The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS A

Monday, June 16, 1975 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

The "Reference Tables for Mathematics" which you may need to answer some questions in this examination are stapled in the center of this booklet.

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DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

Directions (1–18): Write in the spaces provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

1 What is the solution set of the equation $2x^2 - 10 = 0$?

(1) $\{\sqrt{5}, -\sqrt{5}\}$
(2) $\{5, -5\}$
(3) $\{\sqrt{10}, -\sqrt{10}\}$
(4) $\{10, -10\}$

2 If $y = |x|$, as the value of x increases from -5 to 5, the value of y will

(1) increase, only
(2) decrease, only
(3) first increase and then decrease
(4) first decrease and then increase

3 In $\triangle ABC$, $b = 2$, $c = 3$, and $\sin C = 0.63$. What is the value of $\sin B$?

(1) 0.21
(2) 0.31
(3) 0.42
(4) 0.67

4 The solutions of the system of equations, $xy = 4$ and $y = x$, are found in which quadrant or quadrants?

(1) I, only
(2) I and III
(3) III, only
(4) II and IV

5 If $x + 3$ is a factor of $x^2 + 11x + h$, the value of h is

(1) 24
(2) 27
(3) 33
(4) 42

6 The identity
$\sin B (\cos B + \sin B) = (\cos B + \sin B) \sin B$
is an illustration of the

(1) distributive law of multiplication of real numbers with respect to addition
(2) identity element with respect to multiplication of real numbers
(3) commutative law of multiplication of real numbers
(4) associative law of addition of real numbers

7 The expression $\log 2xy$ is equivalent to

(1) $2(\log x + \log y)$
(2) $2(\log x)(\log y)$
(3) $2 \log x + \log y$
(4) $\log 2 + \log x + \log y$

8 At a party there are b boys and g girls. After two boys leave, what is the ratio of the number of boys remaining to the number of children remaining at the party?

(1) $\frac{b-2}{g}$
(2) $\frac{b-2}{b+g}$
(3) $\frac{b}{g+2}$
(4) $\frac{b-2}{b+g-2}$

9 If x and y are positive acute angles and if $\sin x = \frac{4}{5}$ and $\cos y = \frac{8}{17}$, which is the value of $\cos (x + y)$?

(1) $\frac{84}{85}$
(2) $\frac{77}{85}$
(3) $-\frac{36}{85}$
(4) $-\frac{13}{85}$

10 Given the graphs of the equations:

(A) $y = 2 \sin \frac{1}{2}x$
(B) $y = 3 \sin \frac{1}{2}x$
(C) $y = 3 \sin 2x$

Which two of these curves, if any, have the same maximum value?

(1) A and B
(2) A and C
(3) B and C
(4) no two

11 In which equation is the sum of the roots equal to $\frac{5}{3}$?

(1) $3x^2 + 5x - 9 = 0$
(2) $3x^2 - 5x + 9 = 0$
(3) $3x^2 + 7x - 5 = 0$
(4) $3x^2 + 9x + 5 = 0$

12 What is the numerical value of $\sin \frac{\pi}{6} + \cos \frac{\pi}{2}$?

(1) $\frac{1}{2}$
(2) $\frac{3}{2}$
(3) $\frac{\sqrt{3}}{2}$
(4) $\frac{2+\sqrt{3}}{2}$

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13 The expression $\dfrac{2(1 + \cos A)}{\sin^2 A + \cos A + \cos^2 A}$ is equivalent to

(1) 1
(2) 2
(3) $\dfrac{2}{\sin A}$
(4) $\dfrac{2}{\cos A}$

14 The graph of the equation $2x^2 = 5 - 2y^2$ is
(1) a circle
(2) a straight line
(3) a parabola
(4) a hyperbola

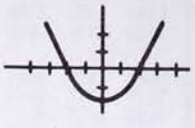
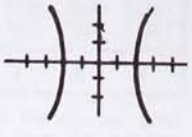
15 The slope of a line which is perpendicular to the line that passes through the points (3,1) and (3,5) is
(1) undefined
(2) 0
(3) 3
(4) -3

