The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, June 19, 1967 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 Factor completely: $3\tan^2 x - 12$

2 Write the number 4.86×10^{-3} in ordinary decimal notation.

3 Express $\log \dfrac{a^2}{\sqrt[3]{b}}$ in terms of $\log a$ and $\log b$.

4 Express in radical form the positive value of $\sin(\text{arc}\cos \frac{1}{2})$.

5 Solve for x: $2ax - b = 2b - ax$

6 Express in radians an angle of 144°. [Leave answer in terms of π.]

7 In triangle ABC, $a = 5$, $b = 4$, and $\cos C = \frac{1}{8}$. Find the length of side c.

8 Express tan 240° as a function of an angle between 0° and 45°.

9 In triangle ABC, angle $A = 45°$, angle $B = 30°$, and side $a = 20$. Express in radical form the length of side b.

10 Solve for x: $4^{x-1} = 2^{4x}$

11 If $\cos A = 0.6046$, find A to the *nearest minute.*

12 If, in triangle ABC, $a = 4$, $b = 3$, and $C = 150°$, find the area of the triangle.

13 Express $\dfrac{\frac{a}{4} - \frac{1}{2}}{\frac{a^2}{4} - 1}$ in *simplest* form.

14 The first three terms of a geometric progression are 2, 6, and 18. If the sum of n terms is 728, find n.

15 The cost of a telegram is a cents for the first 10 words and b cents for each additional word. If n is greater than 10, write an expression for the cost of a telegram containing n words.

Directions (16–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

16 Which equation is an identity?
(1) $\sin x + \cos x = 0$
(2) $\sin x + \cos x = 1$
(3) $\sin^2 x = 1 - \cos^2 x$
(4) $\cos x = \dfrac{1}{\sin x}$

17 What is the maximum value of the function $3\cos 2x$?
(1) 1
(2) 2
(3) 3
(4) π

18 In which equation is the sum of the roots $\dfrac{1}{3}$ and the product of the roots $-\dfrac{4}{3}$?
(1) $3x^2 - x - 4 = 0$
(2) $3x^2 - 4x + 1 = 0$
(3) $3x^2 + x - 4 = 0$
(4) $3x^2 - 4x - 1 = 0$

19 If $(3x - 2)$, $(5x + 3)$, and $(2x - 22)$ are three consecutive terms of an arithmetic progression, what is the value of x?
(1) -3
(2) -12
(3) 3
(4) -6

20 Which value of x satisfies the equation $(\sqrt{3} - 1)x = 1$?

(1) $\sqrt{3} + 2$
(2) $\dfrac{\sqrt{3} + 1}{2}$
(3) $\sqrt{3}$
(4) $\dfrac{\sqrt{3}}{2}$

21 What is an equation of the line which passes through the point $(-1,2)$ and which has a slope of $\frac{1}{2}$?
(1) $x - 2y = -5$
(2) $x + 2y = 5$
(3) $x + 2y = 10$
(4) $x - 2y = -10$

22 Which is an *irrational* number?

(1) $\sqrt{\dfrac{16}{36}}$
(2) $\sqrt{90}$
(3) $\sqrt[3]{125}$
(4) $-\sqrt{4}$

23 The expression $\sqrt[3]{-64} + 3\sqrt{-49}$ is equivalent to
(1) $4 + 21i$
(2) $-4 - 21i$
(3) $-4 + 21i$
(4) $29i$

24 What is an equation of the axis of symmetry of the graph of $y = 2x^2 - x - 5$?
(1) $x = \frac{1}{4}$
(2) $x = 4$
(3) $y = \frac{1}{4}$
(4) $y = 4$

25 If $\cos x - 2 \sin x = 1$ and $\sin x = -\frac{4}{5}$, then angle x is in quadrant
(1) I
(2) II
(3) III
(4) IV

26 If A is a positive acute angle and $\cos A = \frac{4}{5}$, what is the value of $\cos 2A$?

(1) 1
(2) $\dfrac{7}{25}$
(3) $\dfrac{9}{25}$
(4) $\dfrac{24}{25}$

27 In triangle ABC, if $A = 30°$, $a = 15$, and $b = 12$, then triangle ABC must be
(1) acute
(2) obtuse
(3) right
(4) isosceles

