The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Monday, June 19, 1961 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name of teacher.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express $3\tan^2 x - 12$ as the product of three factors. 1.....

2 Express $\dfrac{3}{4+\sqrt{2}}$ as an equivalent fraction with a rational denominator. 2.....

3 If $x = 9$, what is the value of $4x^{\frac{1}{2}} + (x^0)^{-2}$? 3.....

4 Write an equation of the line which is parallel to the line $2x - y = 5$ and which passes through the point $(0, -2)$. 4.....

5 Express $3\sqrt{-8} + \sqrt{-2}$ as a monomial in terms of i. 5.....

6 Solve the equation $2 - \sqrt{6\cos x + 1} = 0$ for the smallest positive value of x. 6.....

7 If x varies directly as the square of y and if $x = 12$ when $y = 4$, find the value of x when $y = 6$. 7.....

8 The sum of the first 9 terms of an arithmetic progression is 126. If the last term is 24, what is the first term of the progression? 8.....

9 If the graph of the equation $y = x^2 - 6x + k$ is tangent to the x-axis, find the value of k. 9.....

10 The sum of the roots of the equation $x^2 + px + q = 0$ is 3 and one of the roots is 5. What is the value of q? 10.....

11 Simplify completely: $\dfrac{\frac{a}{b} - \frac{b}{a}}{\frac{a}{b} - 1}$ 11.....

[OVER]

12 Find the antilogarithm of 8.7004 — 10. 12.....

13 Find log cos 42° 23'. 13.....

14 In triangle ABC, $a = 10$, $c = 6$ and $A = 30°$. What is the value of sin C? 14.....

15 In triangle ABC, $b = 4$, $c = 5$ and $\cos A = -\frac{1}{3}$. Find a. 15.....

16 A central angle of 2 radians intercepts an arc of 3 inches. Find the number of inches in the radius of the circle. 16.....

17 In triangle ABC, $a = 10$, $b = 10$ and $A = 70°$. Find to the *nearest integer* the area of the triangle. 17.....

18 If x is a positive acute angle, express cot x in terms of cos x. 18.....

19 What is the positive value of sin (2 arc sin $\frac{3}{5}$)? 19.....

20 Find in degrees the value of x greater than 0° and less than 360° which satisfies the equation $2 \tan x + \sin x \tan x = 0$. 20.....

Directions (21–30): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

21 The period of the curve $y = 3 \cos 2x$ is

(1) $\frac{2\pi}{3}$
(2) π
(3) 3
(4) 4π

21.....

22 Which of the following is the equation of a hyperbola?

(1) $x^2 = 10 - y^2$
(2) $x = y^2 - 9$
(3) $xy = -6$
(4) $4x^2 + y^2 = 9$

22.....

23 Which of the following is equal to sin 50°?

(1) sin 140°
(2) sin 230°
(3) cos 310°
(4) cos (—40°)

23.....

24 If $\log x^2 = 0.6522$, then $\log 10x$ is

(1) 1.3261
(2) 3.2610
(3) 2.3044
(4) 1.3044

24.....

25 If $\frac{1}{a} = \frac{1}{b} - \frac{1}{c}$, then b equals

(1) $a + c$
(2) $\frac{ac}{c - a}$
(3) $\frac{a + c}{ac}$
(4) $\frac{ac}{a + c}$

25.....

26 The value of $\tan \frac{\pi}{3} + \cos \pi$ is

(1) $\frac{\sqrt{3} + 3}{3}$
(2) $\frac{\sqrt{3} - 3}{3}$
(3) $\sqrt{3} + 1$
(4) $\sqrt{3} - 1$

26.....

27 Which of the following is an example of an identity?

(1) $\sec A \csc A = 1$
(2) $\sin A + \cos A = 1$
(3) $\sec^2 A - 1 = \tan^2 A$
(4) $\cos^2 A - \sin^2 A = 1$

27.....

28 As angle A increases from 180° to 270°,

(1) sin A increases
(2) cos A decreases
(3) tan A decreases
(4) cot A decreases

28.....

29 If $\cos x = 0.28$, the positive value of $\cos \frac{1}{2} x$ is

(1) 0.8
(2) 0.6
(3) 0.08
(4) 0.14

29.....

