6]

b Using the graph made in answer to part *a*, find to the *nearest tenth* the roots of the equation $x^2 + 3x - 2 = 1$. [4]

*37 Solve the following equation for all values of x greater than 0° but less than 90°: [10]

$$\frac{\sin 5x + \sin x}{\cos 5x + \cos x} = \frac{1}{3}\sqrt{3}$$

*This question is based upon an optional topic in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING ELEVENTH YEAR MATHEMATICS

Tuesday, August 23, 1960 — 12 m. to 3 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $\frac{3\sqrt{5}+3}{4}$

(2) 7

(3) 0.7524

(4) 0.7033

(5) 59

(6) $\frac{\sin A}{\sqrt{1-\sin^2 A}}$ *or* $\frac{\sin A\sqrt{1-\sin^2 A}}{1-\sin^2 A}$

(7) 120° *or* $\frac{2\pi}{3}$

(8) 8

(9) $\frac{3}{4}$

(10) $6\sqrt{2}$

(11) $-\cos 80°$ *or* $-\sin 10°$

(12) $-\frac{1}{2}$

(13) $(5\cos x+4)(3\cos x-1)$

(14) -3

(15) 176

(16) $\frac{ab}{b-a}$

(17) -3

(18) $15\sqrt{3}$

(19) $\frac{3}{4}$

(20) -2

(21) a

(22) c

(23) 3

(24) 4

(25) 4

(26) 3

(27) 2

(28) 4

(29) 1

(30) 3

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(31) 4.6 and —0.1 [10]

(32) Analysis [5]
6220 [5]

(33) Analysis [6]
36 [4]

(34) *b* 95 [4]

(36) *b* Allow 0.7, 0.8 *or* 0.9
and —3.7, —3.8 *or* —3.9 [4]

(37) 10°, 70° [10]