June 21, 1983

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided.

1. Express $\sqrt{-16} + \sqrt{-25}$ as a monomial in terms of i.   1_____

2. If $\frac{1}{2}$ of a certain job can be done in 4 days, how much of the job can be done in one day?   2_____

3. Solve the following system of equations for y:
$$2x + y = 4$$
$$x - 2y = 2$$
          3_____

4. Find the amplitude of the graph of the function $y = 4 \sin 2x$.   4_____

5. In right triangle ABC, $m\angle C = 90$, and $\sec A = 13/12$. Find the numerical value of $\csc A$.   5_____

6. Find the two binomial factors of $ax + xy + ay + y^2$.   6_____

7. If $f(x) = 4 \sin x + 3$, find the numerical value of $f(30°)$.   7_____

8. Solve for x in terms of a, b, and c: $a = \dfrac{2x}{b} + c$ 8_____

9. Write an equation of the line that passes through the points $(0,0)$ and $(2,4)$.   9_____

10. If y varies directly as x, and $y = 10$ when $x = \frac{1}{5}$, find the value of y when $x = \frac{1}{2}$.   10_____

11. In how many points do the graphs of $x^2 + y^2 = 4$ and $y = -2$ intersect?   11_____

12. How many distinct triangles can be constructed if $m\angle A = 60$, $a = 9$, and $b = 10$?   12_____

13. If $\log \tan x = 9.5267 - 10$, find angle x to the *nearest minute*.   13_____

14. If the graph of the function $f(x) = x^2 - 6x + k$ is tangent to the x-axis, find the value of k.   14_____

Directions (15-30): Write in the space provided the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

15. For which ordered pair is the fraction $\dfrac{3}{x - y}$ undefined?

(1) $(1,-1)$ (2) $(3,2)$ (3) $(-2,-3)$ (4) $(1,1)$ 15____

16. If $2^{2x+1} = 32$, what is the value of x? (1) 1 (2) 2 (3) 3 (4) 5   16____

17. The expression 2.3×10^{-5} is equal to   (1) 0.0000023
(2) 0.000023   (3) 230,000   (4) 2,300,000   17____

18. Which represents an irrational number?   (1) $\sin \dfrac{\pi}{4}$

(2) $\sin \dfrac{\pi}{2}$ (3) $\sqrt[3]{8}$ (3) $\sqrt{4}$ 18____

19. Which of the following functions both have negative values when the angle lies in Quadrant II?   (1) cosine and secant   (2) sine and tangent   (3) cosecant and cotangent   (4) cosecant and cosine   19____

20. The expression $\dfrac{\sin \theta}{\tan \theta}$ is equivalent to   (1) $-\cos \theta$
(2) $\cos \theta$ (3) $1 - \cos \theta$ (4) $1 + \cos \theta$ 20____

21. The expression $\dfrac{\sqrt{2}}{3} + \dfrac{2}{\sqrt{2}}$ is equivalent to   (1) $\dfrac{3\sqrt{2}}{5}$
(2) $\dfrac{4\sqrt{2}}{3}$ (3) $3\sqrt{2}$ (4) $\dfrac{2}{3}$ 21____

22. In triangle ABC, $\sin A = 0.8$ and $a = 4$. The value of the ratio $\dfrac{\sin B}{b}$ is   (1) 1/5   (2) 1/2   (3) 2/1   (4) 5/1   22____

23. The expression $\tan 320°$ is equivalent to   (1) $\tan 40°$
(2) $\cot 50°$ (3) $-\tan 40°$ (4) $-\cot 40°$ 23____

24. The expression $\log x - \frac{1}{2} \log y$ is equivalent to   (1) $\log \dfrac{x}{\sqrt{y}}$
(2) $\log \dfrac{2x}{y}$ (3) $\log y\sqrt{x}$ (4) $\log (x - \sqrt{y})$ 24____

25. If $\sqrt{x + 1} + 2 = 0$, what is the solution set for all real values of x?   (1) $\{-1\}$ (2) $\{2\}$ (3) $\{3\}$ (4) $\{\ \}$ 25____

26. The expression $\left(\dfrac{x^2 y^3}{ab^0} \right)^{-1}$ is equivalent to   (1) $x^{1/2} y^{1/2}$
(2) $ax^2 y^3$ (3) $\dfrac{a}{x^2 y^3}$ (4) $\dfrac{x^2 y^3}{a}$ 26____

27. The expression Arc tan $(\sqrt{3})$ is equal to   (1) 60°   (2) $-60°$
(3) 30°   (4) $-30°$ 27____

28. If $f(x) = |x^2 - 2|$, the value of $f(-1)$ is   (1) 1
(2) -1 (3) 3   (4) -3 28____

29. If a central angle of 2 radians intercepts an arc of 6 centimeters, the number of centimeters in the radius of the circle is   (1) 1/3   (2) $\pi/3$ (3) 3   (4) 3π 29____

30. In triangle ABC, $a = 6$, $b = 8$, and $\cos C = 3/8$. What is the length of side c?   (1) $\sqrt{118}$ (2) $\sqrt{82}$ (3) 10   (4) 8   30____

Part II

Answer four questions from this part.
Show all work unless otherwise directed.

31. *a* Find, to the *nearest tenth*, the roots of the equation $2x^2 - 4 = 5x$. [8]
     b If, in the equation given in part *a*, x is replaced with $\sin \theta$, determine the quadrant(s) in which angle θ may lie.   [2]

32. The members of a running club planned to contribute equally to raise $900 to sponsor a marathon run. When it was discovered that $1,200 would be needed instead, three members of the club withdrew. Each of the remaining members had to increase his contribution by $100 to raise the necessary amount. How many members were there in the club originally?   [10]

33. *a* A pendulum of length L makes a complete oscillation (swing and return) in t seconds, where $t = \sqrt{\dfrac{L}{24.8}}$. Using logarithms, find the value of t, to the *nearest hundredth*, if $L = 8.5$.   [6]
     b If $a^x = b$, solve for x in terms of log a and log b.   [2]
     c If log $2 = p$ and log $3 = q$, express log 12 in terms of p and q.   [2]

34. *a* Sketch and label the graph of $y = \tan x$ for $-\pi \le x \le \pi$.   [4]
     b On the same set of axes used in part *a*, sketch and label the graph of $y = 2 \cos x$ for $-\pi \le x \le \pi$.   [4]
     c From the graphs made in answer to parts *a* and *b*, determine the number of values of x in the interval $-\pi \le x \le \pi$ which satisfy the equation $\tan x = 2 \cos x$.   [2]

35. An ice skater starts from point A and skates in a straight line across a frozen lake to point B, which is 630 meters away. Then she skates directly to point C. If the measure of angle ABC is 47° 30' and the measure of angle CAB is 31° 20', find, to the *nearest meter*, the distance from C to A.   [10]

36. *a* Beginning with the formula for cosine $(A + B)$, derive a formula for cos $2A$ in terms of cos A.   [5]
     b Prove that the following equality is an identity:
        $\sin^2 x(1 + \tan^2 x) = \tan^2 x$ [5]

*37. *a* On the same set of axes graph the system of inequalities, A and B, where: $A = \{(x,y) \mid xy \le 8\}$
                $B = \{(x,y) \mid y > 3x - 6\}$ [4, 3]
     b Label the solution set of the system, $A \cap B$, with an R.   [1]
     c Choose a point within solution set R and check its coordinates in each inequality.   [2]
* This question is based on an optional topic in the syllabus.