The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, June 18, 1962 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express as a monomial in terms of i the sum of $5\sqrt{-16}$ and $3i$. 1.....

2 Express $\dfrac{3}{5-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 2.....

3 In acute triangle ABC, $a = 8$, $b = 10$ and the area is 20. Find the number of degrees in angle C. 3.....

4 If $x = 5$, find the numerical value of $2x^0 + (x-1)^{-\frac{1}{2}}$. 4.....

5 Write an equation of the line which is parallel to the line $y = 3x - 7$ and which passes through the point $(-2, 1)$. 5.....

6 One root of the equation $x^2 - 6x + k = 0$ is 2. What is the value of k? 6.....

7 A circle has a radius of 4 inches. Find the number of inches in the length of the arc intercepted by a central angle of 2 radians. 7.....

8 If x varies inversely as the square of y and if $x = 18$ when $y = 8$, find x when $y = 3$. 8.....

9 In triangle ABC, $a = 4$, $b = 3$ and $c = \sqrt{13}$. Find $\cos C$. 9.....

10 Find the positive value of $\tan\left(\text{arc } \sin \dfrac{5}{13}\right)$. 10.....

11 Find the logarithm of 0.7538. 11.....

[OVER]

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12 Find to the *nearest minute* the positive acute angle whose cosine is 0.3672. 12.....

13 In triangle ABC, $\sin A = 0.42$, $\sin B = 0.18$ and $a = 7$. Find b. 13.....

14 If x is a positive obtuse angle, express $\tan x$ in terms of $\sin x$. 14.....

15 The third term of an arithmetic progression is -20 and the 35th term is 28. Find the common difference. 15.....

16 In a certain factory, n identical machines work at a uniform rate for b hours to complete a production job. In the same factory, r identical machines of the same type as the former, working at the same rate, would require c hours to complete the same job. Express c in terms of n, b and r. 16.....

17 Find a value of x greater than 0 and less than 2π which satisfies the equation $\sin x \cos x - 2 \sin x = 0$. 17.....

18 Express in *simplest form*: $\dfrac{\frac{a}{b} + c}{\frac{a}{c} + b}$ 18.....

19 Express $\sin 242°$ as a function of a positive acute angle. 19.....

20 The first three terms of a geometric progression are ab^2, ab^x and ab^8. What is the value of x? 20.....

Directions (21–30): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

21 If $k = \dfrac{\pi}{2}$, the value of $\sin \dfrac{k}{2} + \sin k$ is

(1) 1

(2) $\dfrac{\sqrt{2}}{2}$

(3) $\dfrac{\sqrt{2} - 2}{2}$

(4) $\dfrac{\sqrt{2} + 2}{2}$

21.....

22 An equation of the axis of symmetry of the graph of $y = 2x^2 - 3x + 7$ is

(1) $y = \dfrac{3}{4}$

(2) $y = \dfrac{3}{2}$

(3) $x = \dfrac{3}{4}$

(4) $x = \dfrac{3}{2}$

22.....

23 The graph of the equation $y = 3x^2 - 7x + 5$
(1) is tangent to the x-axis
(2) intersects the x-axis in two distinct points
(3) intersects the x-axis in three distinct points
(4) has no points in common with the x-axis

23.....

24 The equation $\sqrt{x+6} + x = 6$ has
(1) neither 3 nor 10 as a root
(2) 10 as its only root
(3) 3 as its only root
(4) both 3 and 10 as roots

24.....

25 An example of an identity is
(1) $(1 + \sec x)(1 - \sec x) = \tan^2 x$
(2) $(\sec x + 1)(\sec x - 1) = \tan^2 x$
(3) $\sec^2 (90° - x) = \tan^2 x$
(4) $1 + \sec^2 x = \tan^2 x$

25.....

26 Log $\sqrt{\frac{100}{n}}$ equals
(1) $1 - \frac{\log n}{2}$
(2) $10 - \frac{\log n}{2}$
(3) $2 - \log n$
(4) $\frac{2}{\log n}$

26.....