16 The *smallest* positive value of x which satisfies the equation $2\cos^2 x + \cos x - 1 = 0$ is

(1) 0
(2) $\dfrac{\pi}{6}$
(3) $\dfrac{\pi}{4}$
(4) $\dfrac{\pi}{3}$

17 If $m\angle A = 35$, $b = 30$, and $a = 20$, then what type of triangle, if any, can be constructed?
(1) a right triangle, only
(2) two distinct triangles
(3) one obtuse triangle, only
(4) no triangle

18 Which is the graph of a quadratic relation for which the domain consists of all the real numbers?

(1) 
(2) 
(3) 
(4) 

Directions (19–30): Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

19 If $2^x = 7$, then what is the value of x to the *nearest tenth*?

20 A boy has \$2.00 in nickels and quarters. If the ratio of nickels to quarters is 3 to 1, how many quarters does he have?

21 Express, in degrees, an angle of $\dfrac{7\pi}{3}$ radians.

22 Find the solution set of $4^{x-1} = 2^x$.

23 Express $\dfrac{1 - \dfrac{1}{x}}{x - 2 + \dfrac{1}{x}}$ as an equivalent fraction in *simplest form*.

24 If $S = \dfrac{n}{2}(a + \ell)$, express n in terms of a, ℓ, and S.

25 Solve for $\sin x$: $\sqrt{1 - \sin x} = \frac{1}{2}$

26 Find the value of cos 43° 47′ to *four decimal places*.

27 Express, in terms of π, the value of Arc sin (1).

28 If x is a positive acute angle and $\cos x = \dfrac{\sqrt{21}}{5}$, find the numerical value of $\sin x$.

29 In triangle ABC, $b = 6$, $c = 10$, and $m\angle A = 30$. Find the area of triangle ABC.

30 Express cot 208° as a function of a positive acute angle.

GO RIGHT ON TO THE NEXT PAGE.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find, to the *nearest tenth*, the roots of the equation

$$4x^2 = 5x + 1 \quad [8]$$

b If, in part *a*, $x = \cos \theta$, determine the quadrant or quadrants in which angle θ lies. [2]

32 *a* On the same set of axes, sketch the graphs of $y = 2 \sin x$ and $y = \cos 2x$ for values of x in the interval $-\pi \leq x \leq \pi$.
[*Label each curve with its equation.*] [8]

b For how many values of x in the interval $-\pi \leq x \leq \pi$ does $2 \sin x = \cos 2x$? [2]

33 Write an equation or a system of equations that can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a Two motorists start from the same point at the same time and travel in opposite directions. One motorist travels 15 miles per hour faster than the other. In 5 hours they are 550 miles apart. Find the rate of speed for *each* motorist. [5]

b A chemist has one solution which is 35% pure acid and another which is 75% pure acid. How many cubic centimeters of *each* solution must be used to produce 80 cubic centimeters of solution which is 50% acid? [5]

34 Using logarithms, compute the value of N to the *nearest tenth*:

$$N = \frac{\sqrt[3]{(3.45)^2 \tan 15^\circ}}{0.18} \quad [10]$$

35 Answer *both a* and *b*.

a Starting with the formula for $\cos (x + y)$, derive the formula for $\cos 2x$ in terms of $\sin x$. [4]

b Prove the identity:

$$\frac{2 \sin^2 x}{\sin 2x} + \frac{1}{\tan x} = \sec x \csc x \quad [6]$$

36 Answer *either a* or *b* but *not* both.

a A diagonal of a parallelogram is 50 centimeters long and makes angles of 37° 10′ and 49° 20′, respectively, with the sides. Find the length of the *shorter* side of the parallelogram to the *nearest centimeter*. [10]

OR

b Two motorboats take off from a point T at the same time. One boat travels at 10 miles per hour to a point R and the other at 20 miles per hour to a point S. If the angle STR between the two paths of travel measures 105°, find, to the *nearest mile*, the distance between R and S after *two hours*. [10]

*37 Solve the following system of equations for x, y, and z. Check your solution.

$$\begin{aligned} 2x + y + z &= 2 \\ 4x - 2y - 3z &= -2 \\ 8x + 3y + 2z &= 5 \end{aligned} \quad [8,2]$$

* This question is based on one of the optional topics in the syllabus.

