The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR
MATHEMATICS

Monday, June 19, 1972 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Find the number of degrees in the positive acute angle x that satisfies the equation $\tan^2 x - 1 = 0$.

2 Express an angle of $240°$ in *radian measure*.

3 What is the numerical value of the product $\left(\cos \dfrac{\pi}{6}\right)\left(\sin \dfrac{\pi}{2}\right)$?

4 Solve for d: $t = a + (n - 1)d$

5 Find the value of $[(4)^{\frac{3}{2}}]^{-1}$.

6 On a trip a man drove 30 miles per hour for 2 hours. He then changed his rate to 40 miles per hour and drove for 3 more hours. Find his average rate, in miles per hour, for the whole trip.

7 If $x + 2$ is a factor of $x^2 + kx - 8$, find the numerical value of k.

8 In acute triangle ABC, $a = 5$, $b = 6$, and $\sin B = \frac{3}{5}$. Find the number of degrees in angle A.

9 In a set of rectangles, the length of each varies inversely as the width. If the length of a rectangle in the set is 16 feet when its width is 3 feet, find the number of feet in the width of a rectangle whose length is 8 feet.

10 Find the value of $\cos (\text{Arc sin } \frac{1}{2} + \text{Arc cos } \frac{1}{2})$.

11 Determine the value of $\tan 36°44'$, with accuracy to *four decimal places*.

12 Express $\sin (-42°)$ as a function of a positive acute angle.

13 Find the length of side c in $\triangle ABC$ if $a = 3$, $b = 5$, and $\cos C = -\frac{1}{2}$.

Directions (14–30): Write in the space provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

14 A number whose square is less than zero is called
(1) a whole number
(2) an imaginary number
(3) a negative real number
(4) a negative rational number

15 Which value of x satisfies the inequality
$3 + x < 3x - 5$?
(1) $x < 4$　　　　(3) $x > 4$
(2) $x \leqq 4$　　　　(4) $x \geqq 4$

16 For all values of x for which the expressions are defined, $\dfrac{1}{\dfrac{1}{x-1}+1}$ is equivalent to

(1) $\dfrac{x}{x-1}$　　　　(3) $\dfrac{1-x}{x}$

(2) $\dfrac{x-1}{x}$　　　　(4) $\dfrac{x}{1-x}$

17 The expression $\sin (A + B) + \sin (A - B)$ is equivalent to
(1) $2 \sin A \cos B$　　(3) $2 \cos A \cos B$
(2) $2 \sin B \cos A$　　(4) $-2 \sin A \sin B$

18 If the straight line having the equation $y = \frac{1}{2}$ is drawn on the same set of axes as the graph of $y = \frac{1}{2} \sin x$, the number of intersections between 0 and 2π would be
(1) 1　　　　(3) 0
(2) 2　　　　(4) 4

19 The graph of $ax^2 + by^2 = c$, in which a, b, and c are real numbers, is an ellipse if
(1) $a = b$, $a > 0$, $b < 0$, $c > 0$
(2) $a = b$, $a > 0$, $b > 0$, $c < 0$
(3) $a \neq b$, $a > 0$, $b > 0$, $c > 0$
(4) $a \neq b$, $a > 0$, $b < 0$, $c > 0$

20 If the solution for 26×13 were obtained as follows: $26 \times 13 = 26 \times (10 + 3) = 26 \times 10 + 26 \times 3 = 260 + 78 = 338$, which law was demonstrated?
(1) the commutative law for addition
(2) the associative law for multiplication
(3) the associative law for addition
(4) the distributive law for multiplication over addition

21 What is the domain of the function defined by the equation $f(x) = \sqrt{x - 4}$?
(1) $\{x \mid x \geq 4\}$
(3) $\{x \mid x \geq 0\}$
(2) $\{x \mid x \leq 4\}$
(4) $\{x \mid x \leq 0\}$

22 If the discriminant of a quadratic equation with rational coefficients is equal to -4, the roots are
(1) imaginary
(2) real and irrational
(3) real, rational, and unequal
(4) real, rational, and equal

23 If $\cos x > 0$, which must always be true?
(1) $\sin x > 0$
(3) $\sec x > 0$
(2) $\tan x > 0$
(4) $\csc x > 0$

24 The additive inverse of $2 - \sqrt{3}$ is
(1) 1
(3) $2 + \sqrt{3}$
(2) 0
(4) $-2 + \sqrt{3}$

25 Which statement concerning the roots of the equation $\sqrt{x^2 - 3x + 3} = 1$ is true?
(1) The equation has 1 as its only root.
(2) The equation has 2 as its only root.
(3) The equation has both 1 and 2 as its roots.
(4) The equation has neither 1 nor 2 as its roots.