27 An illustration of a commutative law is
(1) $c(a + b) = ca + cb$
(2) $a(bc) = (ab)c$
(3) $ab = ba$
(4) $a + 2a = 3a$

27.....

28 If $\cos x = 0.6$, the positive value of $\tan \frac{1}{2}x$ is
(1) 0.9
(2) 2.0
(3) 1.4
(4) 0.5

28.....

29 If the graphs of the equations $xy = 12$ and $x - y = 3$ were drawn on the same set of axes, the total number of points common to these graphs would be
(1) 1
(2) 2
(3) 0
(4) 4

29.....

30 Which of the following is the equation of an ellipse?
(1) $9x^2 = 4y^2 + 36$
(2) $xy = -8$
(3) $x^2 = 25 - 4y^2$
(4) $x^2 = y - 16x + 4$

30.....

[OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find to the *nearest tenth* the roots of the equation $3x^2 = 7x - 1$. [10]

32 *a* On the same set of axes *sketch* the graphs of $y = \sin \frac{1}{2}x$ and $y = 2 \cos x$ as x varies from 0 to 2π radians. [Label each curve with its equation.] [4, 4]

b What is the *period* of the curve $y = \sin \frac{1}{2}x$? [1]

c How many values of x between 0 and 2π radians satisfy the equation $\sin \frac{1}{2}x = 2 \cos x$? [1]

33 The members of a boys' club agreed to contribute equally to provide a fund of \$240. If there had been 3 more boys in the club, each member would have had to pay \$4 less than in the original agreement. How many members were actually in the club? [5, 5]

34 *a* Starting with the formula for $\tan (x + y)$, *derive* a formula for $\tan 2x$ in terms of $\tan x$. [3]

b Using the formula obtained in part *a*, show that $\tan 120° = -\sqrt{3}$. [3]

c Prove the identity: $\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = \dfrac{2}{\sin 2x}$ [4]

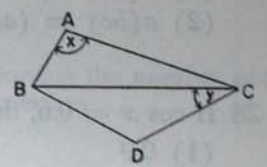
35 *a* Using logarithms, find to the *nearest thousandth* the value of $\sqrt[3]{0.3515}$. [5]

b Given $\log 3 = x$ and $\log 5 = y$.

(1) Express $\log 45$ in terms of x and y. [3]

(2) Express $\log\sqrt{\frac{3}{5}}$ in terms of x and y. [2]

36 In the accompanying diagram, $DB \perp BA$ and $BD = DC$. Angle $BAC = x$ and angle $BCD = y$. Show that $BD = \dfrac{AC \sin x}{2 \cos^2 y}$. [10]



37 Two forces of 62 pounds and 49 pounds, respectively, act on a body at an angle of 53° 20′ with each other. Find to the *nearest ten minutes* the angle formed by the resultant and the greater force. [3, 7]

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

Monday, June 18, 1962 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $23i$

(2) $\frac{15+3\sqrt{2}}{23}$

(3) 30

(4) $2\frac{1}{2}$

(5) $y=3x+7$

(6) 8

(7) 8

(8) 128

(9) $\frac{1}{2}$

(10) $\frac{5}{12}$

(11) 9.8773 — 10

(12) 68° 27′

(13) 3

(14) $-\frac{\sin x}{\sqrt{1-\sin^2 x}}$

(15) $\frac{3}{2}$

(16) $\frac{nb}{r}$

(17) π

(18) $\frac{c}{b}$

(19) $-\sin 62°$ *or* $-\cos 28°$

(20) 5

(21) 4

(22) 3

(23) 4

(24) 3

(25) 2

(26) 1

(27) 3

(28) 4

(29) 2

(30) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 2.2 and 0.2 [10]

(32) *b* 4π or $720°$ [1]
c two [1]

(33) Analysis [5]
12 [5]

(35) *a* 0.706 [5]
b (1) $2x + y$ [3]
(2) $\frac{1}{2}(x - y)$ [2]

(37) Analysis [3]
23° 20′ [7]