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, June 16, 1975 — 1:15 to 4:15 p.m., only

A

Part I Score: .....

Rater's Initials: .....

ANSWER SHEET

Pupil ..... Teacher .....

School .....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.	11.	21. .....
2.	12.	22. .....
3.	13.	23. .....
4.	14.	24. .....
5.	15.	25. .....
6.	16.	26. .....
7.	17.	27. .....
8.	18.	28. .....
9.	19.	29. .....
10.	20.	30. .....

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The declaration below should be signed when you have completed the examination.

I do hereby affirm, at the close of this examination, that I had no unlawful knowledge of the questions or answers prior to the examination, and that I have neither given nor received assistance in answering any of the questions during the examination.

_____

Signature

FOR TEACHERS ONLY

ELEVENTH YEAR MATHEMATICS

11 A

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Use only *red* ink or *red* pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

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SCORING KEY

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 1–18, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 1

(2) 4

(3) 3

(4) 2

(5) 1

(6) 3

(7) 4

(8) 4

(9) 3

(10) 3

(11) 2

(12) 1

(13) 2

(14) 1

(15) 2

(16) 4

(17) 2

(18) 3

(19) 2.8

(20) 5

(21) 420

(22) $\{2\}$ *or* $x = 2$

(23) $\frac{1}{x - 1}$

(24) $\frac{2S}{a + l}$

(25) $\frac{3}{4}$

(26) 0.7220

(27) $\frac{\pi}{2}$

(28) $\frac{2}{5}$

(29) 15

(30) cot 28° *or* tan 62°

[OVER]

Part II

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(31) *a* 1.4, —.2 [8]

b II and III [2]

(32) *b* two [2]

(33) *a* x = slower rate

$5x + 5(x + 15) = 550$ [5]

b x = amount of 35% acid

y = amount of 75% acid

$x + y = 80$

$.35x + 75y = .50(80)$ [5]

(34) 8.2 [10]

(36) *a* 30 [10]

OR

b 49 [10]

(37) $x = \frac{1}{2}$, $y = -1$, $z = 2$ [8]

Check [2]

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

B

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Part I

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1 Express 144° in radian measure.

2 Express $\frac{1}{3+\sqrt{2}}$ as an equivalent fraction with a rational denominator.

3 Solve for x: $2^{2x+1} = 8$

4 Solve for x: $\frac{1}{x} + \frac{1}{2} = \frac{1}{6}$

5 Solve for $\sin A$:

$$2 \sin A + 3 \cos A = 2$$
$$4 \sin A + 3 \cos A = 1$$

6 If the number 0.0000072 is written in the form 7.2×10^n, what is the value of n?

7 If the graphs of the equations $y = x + 2$ and $x^2 + y^2 = 16$ are drawn on the same set of axes, what is the total number of points in which the two graphs will intersect?

8 What is the numerical value of $\sin \frac{7\pi}{6}$?

9 Express in *simplest form*: $\dfrac{\frac{a^2}{4} - 1}{1 - \frac{a^2}{4}}$

10 In $\triangle ABC$, $b = 7$, $c = 4$, and $\cos A = \frac{5}{7}$. Find the numerical value of a.

11 Express $ax + 2x - ab - 2b$ as the product of two binomials.

12 The rate at which a man travels from city A to city B varies inversely as the time it takes to make the trip. If the man can make the trip in 2½ hours at 60 kilometers per hour, how many kilometers per hour must he travel in order to make the trip in 3 hours?

