The University of the State of New York

Centennial of Regents Examinations

1865–1965

11

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Friday, June 18, 1965—1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Express in terms of i the sum of $\sqrt{-81}$ and $2\sqrt{-25}$.

2 If $\tan A = -1$ and if A is an angle greater than $0°$ and less than $180°$, find the number of degrees in A.

3 In $\triangle ABC$, $b = 8$, $c = 5$ and $\cos A = -0.4$. Find the value of a.

4 Solve the following set of equations for $\sin A$:
$\sin A + 2 \cos B = -\frac{1}{3}$
$2 \sin A - \cos B = 1$

5 Solve for $\cos x$: $4 \cos^2 x - 4 \cos x + 1 = 0$

6 Find the amplitude of the graph of the equation $y = \frac{1}{2} \sin 4x$.

7 A central angle of 3.5 radians intercepts an arc of 14 inches on a circle. Find the number of inches in the radius of the circle.

8 Find the 20th term of the arithmetic progression $-35, -25, -15, \ldots$.

9 If d varies directly as the square of t and if $d = 144$ when $t = 3$, find d when $t = 2$.

10 Express in simplest form: $\dfrac{x + \dfrac{2}{x} - 3}{x - 1 - \dfrac{2}{x}}$

11 One root of the equation $3x^2 - 10x + k = 0$ is $\frac{1}{3}$. Find the value of k.

12 If the third term of a geometric progression is 21 and the fifth term is 189, find the first term.

13 Express $\dfrac{3}{4 + \sqrt{5}}$ as an equivalent fraction with a rational denominator.

14 If $\log N = 2.3814$, find N to the *nearest tenth*.

15 Factor completely: $2a^3 - 7a^2 + 6a$

16 If $\tan x = 2$ and $\tan y = -\frac{1}{2}$, find the value of $\tan (x + y)$.

17 Given that x is a positive angle less than $360°$. In what quadrant does $\frac{1}{2}x$ lie if $\sin x$ is negative and $\tan x$ is positive?

18 If $\log x = d$ and $\log z = h$, express $\log yz^3$ in terms of d and h.

19 If, in $\triangle ABC$, $A = 27°$ and $B = 54°$, find to the *nearest hundredth* the value of $\frac{a}{b}$.

20 Express cos 239° as a function of a positive acute angle.

21 If the graphs of the equations $x^2 + y^2 = 9$ and $x - 2y + 3 = 0$ are drawn on the same set of axes, find the total number of points common to these graphs.

Directions (22–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

22 If n is an odd integer, which of the following represents an even integer?
(1) $3n$ (3) $n^2 + 1$
(2) n^3 (4) $n + 2$

23 The expression $\left(\frac{Z^6}{27}\right)^{\frac{1}{3}}$ is equivalent to
(1) $\frac{Z^2}{9}$ (2) $\frac{Z^2}{3}$ (3) $\frac{Z^3}{9}$ (4) $\frac{Z^3}{3}$

24 The roots of the equation $3x^2 + kx - 4 = 0$ are real, rational and unequal if k is equal to
(1) 1 (2) 2 (3) 0 (4) 6

25 If $\tan t = \sqrt{2}$, $\tan 2t$ equals
(1) $\sqrt{2}$ (2) $2\sqrt{2} - 4$ (3) $2\sqrt{2}$ (4) $-2\sqrt{2}$

26 The average of three numbers is $3k$. If two of these three numbers are 3 and 6, the remaining number is
(1) $3k - 9$ (3) $k - 9$
(2) $9k - 9$ (4) $k - 1$

27 The expression $\sin A(\cos A + \tan^2 A) = \sin A \cos A + \sin A \tan^2 A$ is an illustration of the
(1) commutative postulate for multiplication
(2) associative postulate for multiplication
(3) distributive postulate for multiplication with respect to addition
(4) associative postulate for addition

28 Which statement of equality is an identity?
(1) $(a + b)^2 = a^2 + b^2$
(2) $\sin^2 x - 1 = \cos^2 x$
(3) $\frac{\sin 4x}{2} = \sin 2x \cos 2x$
(4) $\cos \frac{x}{2} = \frac{1}{2} \cos x$

29 The equation $x + \sqrt{x^2 + 3} = 3x$ has
(1) both +1 and −1 as its roots
(2) +1 as its only root
(3) −1 as its only root
(4) neither +1 nor −1 as its roots

30 As x increases from π to 2π radians, $\cos x$
(1) increases throughout the interval
(2) decreases throughout the interval
(3) increases and then decreases
(4) decreases and then increases

