The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, June 16, 1969—1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Given $s = \dfrac{a}{1 - r}$. Express in *simplest form* the value of s when $a = 3$ and $r = \frac{2}{3}$.

2 Express the sum of $\sqrt{-16}$ and $3\sqrt{-25}$ in terms of i.

3 Solve for h: $A = \dfrac{h}{2}(b + c)$

4 Factor completely: $4a \cos^2 x - a \cos x$

5 Find angle A to the *nearest minute* if $\tan A = 1.3206$.

6 Write an equation of the line having a slope of 2 and intersecting the x-axis at (3,0).

7 Express in degrees the positive acute angle θ which satisfies the equation $2 \cot \theta \sin \theta - 1 = 0$.

8 Find the area of $\triangle ABC$ if $\angle A = 150°$, $b = 6$, and $c = 10$.

9 If θ is a positive acute angle and $\cos \theta = \frac{1}{8}$, what is the numerical value of $\cos \frac{1}{2}\theta$?

10 If two sides of a triangle are 5 and 8 and the cosine of the included angle is $-\frac{2}{5}$, find the length of the third side.

Directions (11–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

11 What is the positive value of m in the equation $4x^2 + mx + 9 = 0$ which makes the roots of the equation real, equal, and rational?

(1) 12
(2) 9
(3) 3
(4) 4

12 If the coordinates of the vertex of a parabola are (5,—11) and the axis of symmetry passes through the point (5,2), what is the equation of the axis of symmetry?

(1) $x = 2$
(2) $x = 5$
(3) $x = -11$
(4) $y = 5$

13 The expression $\dfrac{3 + \sqrt{2}}{3 - \sqrt{2}}$ is equivalent to

(1) $\dfrac{17\sqrt{2}}{7}$
(2) $\dfrac{11 + 6\sqrt{2}}{7}$
(3) $\dfrac{7 - 6\sqrt{2}}{7}$
(4) $\dfrac{11}{7}$

14 The complex number $a + bi$ for which a and b are real numbers is a *pure* imaginary number if

(1) $a \neq 0$ and $b \neq 0$
(2) $a = 0$ and $b = 0$
(3) $a \neq 0$ and $b = 0$
(4) $a = 0$ and $b \neq 0$

15 The expression $\cos 210°$ is equivalent to

(1) $\sin 30°$
(2) $\cos 30°$
(3) $-\sin 30°$
(4) $-\cos 30°$

16 In oblique triangle ABC, if angles A and B and side a are given, which formula would be used to solve for side c?

(1) $\frac{b}{\sin B} = \frac{c}{\sin C}$

(2) $c^2 = a^2 + b^2 - 2ab \cos C$

(3) $\frac{a}{\sin A} = \frac{c}{\sin C}$

(4) $c^2 = a^2 + b^2$

17 If one root of $3x^2 - 3cx + 2 = 0$ is $\frac{1}{3}$, what is the value of c?

(1) $\frac{5}{3}$ (3) $\frac{7}{3}$

(2) 2 (4) $\frac{5}{9}$

18 If placed in standard position, an angle of $\frac{11\pi}{6}$ radians has the same terminal side as an angle of

(1) $-150°$ (3) $150°$

(2) $-30°$ (4) $240°$

19 The numerical value of 4^{x-1} when $x = \frac{1}{2}$ is

(1) 1 (3) $\frac{1}{2}$

(2) 2 (4) $-\frac{1}{2}$

20 The set of real numbers has the commutative property under the operations of

(1) addition, subtraction, and multiplication
(2) addition and subtraction, but not multiplication
(3) subtraction and multiplication, but not addition
(4) addition and multiplication, but not subtraction

21 A value of x for which the expression $\frac{1}{1 - \tan x}$ is undefined is

(1) $0°$ (3) $45°$

(2) $30°$ (4) $60°$

22 If r varies inversely as s, then their

(1) difference is constant
(2) sum is constant
(3) quotient is constant
(4) product is constant

23 If $\sec^2 x = 2$, then what is the value of $\tan^2 x$?

(1) 1 (3) 3

(2) 2 (4) 0

24 If $\sin \frac{\pi}{2} = x + 1$, then x is equal to

(1) 0 (3) -2

(2) 2 (4) $\frac{\pi}{2} - 1$

25 If the domain and range are subsets of the real numbers, which one of the following relations is *not* a function?

(1) $\{(x,y) \mid y = 2x\}$
(2) $\{(x,y) \mid y \geqq x\}$
(3) $\{(x,y) \mid y = x^2\}$
(4) $\{(x,y) \mid y = \sin x\}$

26 If principal values are used, then the ratio $\frac{\text{arc sin } \frac{1}{2}}{\text{arc tan } 1}$ is equal to

(1) 1 (3) $\frac{2}{3}$

(2) 2 (4) $\frac{1}{2}$

27 The graph of the equation $4y^2 = 25 - 4x^2$ is

(1) a hyperbola (3) a parabola
(2) an ellipse (4) a circle

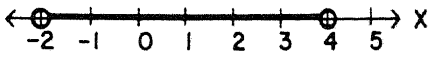
28 If $\log a = p$, then $\log 10a^2$, expressed in terms of p, is equivalent to