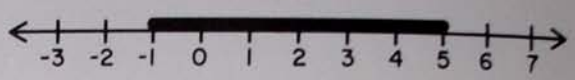
Directions (13-30): Write in the spaces provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

13 If $\sqrt{5x-1}$ is a real number, then which may be a value of x?

(1) -1 (3) 3

(2) 0 (4) $\frac{1}{10}$

14 The accompanying diagram represents the solution set of which inequality?



(1) $|x - 2| > 3$ (3) $|x + 2| \geq 3$

(2) $|x + 2| < 3$ (4) $|x - 2| \leq 3$

15 Which relation is also a function?

(1) $x^2 + y^2 = 25$ (3) $2x + 5y = 10$

(2) $2x^2 + 5y^2 = 50$ (4) $4x^2 - 9y^2 = 36$

16 If $\log_4 16 = x$, then the value of x is

(1) ½ (3) ¼

(2) 2 (4) 4

17 The expression $\frac{\sin 2x}{\cos 2x}$ is equivalent to

(1) $\tan x$ (3) $\frac{\sin x}{\cos x - 1}$

(2) $\tan 2x$ (4) $\frac{2}{\sin x - \cos x}$

18 Which expression is *not* equal to 50?

(1) $(\sqrt{50})^2$ (3) $(5\sqrt{2})^2$

(2) $(-\sqrt{50})^2$ (4) $(\sqrt{-50})^2$

19 The sum of the roots of the equation $2x^2 - 5x + 7 = 0$ is

(1) $\frac{5}{2}$ (3) $-\frac{5}{4}$

(2) $\frac{5}{4}$ (4) $\frac{7}{2}$

1,004

20 Which is an equation of the straight line that passes through the points $(-2,5)$ and $(-6,-3)$?

(1) $y = -2x$

(2) $y = -\frac{5}{2}x$

(3) $y = 2x + 9$

(4) $y = \frac{1}{2}x + 6$

21 In $\triangle ABC$, $a = 8$, $m\angle A = 45$, and $m\angle C = 30$. What is the length of side c?

(1) $4\sqrt{2}$

(2) $8\sqrt{2}$

(3) $\frac{8\sqrt{2}}{3}$

(4) 4

22 Which trigonometric function is negative and decreasing throughout the second quadrant?

(1) sine
(2) cosine
(3) tangent
(4) cosecant

23 The *smallest* positive angle whose cosine is -0.8718 is

(1) 119° 20′
(2) 150° 40′
(3) 209° 20′
(4) 240° 20′

24 If $\csc 70° = a$, then what is $\sin 110°$, expressed in terms of a?

(1) a

(2) $-a$

(3) $-\frac{1}{a}$

(4) $\frac{1}{a}$

25 The roots of the equation $x^2 - 4x + 7 = 0$ are

(1) imaginary
(2) real, rational, and equal
(3) real, rational, and unequal
(4) real, irrational, and unequal

26 Which value of x satisfies the equation $\cos^2 x - \cos x - 2 = 0$?

(1) 0°
(2) 90°
(3) 180°
(4) 270°

27 The tens digit of a two-digit number is twice the units digit. If the units digit is represented by u, the number, expressed in terms of u, is

(1) $13u$
(2) $21u$
(3) $3u$
(4) $11u + 20$

28 In radian measure, Arc cos $\frac{1}{2}$ + Arc tan 1 is

(1) $\frac{5\pi}{12}$

(2) $\frac{\pi}{2}$

(3) $\frac{2\pi}{3}$

(4) $\frac{7\pi}{12}$

29 The expression $\sin 75°$ equals

(1) $\sin 30° + \sin 45°$
(2) $\frac{1}{2} \sin 150°$
(3) $\sin 45° \cos 30° + \cos 45° \sin 30°$
(4) $\sin 45° \cos 30° - \cos 45° \sin 30°$

30 In $\triangle ABC$, $a = 4$, $b = 6$, and $m\angle C = 60$. What is the area of the triangle?

(1) $6\sqrt{3}$
(2) 6
(3) 12
(4) $12\sqrt{3}$

☞ GO RIGHT ON TO THE NEXT PAGE.

