The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR

MATHEMATICS

Monday, June 18, 1979 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" which you may need to answer some questions in this examination are stapled in the center of this booklet.

When you have completed the examination, you must sign the statement printed at the end of the answer paper, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer paper cannot be accepted if you fail to sign this declaration.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Express 46.2×10^{-3} as a decimal.

2 If one root of the equation $x^2 - 12x + k = 0$ is 4, what is the value of k?

3 Solve for x: $3^{x+1} - 5 = 22$

4 Find the two binomial factors of $ax + bx + ay + by$.

5 Find the solution set of the equation $\sqrt{3x + 1} - 3 = 1$.

6 If $\cos A = \sin 23°$, find the number of degrees in acute angle A.

7 In triangle ABC, $b = 5$, $c = 4$, and $\cos A = \frac{1}{8}$. Find the numerical value of a.

8 If $\sin x = a$ and $\cos x = b$, express $\sin 2x$ in terms of a and b.

9 What is the measure of the positive acute angle A which satisfies the equation $4 \sin^2 A - 1 = 0$?

10 The point $(\frac{\pi}{2}, k)$ lies on the graph of the function $y = 2 \sin x$. What is the value of k?

11 Find, to the *nearest minute*, the positive acute angle whose tangent is 0.6273.

12 In triangle ABC, $\sin A = 0.2$, $m\angle B = 30$, and $b = 5$. What is the length of side a?

13 Using the following system of equations, write an equation in x and y which does *not* contain a:
$$x = 2a + 3$$
$$y = 3a + 2$$

Directions (14–30): Write in the space provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

14 Which value of x satisfies the inequality $|x| + 2 < 5$?
(1) -5 (3) -3
(2) -2 (4) 4

15 The expression $\frac{x}{x + 2} - \frac{x}{x - 4}$ is meaningless when x is equal to
(1) -2 (3) 3
(2) 2   (4) -4

16 Which is an equation of a line whose slope is undefined?
(1) $y = x$ (3) $x = -5$
(2) $y = -5$ (4) $x + y = 5$

17 The product of $\frac{x^2 - 1}{x}$ and $\frac{4x^2}{x + 1}$ is
(1) $\frac{x - 1}{4x^3}$ (3) $4x(x + 1)$
(2) $\frac{(x^2 - 1)(x + 1)}{4x^3}$ (4) $4x(x - 1)$

18 Which expression represents the reciprocal of $2 + \frac{1}{x}$?
(1) $2 + x$ (3) $\frac{x}{2x + 1}$
(2) $\frac{1 + 2x}{x}$ (4) $\frac{x + 2}{2x}$

19 The expression $\frac{5 - \sqrt{3}}{3}$ is equivalent to
(1) $\frac{9}{5 - \sqrt{3}}$ (3) $\frac{1}{15 - \sqrt{3}}$
(2) $\frac{22}{3(5 + \sqrt{3})}$ (4) $\frac{22}{5 + \sqrt{3}}$

20 If $x^8 - y^8$ is divided by $x^4 - y^4$, then the quotient is
(1) $x^2 + y^2$ (3) $x^4 + x^2y^2 + y^4$
(2) $x^4 + y^4$ (4) $x^4 - x^2y^2 + y^4$

21 In a rectangle with perimeter p, the length is three more than the width. What is the width, expressed in terms of p?

(1) $\dfrac{p}{4}$

(3) $\dfrac{p+6}{4}$

(2) $\dfrac{p}{4} - 3$

(4) $\dfrac{p-6}{4}$

22 Which statement is true?
(1) $\log (A - B) = \log A - \log B$

(2) $\log (A - B) = \log \left(\dfrac{A}{B}\right)$

(3) $\log \left(\dfrac{A}{B}\right) = \dfrac{\log A}{\log B}$

(4) $\log \left(\dfrac{A}{B}\right) = \log A - \log B$

23 If $f(x) = x^2 - 2x + 4$, what is $f(i)$ where $i = \sqrt{-1}$?
(1) $1 - 2i$ (3) $3 - 2i$
(2) $-2i$ (4) $5 - 2i$

24 Which is an equation of the axis of symmetry of the graph of $y = x^2 - 4x + 1$?
(1) $y = 2$ (3) $y = -2$
(2) $x = 2$ (4) $x = -2$

25 Which statement is represented by the formula $yx^2 = k$?
(1) x varies inversely as the square of y.
(2) y varies directly as the square of x.
(3) y varies inversely as the square of x.
(4) x varies directly as the square of y.