(1) $1 + 2p$ (3) $20p$
(2) $2p$ (4) $\log 10 + p^2$

29 Which equation represents a curve whose period is π?

(1) $y = \frac{1}{2} \sin x$ (3) $y = \frac{1}{4} \sin 2x$
(2) $y = 2 \sin \frac{1}{2}x$ (4) $y = 2 \sin x$

30 Which inequality expresses the solution set of this graph?



(1) $|x + 1| \leqq 3$ (3) $|x - 1| > 3$

(2) $|x - 1| < 3$ (4) $|x + 1| \geqq 3$

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Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the roots of the equation $3x^2 - 5x = 4$. [8]

b If in part *a*, $x = \cos \theta$, determine the quadrant(s) in which angle θ lies. [2]

32 *a* On the same set of axes sketch the graphs of $y = \frac{1}{2} \cos x$ and $y = \sin 2x$ for values of x in the interval $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$. [Label each curve with its equation.] [4,4]

b For what value(s) of x in the interval $0 < x < \pi$ does $\frac{1}{2} \cos x + \sin 2x = 0$? [2]

33 Write an equation or a system of equations which can be used to solve *each* of the following problems. In each case state what the variable or variables represent.

[*Solution of the equations is not required.*]

a A group of boys purchased a car for \$300. Had there been twice as many boys, each would have had to pay \$30 less. How many boys were in the original group? [5]

b A beaker contains 20 cubic centimeters (cc.) of a 20% solution of acid. How many cc. of this solution must be drawn off and replaced by pure acid so that the resulting solution is 40% acid? [5]

34 Using logarithms, find to the *nearest hundredth* the value of N if

$$N = \frac{\sqrt{(552)\ (.00125)}}{(2.3)^2 \tan 20^\circ\ 30'}.$$ [10]

35 *a* Starting with the formula for $\cos (x + y)$, *derive* the formula for $\cos 2x$ in terms of $\cos x$. [4]

b For all values of x for which the expression is defined, show that the following is an identity: [6]

$$\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = 2 \cot x \sec x$$

36 Answer *either* *a* or *b* but *not* both:

a In triangle ABC, side AB is 22 inches, side AC is 15 inches, and angle C is 112° 20′. Find to the *nearest ten minutes* angle A. [10]

OR

b Airplane A leaves Chicago, Illinois, on a flight plan to New York City at the same time that airplane B leaves St. Louis, Missouri, on a flight plan to New York City. The directions of the two flight plans make an angle of 55° 40′ with each other. At a given instant, airplane A is 120 miles from New York City while airplane B is 200 miles away. Find to the *nearest mile* the distance between the planes at this instant. [10]

*37 *a* Graph $\{(x,y) \mid xy \leq 8 \text{ and } y < 2x\}$ on the same set of axes. [6,2]

b Indicate the solution set. [2]

*This question is based on an optional topic in the syllabus.

Part I Score:.....

Rater's Initials:

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ANSWER SHEET

Pupil.....Teacher.....

School.....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	11.....	21.....
2.....	12.....	22.....
3.....	13.....	23.....
4.....	14.....	24.....
5.....	15.....	25.....
6.....	16.....	26.....
7.....	17.....	27.....
8.....	18.....	28.....
9.....	19.....	29.....
10.....	20.....	30.....

Your answers for Part II should be placed on paper provided by the school.

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FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

Monday, June 16, 1969 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 11–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 9

(2) $19i$

(3) $\frac{2A}{b+c}$

(4) $a \cos x(4 \cos x - 1)$

(5) $52° \ 52'$

(6) $y = 2x - 6$

(7) 60

(8) 15

(9) $\frac{3}{4}$

(10) 11

(11) 1

(12) 2

(13) 2

(14) 4

(15) 4

(16) 3

(17) 3

(18) 2

(19) 3

(20) 4

(21) 3

(22) 4

(23) 1

(24) 1

(25) 2

(26) 3

(27) 4

(28) 1

(29) 3

(30) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 2.3, —0.6 [8]
b 2 and 3 [2]

(32) *b* $\frac{\pi}{2}$ [2]

(33) *a* x = number of boys in original group
y = amount each paid
$xy = 300$
$2x(y - 30) = 300$ [5]

b x = number of cc. to be drawn off and replaced by pure acid
$.20(20 - x) + x = .40(20)$ [5]

(34) 0.42 [10]

(36) *a* 28° 30′ [10]
b 165 [10]

DO YOU KNOW . . .

. . . that 400 classroom teachers were involved in preparing Regents examinations last year?

- Teachers wrote the questions.
- Other teachers assembled the examinations.
- Still other teachers reviewed the finished product.

And a committee of principals approved all of the examinations before they went to the printer.