June 21, 1982

## Part I

*Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of $\pi$ or in radical form. Write your answers in the spaces provided on the answer sheet.*

1. For what value of $x$ is the function $y = \frac{4}{x}$ undefined? 1_____

2. If $\cos\theta$ is positive and $\sin\theta$ is negative, in which quadrant does angle $\theta$ lie? 2_____

3. If $y$ varies directly as $x$ and $y = 10$ when $x = \frac{1}{2}$, what is the value of $y$ when $x = 2$? 3_____

4. What is the *smallest* integral value of $x$ for which the expression $\sqrt{7 - 2x}$ is an imaginary number? 4_____

5. If $f(x) = 2x^0 + x^{-2}$, find $f(3)$. 5_____

6. Factor completely: $5x^3 - 20x$ 6_____

7. In a circle, a central angle of 1 radian intercepts an arc 9 centimeters long. How many centimeters are in the length of the radius of the circle? 7_____

8. Find the positive value of $k$ that will make the roots of the equation $9x^2 + kx + 1 = 0$ real, rational, and equal. 8_____

9. In triangle $RST$, $r = 10$, $s = 15$, and $\sin R = \frac{1}{5}$. Find the numerical value of $\sin S$. 9_____

10. The graphs of $y = \sin x$ and $y = \cos x$ are drawn on the same set of axes. In how many points will the graphs intersect if $0 \leq x \leq 2\pi$? 10_____

11. In triangle $ABC$, if $a = 5$, $b = 7$, and $c = 8$, what is the value of $\cos C$? 11_____

12. For what value of $b$ will the equation of $bx^2 - bx - 2 = 0$ have 2 as a root? 12_____

13. Find the value of tan 27° 43′ to four decimal places. 13_____

*Directions* (14-30): Write in the space provided on the answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

14. The sum of $\sqrt{-8}$ and $\sqrt{-18}$ in terms of $i$ equals
(1) $5i$ (2) $13i$ (3) $5i\sqrt{2}$ (4) $13i\sqrt{2}$ 14____

15. Given: $\frac{x + 20}{x - 10} = b$

If $b = 2$, then $x$ is equal to (1) 0 (2) 10 (3) 30 (4) 40 15____

16. Which value of $x$ is in the solution set of the inequality $x > 2x - 1$? (1) 1 (2) $-1$ (3) 3 (4) $\frac{3}{2}$ 16____

17. The equation $\sqrt{x + 18} + 2 = x$ is satisfied when $x$ is equal to (1) both 7 and $-2$ (2) $-2$, only (3) 7, only (4) neither 7 nor $-2$ 17____

18. The product of which of the following pairs of complex numbers may be expressed as a real number? (1) $(1 + 2i)(1 - 2i)$ (2) $(2 + 3i)(2 + 3i)$ (3) $(2 + 5i)(5 - 2i)$ (4) $(1 + 3i)(1 + 3i)$ 18____

19. The secant of an angle can *not* be equal to (1) (2) 2 (3) 3 (4) 0 19____

20. The expression Arc sin (sin 150°) is equal to (1) $-30°$ (2) 30° (3) 60° (4) 150° 20____

21. The expression $3 \log x - \frac{1}{2} \log y$ is equivalent to (1) $\log \frac{x^3}{\sqrt{y}}$ (2) $\log x^3 - \log \frac{y}{2}$ (3) $\log \frac{3x}{2y}$ (4) $\log \frac{\sqrt[3]{x}}{2y}$ 21____

22. The expression $\frac{\cot x}{\csc x}$ is equivalent to (1) $\cos x$ (2) $\csc x$ (3) $\sec x$ (4) $\sin x$ 22____

23. Which value of $x$ satisfies the equation $6 \tan x - 2 = 4$? (1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$ 23____

24. Which equation defines a relation that is *not* a function? (1) $y = \sin x$ (2) $y = x^2$ (3) $y^2 = -x^2 + 9$ (4) $y = \frac{1}{x}$ 24____