28 The surface area of a sphere varies directly as the square of its radius. If the surface area is 64π square inches when the radius is 4 inches, what is the surface area, in square inches, when the radius is 12 inches?
(1) 112π
(2) 144π
(3) 192π
(4) 576π

29 When drawn on the same set of axes, what is the number of intersections of the graphs of $y = \sin x$ and $y = \cos x$ as x varies from $\dfrac{3\pi}{2}$ to 2π radians?
(1) 1
(2) 2
(3) 3
(4) 0

30 What is the value of $\tan \dfrac{\pi}{4} + \cos \dfrac{\pi}{3} + \cos \pi$?

(1) 0
(2) $\dfrac{1}{2}$
(3) $\dfrac{\sqrt{3}}{2}$
(4) $\dfrac{5}{2}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* In the following equation, solve for tan x to the *nearest tenth*: [7]

$$\tan^2 x + 2 \tan x = 4$$

b Using the results obtained in part *a*, find to the *nearest degree* the value of x in the second quadrant for which $\tan^2 x + 2 \tan x = 4$. [3]

32 Using logarithms, find to the *nearest degree* the value of the acute angle x for which

$$\cos x = \frac{\sqrt[3]{0.064 \sin 22^\circ}}{0.932}. \quad [10]$$

33 *a* Draw the graph of the equation $y = x^2 - 2x - 2$, using all integral values of x from $x = -2$ to $x = 4$, inclusive. [6]

b Using the graph made in answer to part *a*, estimate to the *nearest tenth* the values of x which satisfy the equation $x^2 - 2x - 2 = 0$. [2]

c Using the graph made in answer to part *a*, find the minimum value of k for which the roots of the equation $x^2 - 2x - 2 = k$ are real. [2]

34 *a* Starting with the formulas for $\sin (A + B)$ and $\cos (A + B)$, derive the formula for $\tan (A + B)$ in terms of $\tan A$ and $\tan B$. [5]

b Show that the following equality is an identity: [5]

$$\frac{\tan \theta - \cot \theta}{\tan \theta + \cot \theta} = 2 \sin^2 \theta - 1$$

35 Three positive integers are in arithmetic progression with a common difference of 3. If the first number is decreased by 2, the second number increased by 7, and the third number multiplied by 5 and increased by 4, the new numbers, in the given order, form a geometric progression. Find the three original integers. [*Only an algebraic solution will be accepted.*] [10]

36 Answer *either a* or *b* but *not* both:

a Two ships leave a harbor at the same time, one sailing in a direction S 70° E (110°) at a rate of 18 miles per hour and the other in a direction S 85° W (265°) at a rate of 21 miles per hour. Find, to the *nearest mile*, the distance between them at the end of one hour. [4, 6]

OR

b In triangle ABC, angle $A = 38^\circ 40'$, $a = 16$, $b = 20$, and angle B is an obtuse angle. Find angle C to the *nearest ten minutes*. [10]

*37 In $\triangle ABC$, $\dfrac{\sin A + \sin B}{\cos A + \cos B} = 1$. Using the formulas for the sum of two sines and the sum of two cosines, show that $A + B = 90^\circ$. [10]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
ELEVENTH YEAR MATHEMATICS

Monday, June 19, 1967 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 16–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) $3(\tan x - 2)(\tan x + 2)$

(2) .00486

(3) $2 \log a - \frac{1}{3} \log b$

(4) $\frac{\sqrt{3}}{2}$

(5) $\frac{b}{a}$

(6) $\frac{144\pi}{180}$ *or* $\frac{4\pi}{5}$

(7) 6

(8) cot 30°

(9) $\frac{20}{\sqrt{2}}$ *or* $10\sqrt{2}$

(10) -1

(11) 52° 48′

(12) 3

(13) $\frac{1}{a+2}$

(14) 6

(15) $a + b(n - 10)$

(16) 3

(17) 3

(18) 1

(19) 4

(20) 2

(21) 1

(22) 2

(23) 3

(24) 1

(25) 3

(26) 2

(27) 2

(28) 4

(29) 4

(30) 2

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 1.2 and —3.2 [7]
b 107° [3]

(32) 72° [10]

(33) *b* 2.6, 2.7, *or* 2.8;
—.6, —.7, *or* —.8 [2]
c —3 [2]

(35) 6, 9, 12 [10]

(36) *a* 38 [10]
b 12° 40′ [10]