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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Tuesday, June 16, 1970 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 What is the area of $\triangle ABC$ if $a = 20$, $b = 12$, and $C = 30°$?

2 Express in degrees an angle of $\dfrac{4\pi}{5}$ radians.

3 Find in degrees the acute angle x for which $\sin x = \cos (x + 10°)$.

4 In $\triangle ABC$, $a = 7.0$, $b = 20$, and $\sin A = 0.21$. Find the value of $\sin B$.

5 If $|x| = 4$ and $x + 3 > 0$, find the solution set.

6 Solve for the positive value of $\cos x$:
$2 \cos^2 x + \cos x - 1 = 0$

7 Solve for x: $\dfrac{2x + 1}{5} - \dfrac{3x - 7}{2} = 7$

8 Find the value of $\sin (\text{Arc cos } \%_{17})$.

9 Express tan 307° as a function of a positive acute angle.

10 Find the value of cos 29° 36'.

11 After a car's brakes are applied, the distance d a car travels before stopping varies directly as the square of its velocity v. If d is 32 feet when v is 40 m.p.h., find the distance d in feet when v is 60 m.p.h.

12 Write an equation of the straight line that is parallel to the line whose equation is $5x - y = 3$ and passes through the point $(0, -5)$.

13 In triangle ABC, $a = 4$, $b = 5$, and $\cos C = -\frac{1}{5}$. Find the value of c.

14 For all a and b, what is the additive inverse of the complex number $a + bi$?

15 In which quadrant does an angle lie whose cosecant is negative and whose secant is positive?

16 If the sum of the roots of $x^2 - x - 30 = 0$ is added to the product of the roots of this equation, what is the numerical value of the result?

17 The expression $\dfrac{ay + xy}{a^2 + 3ax + 2x^2}$, in which $a \neq -x$ and $a \neq -2x$, is equivalent to

(1) $\dfrac{-y}{2x}$

(3) $\dfrac{y}{a - 2x}$

(2) $\dfrac{y}{2x}$

(4) $\dfrac{y}{a + 2x}$

18 If $ar^n - rx = 0$, then an expression for x in terms of a, n, and r is

(1) $x = ar^n$

(3) $x = ar^{n-1}$

(2) $x = ar^{n+1}$

(4) $x = ar^{2n}$

19 Which equation can be used in finding n, when n is the *smallest* of three consecutive odd integers whose sum is s?

(1) $(n) + (n + 1) + (n + 2) = s$
(2) $(n) + (n + 1) + (n + 3) = s$
(3) $(n) + (n + 2) + (n + 4) = s$
(4) $(n) + (3n) + (5n) = s$

20 The graphs of $y = x^2$ and $y = 2x$ intersect in two points, one of which is the origin. What are the coordinates of the other point?

(1) $(1,2)$

(3) $(2,1)$

(2) $(2,4)$

(4) $(4,2)$

21 The expression $\dfrac{3}{\sqrt{3} - \sqrt{2}}$ is equal to

(1) $3\sqrt{6}$

(3) $\dfrac{(\sqrt{3} - \sqrt{2})}{2}$

(2) $3\sqrt{3} + 3\sqrt{2}$

(4) $\dfrac{3(\sqrt{3} + \sqrt{2})}{5}$

22 As angle x increases from $\dfrac{\pi}{4}$ to $\dfrac{3\pi}{2}$, $\cos x$

(1) increases throughout the interval
(2) decreases throughout the interval
(3) increases, then decreases
(4) decreases, then increases

23 For all values of θ for which the expression is defined, $\dfrac{\tan \theta}{\sin \theta}$ is equivalent to

(1) $\sec \theta$

(3) $\sin \theta$

(2) $\cot \theta$

(4) $\cos \theta$

24 The roots of $x^2 + 6x - 7 = 0$ are
(1) real, rational, and equal
(2) imaginary
(3) real, irrational, and unequal
(4) real, rational, and unequal

25 Which one of the following equations defines a relation which is *not* a function?

(1) $x^2 + 2 + 1 = y$

(3) $x^2 + y^2 = 16$

(2) $y = \sin x$

(4) $y = \tan x$

26 The graph of the equation $4x^2 - 100 = 25y^2$ is
(1) a hyperbola   (3) an ellipse
(2) a circle   (4) a parabola

27 Which expression is equivalent to $\dfrac{b \sin 2x}{\sin x}$?

(1) $2b$

(3) $2b \cos x$

(2) $2b \sin x$

(4) $\sin x$

28 The expression $2 \log x - \log y$ is equal to

(1) $\log \dfrac{x^2}{y}$

(3) $\dfrac{\log x^2}{\log y}$

(2) $\log \dfrac{2x}{y}$

(4) $\dfrac{\log 2 + \log x}{\log y}$

29 Because of the restrictions on the domain of the relation defined by $f(x) = \dfrac{3}{x - 3} - \dfrac{1}{x}$, which of the following can *not* be a subset of the domain?