25. Which value of $x$ satisfies the equation $\sin (2x + 30°) = \cos (3x + 20°)$? (1) 8° (2) 2° (3) 10° (4) 28° 25____

26. An equivalent expression for $\log_b x = a$ is (1) $b^a = x$ (2) $a^a = b$ (3) $a^b = x$ (4) $x^a = b$ 26____

27. The expression $\tan (x + 45°)$ is equivalent to (1) $\frac{\tan x - 1}{1 + \tan x}$ (2) $\frac{\tan x + 1}{1 - \tan x}$ (3) $\frac{\tan x}{1 + \tan x}$ (4) $\frac{\tan x}{1 - \tan x}$ 27____

28. If $a$, $b$, and $c$ represent real numbers, which property describes the statement $(a + b)c = c(a + b)$? (1) the commutative property of addition (2) the associative property of multiplication (3) the associative property of addition (4) the commutative property of multiplication 28____

29. In triangle $ABC$, if $b = 20$ and $m\angle A = 30$, it is possible to construct two distinct triangles when side $a$ is (1) 8 (2) 10 (3) 15 (4) 20 29____

30. Which is the equation of a line perpendicular to the line whose equation is $3x - 2y = 6$? (1) $3x - 2y = 8$ (2) $3x + 2y = 4$ (3) $2x - 3y = 8$ (4) $2x + 3y = 4$ 30____

## Part II

*Answer four questions from this part. Show all work unless otherwise directed.*

31. *a* Find, to the *nearest tenth*, the roots of the equation $2(x^2 + 1) = 7x$. [8]

*b* If, in the equation given in part *a*, $x$ is replaced with $\cos\theta$, determine the quadrant(s) in which angle $\theta$ may lie. [2]

32. *a* On the same set of axes, sketch and label the graphs of $y = \frac{1}{2}\cos 2x$ and $y = \tan x$ for values of $x$ in the interval $0 \leq x \leq 2\pi$. [4, 4]

*b* Throughout which interval is $\tan x$ less than $\frac{1}{2}\cos 2x$?

(1) $0 < x < \frac{\pi}{2}$ (2) $0 < x < \pi$ (3) $\frac{\pi}{2} < x < \pi$

(4) $\pi < x < 2\pi$ [2]

33. *a* Find all values of $\theta$ in the interval $0 < \theta < 360°$ that satisfy the equation $\sin 2\theta - \sin\theta = 0$. [6]

*b* Prove that the following equality is an identity:

$$\frac{\cos x}{\tan x} = \csc x(1 - \sin^2 x) \quad [4]$$

34. *a* Using logarithms, find the value of $n$ to the *nearest hundredth*: $n = \sqrt[4]{0.461}$ [4]

*b* If $3^n = 8$, find the value of $x$ to the *nearest tenth*. [4]

*c* Find the coordinates of the point that the graphs of $y = \log_2$ and $y = \log_{10} x$ have in common. [2]

35. Write an equation or system of equations which can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

*a* The tens digit of a two-digit number is three times the units digit. The number obtained by reversing the digits is 18 less than the original number. Find the original number. [5]

*b* How many ounces of pure acid must be added to 30 ounces of a 20% solution of acid to make it a 50% solution? [5]

36. Answer *either* *a* or *b*, but *not* both.

*a* In triangle $ABC$, $m\angle A = 50$, $a = 48$, $b = 62$, and angle $B$ is obtuse. Find $m\angle C$ to the *nearest degree*. [10]

*b* Town $A$ is 24 kilometers from town $C$.
Town $B$ is 30 kilometers from town $C$.
If $m\angle ACB = 68$, find, to the *nearest kilometer*, the distance between towns $A$ and $B$. [10]

*37. Solve the system of equations and check.

$$\begin{aligned} 3x + y - 4z + 2 &= 0 \\ x + y + 2z + 1 &= 0 \\ x + 2y + 6z + 2 &= 0 \end{aligned}$$ [8, 2]

* This question is based on an optional topic in the syllabus.