26 The reciprocal of $3 - \sqrt{5}$ is
(1) $\dfrac{3 - \sqrt{5}}{4}$
(3) $\dfrac{3 + \sqrt{5}}{14}$
(2) $\dfrac{3 + \sqrt{5}}{4}$
(4) $\dfrac{3 - \sqrt{5}}{14}$

27 Given the relation $\{(1,2),(2,3),(3,4)\}$. What is the inverse of the relation?
(1) $\{(1,\frac{1}{2}),(2,\frac{1}{3}),(3,\frac{1}{4})\}$
(2) $\{(-1,-2),(-2,-3),(-3,-4)\}$
(3) $\{(3,4),(2,3),(1,2)\}$
(4) $\{(2,1),(3,2),(4,3)\}$

28 For all values of θ for which the expression is defined, $\sin \theta \left(\dfrac{1}{\sin \theta} - \sin \theta \right)$ is equivalent to
(1) $-\cos^2 \theta$
(3) $1 - \cos^2 \theta$
(2) $\cos^2 \theta$
(4) $1 + \cos^2 \theta$

29 If $m\angle A = 60$, $a = 5$, and $b = 15$, then the triangle ABC
(1) must be an acute triangle
(2) must be an obtuse triangle
(3) may be either an acute or an obtuse triangle
(4) cannot be constructed

30 The expression $\log \sin 2x$ is equivalent to
(1) $2 \log \sin x$
(2) $2 (\log \sin x + \log \cos x)$
(3) $\log 2 + \log \sin x + \log \cos x$
(4) $\log \sin x + \log \cos x$

Math. 11-June '72   [3]   [OVER]

Answers to the following questions are to be written on paper provided by the school.

Part II
Answer four questions from this part. Show all work unless otherwise directed.

31 Solve the following equation for all values of θ to the *nearest degree* in the interval $0° < \theta < 360°$:   [10]
$$5 \sin \theta + 2 \cos 2\theta - 3 = 0$$

32 Solve the following system of equations and check in *both* equations: [8,2]
$$x^2 - 2y^2 = 7$$
$$x - 2y = 5$$

33 Answer *both* a and b:
  a On the same set of axes, sketch the graphs of $y = \sin \frac{1}{2}x$ and $y = \cos x$ for all values of x which are in the interval $0 \leqq x \leqq 2\pi$.   [4,4]
  b From the graphs made in part a, determine the number of values of x that satisfy the equation $\sin \frac{1}{2}x = \cos x$ in the interval $0 \leqq x \leqq 2\pi$.   [2]

34 Write an equation or system of equations which can be used to solve *each* of the following problems. In each case, state what the variable or variables represent. [Solution of the equations is *not* required.] [10]
  a A class consists of 14 boys and 19 girls. How many girls should be taken out of this class so that the remaining girls will make up only 30% of the total number of pupils left in the class?
  b The rate of a stream in which a man rows a boat is 2 miles an hour. If it takes the man twice as much time to row upstream a distance of 30 miles as it takes him to return the same distance downstream, what is the rate of the boat in still water?

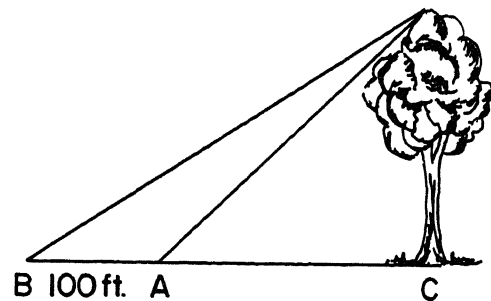35 Using logarithms, find, to the *nearest hundredth*, the value of: [10]
$$\frac{\sqrt[3]{93.1} \sin 29° \ 40'}{7.74}$$

36 Answer a or b but *not* both: [10]
  a A canoe race is to be run over a triangular course marked by buoys A, B, and C. The distance between A and B is 100 yards, that between B and C is 160 yards, and that between C and A is 220 yards. Find, to the *nearest degree*, the angle ABC.

OR

  b From a point, A, on one bank of a river, the angle of elevation of the top of a tree directly across the river on the other bank is $43° \ 20'$. As shown in the figure, point B is 100 feet behind A and in the same straight line as point A and point C (the foot of the tree). From point B the angle of elevation of the top of the tree is $32° \ 30'$.

B 100 ft. A   C

What is the length of $\overline{AC}$ to the *nearest foot*?

*37 a On a graph, indicate the solution set of
$$\{(x,y) \mid xy \geqq 8 \text{ and } y \leqq \tfrac{1}{2}x + 2\}.$$ [9]
  b State the coordinates of one point which is in the solution set of $\{(x,y) \mid y = \frac{1}{2}x + 2\}$, but is *not* included in the solution set of $\{(x,y) \mid xy \geqq 8 \text{ and } y \leqq \frac{1}{2}x + 2\}$.   [1]

* Based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 14–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 45

(2) $\dfrac{4\pi}{3}$

(3) $\dfrac{\sqrt{3}}{2}$ *or* 0.866

(4) $d = \dfrac{t - a}{n - 1}$

(5) ⅛

(6) 36

(7) —2

(8) 30

(9) 6

(10) 0

(11) 0.7463

(12) —sin 42° *or* —cos 48°

(13) 7

(14) 2

(15) 3

(16) 2

(17) 1

(18) 1

(19) 3

(20) 4

(21) 1

(22) 1

(23) 3

(24) 4

(25) 3

(26) 2

(27) 4

(28) 2

(29) 4

(30) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 14, 90, 166   [10]

(32) (3,—1) and (—13,—9)   [8]

(33) *b* 2   [2]

(34) *a* x = number of girls to be taken out

$$19 - x = .3(33 - x) \quad [5]$$

b x = rate of boat in still water

$$2\left(\frac{30}{x + 2}\right) = \frac{30}{x - 2} \quad [5]$$

(35) 0.46   [10]

(36) *a* 114   [10]

OR

b 208   [10]