30 A student is given the data $A = 50°$, $a = 10$ and $b = 6$ and asked to construct a triangle if possible. He should find that

(1) no triangle can be constructed with these parts
(2) a triangle can be constructed in which angle B is acute
(3) a triangle can be constructed in which angle B is obtuse
(4) two different triangles can be constructed, one having angle B acute and the other having angle B obtuse

30.....

[OVER]

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Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find to the *nearest tenth* the roots of the equation $2x^2 - 5x = 1$. [10]

32 *a* On the same set of axes, *sketch* the graphs of $y = 2 \sin x$ and $y = \cos 2x$ as x varies from 0 to 2π radians. [Label each curve with its equation.] [4, 4]

b How many values of x between 0 and 2π radians satisfy the equation $2 \sin x = \cos 2x$? [1]

c What is the amplitude of the function $2 \sin x$? [1]

33 Traveling at a certain average rate, a man went 140 miles in going from A to B. His return trip from B to A was by a route that was 10 miles longer but which allowed him to increase his average rate by 10 miles per hour. If his return trip took one-half hour less time than his trip going from A to B, find his average rate in miles per hour for each trip. [5, 5]

34 *a* Starting with the formulas for $\sin (A - B)$ and $\cos (A - B)$, *derive* the formula for $\tan (A - B)$ in terms of $\tan A$ and $\tan B$. [5]

b Prove the identity: [5]

$$\frac{2 \tan x - \sin 2x}{2 \sin^2 x} = \tan x$$

35 Using logarithms, find to the *nearest tenth* the value of t if

$$t = \frac{(5.38)^2 \sqrt[3]{0.0897}}{\sin 42^\circ}.$$ [10]

36 Point C is 70 miles directly east of point A. Point B is 60 miles from A and the bearing of B from A is 22° 20′ (N 22° 20′ E). Find to the *nearest mile* the distance from B to C. [3, 7]

*37 *a* Construct the graph of the equation $x^2 + y^2 = 1$. [Let the length of 10 squares = 1 unit.] [2]

b Draw the angle θ in standard position with its terminal side passing through the point $(-0.6, -0.8)$. [2]

c Draw the line segments whose directed lengths represent $\sin \theta$, $\cos \theta$ and $\tan \theta$. [Label each line segment to show the function it represents.] [6]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 12 is not expressed to *four significant figures*. For questions 21–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $3(\tan x - 2)(\tan x + 2)$

(2) $\frac{3(4-\sqrt{2})}{14}$

(3) 13

(4) $y = 2x - 2$

(5) $7i\sqrt{2}$

(6) $60°$ *or* $\frac{\pi}{3}$

(7) 27

(8) 4

(9) 9

(10) -10

(11) $\frac{a+b}{a}$

(12) .05017

(13) $9.8684 - 10$

(14) 0.3

(15) 7

(16) $1\frac{1}{2}$

(17) 32

(18) $\frac{\cos x}{\sqrt{1-\cos^2 x}}$ *or* $\frac{\cos x\sqrt{1-\cos^2 x}}{1-\cos^2 x}$

(19) $\frac{24}{25}$

(20) 180

(21) 2

(22) 3

(23) 4

(24) 1

(25) 4

(26) 4

(27) 3

(28) 4

(29) 1

(30) 2

[OVER]

Please refer to the Department's pamphlet Suggestions on the Rating of Regents Examination Papers in Mathematics. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(31) 2.7 and — 0.2 [10]

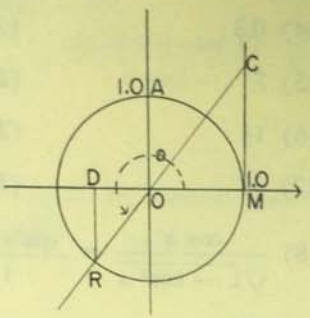
(32) *b* 2 [1]
c 2 [1]

(33) Analysis [5]
40 m.p.h. from *A* to *B* and
50 m.p.h. from *B* to *A* [5]

(35) 19.4 [10]

(36) Analysis [3]
73 [7]

(37) *a* and *b*



[2, 2]

c *DR* represents sin θ [2]
OD represents cos θ [2]
MC represents tan θ [2]