(1) $\{1\}$

(3) $\{$all irrationals$\}$

(2) $\{0,3\}$

(4) $\{\ \}$

30 Using the data angle $A = 35°$, $b = 3$, and $a = 4$, it is possible to construct
(1) two distinct triangles
(2) a right triangle, only
(3) no triangles
(4) an obtuse triangle, only

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 a Find to the *nearest tenth* the roots of the equation $2x^2 + 2x - 1 = 0$. [8]

  b If in part a, $x = \csc \theta$, determine the quadrant(s) in which angle θ lies. [2]

32 a Sketch and label the graph of $y = \sin x$ as x varies from 0 to 2π radians. [3]

  b On the same set of axes sketch and label the graph of $y = \cos 2x$ as x varies from 0 to 2π radians. [5]

  c From a and b, determine the number of values of x between 0 and 2π radians that satisfy the equation $\sin x = \cos 2x$. [2]

33 Write an equation or a system of equations that can be used to solve *each* of the following problems. In each case state what the variable or variables represent. [*Solution of the equation is not required.*]

  a A two digit number is 16 less than 3 times the number obtained by reversing the digits. The tens digit is 2 more than twice the units digit. Find the original number. [5]

  b Around the outside of a picture, whose length is 2 inches more than its width, is a border of uniform width of 3 inches. If the area of the border is 72 square inches, what are the dimensions of the picture? [5]

34 Using logarithms, compute the value of N to the *nearest hundredth*: [10]

$$N = \frac{(5.12) \sqrt{\cos 12° \ 50'}}{\sqrt[3]{7.29}}$$

35 Answer *either* a or b, but *not* both:

  a Two straight roads RT and ST intersect at a town T and form with each other an acute angle of 67°. Towns at R and S are 22 miles and 31 miles respectively from T. Find to the *nearest mile* the distance between towns R and S. [4,6]

OR

  b A man at one point on the street finds that the angle of elevation of the top of a tower is 29° 50'. After walking toward the tower for 200 feet in a straight line, he finds that at the second point the angle of elevation of the top of the tower is 65° 20'. What is the height of the tower to the *nearest foot*? [10]

36 a Starting with the formulas for $\sin \frac{1}{2}\theta$ and $\cos \frac{1}{2}\theta$, *derive* a formula for $\tan \frac{1}{2}\theta$ in terms of $\cos \theta$. [Assume θ is an angle in the first quadrant.] [5]

  b For all values of x for which the expression is defined, show that the following equality is an identity:

$$\frac{\sin x + \tan x}{1 + \sec x} = \sin x \qquad [5]$$

*37 a Solve the following system of inequalities graphically: [8]

$$x^2 + y^2 < 16$$
$$y \geq x + 2$$

  b State the coordinates of *two* points which lie in the region representing the solution set. [2]

  *This question is based on an optional topic in the syllabus.

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ANSWER SHEET

Pupil..Teacher...

School..

Your answers to Part I should be recorded on this answer sheet.

Part I
Answer all questions in this part.

1................................ 11................................ 21................................

2................................ 12................................ 22................................

3................................ 13................................ 23................................

4................................ 14................................ 24................................

5................................ 15................................ 25................................

6................................ 16................................ 26................................

7................................ 17................................ 27................................

8................................ 18................................ 28................................

9................................ 19................................ 29................................

10................................ 20................................ 30................................

Your answers for Part II should be placed on paper provided by the school.

[7]

FOR TEACHERS ONLY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 60	(11) 72	(21) 2
(2) 144	(12) $5x - y = 5$	(22) 4
(3) 40	(13) 7	(23) 1
(4) 0.6	(14) $-a - bi$	(24) 4
(5) $\{4\}$ *or* $x = 4$	(15) 4	(25) 3
(6) ½	(16) -29	(26) 1
(7) -3	(17) 4	(27) 3
(8) $^{15}/_{17}$	(18) 3	(28) 1
(9) $-\tan 53°$ *or* $-\cot 37°$	(19) 3	(29) 2
(10) 0.8695	(20) 2	(30) 4

[OVER]

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Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* .4, —1.4   [8]
     b III and IV   [2]

(32) *c* 3   [2]

(33) *a* t = tens digit
     u = units digit
     $10t + u = 3(10u + t) - 16$
         $t = 2u + 2$ [5]
     b w = width of picture
     $(w + 8)(w + 6) - w(w + 2) = 72$
     or $6(w + 8) + 6w = 72$ [5]

(34) 2.61   [10]

(35) *a* Analysis   [4]
       30   [6]
     b 156   [10]

DO YOU KNOW...

   ... that most questions used on Regents examinations have been tried out in advance in representative classrooms throughout the State?

   Each year more than 40,000 pupils in about 300 schools " pretest " questions intended for use in future Regents examinations. When committees of classroom teachers meet to assemble Regents examinations, the information obtained from this pretesting is to aid them in determining which questions are appropriate, which questions need revision, and which questions should be eliminated.