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find the value or values of tangent x which satisfy the equation $2 \tan x - 4 = \frac{7}{\tan x}$. [*Express the answer in radical form.*] [5]

b To the *nearest degree*, find an angle in quadrant IV which satisfies the equation given in part *a*. [5]

32 *a* Sketch and label the graph of $y = \sin 2x$ where $0 \leq x \leq 2\pi$ radians. [4]

b Using the same set of axes, sketch and label the graph of $y = 2 \cos \frac{1}{2}x$ where $0 \leq x \leq 2\pi$. [4]

c From the graphs drawn in parts *a* and *b*, determine a value of x where $\sin 2x - 2 \cos \frac{1}{2}x = 0$. [2]

33 The members of the Math Team agreed to contribute equally to purchase a small calculator that costs \$72. Four students were dropped from the Math Team so each student had to contribute an additional \$3. How many students were in the original group? [*Only an algebraic solution will be accepted.*] [10]

34 *a* Given the function $\{(x,y) \mid y = \sqrt{1 - \sin^2 x}\}$.

(1) What is the domain of this function? [1]

(2) What is the range of this function? [2]

(3) If $\cos x = \sqrt{1 - \sin^2 x}$, in which quadrant or quadrants must x lie ? [2]

b Prove the identity: $\sin A = \dfrac{\sec A}{\cot A + \tan A}$ [5]

35 Using logarithms, find the value of N to the *nearest tenth*:

$$N = \frac{29.8\sqrt[3]{0.034}}{\sin^2 34^\circ}$$ [10]

36 Answer *either a* or *b*, but *not* both.

a Two forces of 21 pounds and 15 pounds act on a body. The angle between the forces is $62^\circ\ 10'$. Find, to the *nearest pound*, the magnitude of the resultant. [10]

OR

b In triangle ABC, the measure of $\angle A = 24^\circ\ 30'$, the measure of $\angle B = 38^\circ\ 10'$, and $c = 35.5$ feet. Find, to the *nearest tenth* of a foot, the length of $\overline{AC}$. [10]

*37 *a* Graph $\{(x,y) \mid xy < -4 \text{ and } x + 2y \geq 2\}$ on the same set of axes. [8]

b State the coordinates of *one* point in the solution set of this system. [2]

*This question is based on an optional topic in the syllabus.

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Part I

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3	13	23 .....
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5	15	25 .....
6	16	26 .....
7	17	27 .....
8	18	28 .....
9	19	29 .....
10	20	30 .....

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(1) $\frac{4\pi}{5}$

(2) $\frac{3-\sqrt{2}}{7}$

(3) 1

(4) −3

(5) $-\frac{1}{2}$

(6) −6

(7) 2

(8) $-\frac{1}{2}$

(9) −1

(10) 5

(11) $(a+2)(x-b)$

(12) 50

(13) 3

(14) 4

(15) 3

(16) 2

(17) 2

(18) 4

(19) 1

(20) 3

(21) 1

(22) 2

(23) 2

(24) 4

(25) 1

(26) 3

(27) 2

(28) 4

(29) 3

(30) 1

[OVER]

Part II

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(31) *a* $\frac{4 \pm \sqrt{72}}{4}$ *or* $\frac{2 \pm 3\sqrt{2}}{2}$ [5]

b 312° [5]

(32) *c* π *or* 180° [2]

(33) 12 [10]

(34) *a* (1) all real x (any angle) [1]
(2) $0 \leq y \leq 1$ [2]
(3) I and IV [2]

(35) 30.9 [10]

(36) *a* 31 [10]
OR
b 24.7 [10]