26 The value of sin (Arc sin 1 + Arc cos 1) is

(1) 1

(3) $\dfrac{\sqrt{3}}{2}$

(2) $\dfrac{1}{2}$

(4) 0

27 Expressed as a function of a positive acute angle, sec 283° is equal to
(1) cos 13°   (3) sec 77°
(2) sec 13°   (4) csc 77°

28 The value of $\cos \dfrac{\pi}{2} + \sin \dfrac{\pi}{4}$ is

(1) $\dfrac{2 + \sqrt{2}}{2}$

(3) $\dfrac{\sqrt{2}}{2}$

(2) $\sqrt{2}$

(4) $1 + \sqrt{2}$

29 In a circle with its center at the origin, a central angle of 1 radian subtends an arc 5 units long. Which is an equation of the circle?
(1) $x + y = 5$ (3) $x^2 + y^2 = 10$
(2) $x^2 + y^2 = 5$ (4) $x^2 + y^2 = 25$

30 What is the maximum value of the function
$y = \dfrac{1}{2} \sin 2x$?

(1) 1

(3) $\dfrac{1}{2}$

(2) 2

(4) 0

☞ GO RIGHT ON TO THE NEXT PAGE.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the values of tan x which satisfy the equation
$3 \tan^2 x + 2 \tan x - 4 = 0$.   [8]

b Using the answers obtained in part *a*, determine the number of values of x in the interval $0° \le x < 360°$ which satisfy the equation
$3 \tan^2 x + 2 \tan x - 4 = 0$.   [2]

32 *a* Using logarithms, find the value of n to the *nearest thousandth*:

$$n = \frac{\sqrt[3]{0.863}}{56}$$ [6]

b If $\log 2 = a$, $\log 3 = b$, and $\log 5 = c$, express $\log 60$ in terms of a, b, and c.   [4]

33 *a* Sketch the graph of $y = 3 \sin 2x$ in the interval $0 \le x \le 2\pi$.   [6]

b On the same set of axes, draw the graph of $y = -3$.   [2]

c What is a value of x between 0 and 2π radians for which $3 \sin 2x = -3$?   [2]

34 Write an equation or system of equations which can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a The length of a rectangle is 5 units more than twice the width. The number of square units in the area is 4 less than the number of units in the perimeter. What are the dimensions of the rectangle?   [5]

b In a tank, there are 50 liters of water and 20 liters of acid. How many liters of water must be evaporated to make a solution which is 40% acid?   [5]

35 *a* Starting with the formula for $\cos (A + B)$, derive the formula for $\cos (A - B)$.   [4]

b Prove the identity:

$$\cos (60° + B) + \cos (60° - B) = \frac{1}{\sec B}$$ [6]

36 From point A on level ground, the angle of elevation of the top of a vertical control tower BD is 43°. At point C, 200 feet closer to the foot of the tower, the angle of elevation is 56°. Find to the *nearest foot* the height of the tower.   [10]
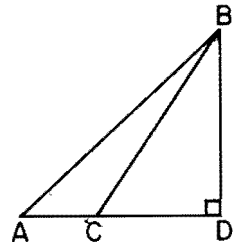
*37 If the integers 3, x, and y form an *increasing* arithmetic progression in that order, and if 3, $x + 4$, and $3y + 9$ form a geometric progression in that order, find the value(s) of x and y. [*Only an algebraic solution will be accepted.*]   [10]

* This question is based on an optional topic in the syllabus.