Answers to the following questions are to be written on paper supplied by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find, in *radical form*, the roots of the equation

$$\frac{5}{x+1} + \frac{x-1}{4} = 2. \quad [10]$$

32 Given the function $y = 3 \sin \frac{x}{2}$.

a Sketch the graph of this function for values of x between -2π and 2π, inclusive. [6]

b What is the amplitude of this function? [2]

c What is a period of this function? [2]

33 a Derive a formula for $\cos 2A$ from the formula for $\cos(A + B)$. [3]

b Using logarithms, find the value of n to the *nearest integer* if $n = \frac{\tan 75°}{\sqrt{0.491}}$. [7]

34 a Draw the graph of the equation $y = -x^2 + 4x$, using all integral values of x from $x = -1$ to $x = 5$, inclusive. [6]

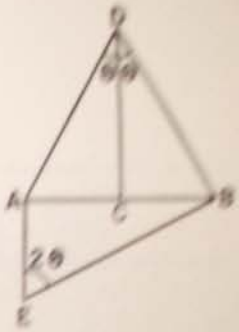
b Draw the axis of symmetry of the graph made in answer to part a and write its equation. [2]

c [After the letter c on your answer paper write the *number* of the expression which best completes the statement.] [2]

The value of K for which the roots of the equation $-x^2 + 4x = K$ are imaginary is

(1) $K = \pm 4$ (2) $K > 4$ (3) $K < 0$ (4) $0 < K < 4$

35 In the figure, $AC = CB = 1$, AE is perpendicular to AB, CD is perpendicular to AB and angle AEB = angle ADB. Show that $EB = \frac{(AD)^2}{DC}$. [10]



36 Answer either a or b but not both:

a Two forces act on a body to produce a resultant of 70 pounds. The 50-pound force makes an angle of 67° 40′ with the resultant force. Find to the *nearest pound* the magnitude of the other force. [10]

OR

b Two lighthouses, A and B, are 22 miles apart. Lighthouse A is due north of lighthouse B. The bearing of a ship from lighthouse B is 41° 40′ (N 41° 40′ E), and the bearing of the same ship from lighthouse A is 142° 20′ (S 37° 40′ E). Find to the *nearest mile* the distance from the ship to lighthouse A. [5, 5]

37 Write the equation or equations that would be used to solve the following problems. In each case state what the letter or letters represent. [*Solution of the equations is not required.*]

a A two-digit number is 2 less than 5 times the sum of its digits. If the digits are reversed, the new number is 9 more than the original number. Find the original number. [5]

b How many quarts of water must be added to 20 quarts of a 15% salt solution in order to obtain a 10% solution? [5]

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ANSWER SHEET

Pupil.....Teacher.....

School.....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	11.....	21.....
2.....	12.....	22.....
3.....	13.....	23.....
4.....	14.....	24.....
5.....	15.....	25.....
6.....	16.....	26.....
7.....	17.....	27.....
8.....	18.....	28.....
9.....	19.....	29.....
10.....	20.....	30.....

Part I Score:.....

Rater's Initials:

.....

Your answers for Part II should be placed on paper supplied by the school.

Tear Here

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SCORING KEY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $19i$

(2) 135

(3) 11

(4) $\frac{1}{3}$

(5) $\frac{1}{2}$

(6) $\frac{1}{2}$

(7) 4

(8) 155

(9) 64

(10) $\frac{x-1}{x+1}$

(11) 3

(12) $\frac{7}{3}$

(13) $\frac{12-3\sqrt{5}}{11}$

(14) 240.7

(15) $a(2a-3)(a-2)$

(16) $\frac{3}{4}$

(17) 2

(18) $d+2b$

(19) 0.56

(20) $-\cos 59°$ *or* $-\sin 31°$

(21) 2

(22) 3

(23) 2

(24) 1

(25) 4

(26) 2

(27) 3

(28) 3

(29) 2

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) $\frac{8 \pm \sqrt{20}}{2}$ *or* $4 \pm \sqrt{5}$ [10]

(32) *b* 3 [2]
c 4π *or* 720° [2]

(33) *b* 5 [7]

(34) *b* $x = 2$ [1]
c 2 [2]

(36) *a* 69 [10]

OR

b Analysis [5]
15 [5]

(37) *a* t = tens digit, u = units digit
$10t + u = 5\ (t + u) - 2$
$10u + t = 10t + u + 9$ [5]

b x = number of quarts of water to be added
$.10(20 + x) = 3$ [5]