The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Friday, June 17, 1960 — 1:15 to 4:15 p.m., only

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form

1 Express as a single term the sum of $6i$ and $\sqrt{-9}$. 1.....

2 If $x = 4$, find the value of $4x^{\frac{1}{2}} + (x^0 + 3)^{-1}$. 2.....

3 Perform the indicated operations and express the result in *simplest* form:
$\left(1 + \frac{1}{x}\right)\left(\frac{1}{x+1} - 1\right)$ 3.....

4 Find the positive value of t which satisfies the equation $2t^2 + 5t - 33 = 0$. 4.....

5 Express $\frac{2}{4-\sqrt{7}}$ as an equivalent fraction with a rational denominator. 5.....

6 If x varies inversely as y and if $x = 12$ when $y = 8$, find x when $y = 10$. 6.....

7 If the roots of the equation $x^2 + kx + t = 0$ are $3 + \sqrt{2}$ and $3 - \sqrt{2}$, find the value of k. 7.....

8 Write an equation of the line which passes through the point $(0, -3)$ and which has the same slope as the line whose equation is $y = 2x + 6$. 8.....

9 Write an equation in x and y by eliminating t from the system:
$x = t - 1$
$y = 3t + 4$ 9.....

10 Find the 57th term of the arithmetic progression 20, 16, 12, ... 10.....

11 If the sum of three consecutive numbers is S, express in terms of S the *smallest* of these numbers. 11.....

12 Find, to the ***nearest minute***, the positive acute angle whose cosine is 0.2500. 12.....

13 Find the positive value of $\sin \frac{1}{2}x$ if $\cos x = 0.02$. 13.....

14 Find the number of radians in a central angle of a circle of radius 6 if the length of the intercepted arc is 12. 14.....

15 In triangle ABC, $a = 5$, $b = 6$ and $c = 8$. Find $\cos A$. 15.....

16 In triangle ABC, $a = 24$, $b = 20$ and $\sin A = 0.24$. Find $\sin B$. 16.....

17 Find in *degrees* the value of x greater than $0°$ and less than $360°$ which satisfies the equation $\tan x - \tan x \cos x = 0$. 17.....

18 If t is greater than zero, find the positive value of $\tan (\text{arc cot } t)$. 18.....

Directions (19–29): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

19 In triangle ABC, $A = 50°$ and $B = 100°$. The area of the triangle is
(1) $\frac{1}{2}ab \sin 50°$ (2) $\frac{1}{2}ab \sin 100°$ (3) $\frac{1}{2}ab$ (4) $\frac{1}{4}ab$ 19.....

20 If $k = 30°$, the value of $\tan 2k + \cos 3k$ is (1) $\sqrt{3}$ (2) $\frac{\sqrt{3}}{3}$
(3) $\sqrt{3} + 1$ (4) $\frac{\sqrt{3}+3}{3}$ 20.....

21 The equation $x + \sqrt{x-2} = 2$ has (1) both 2 and 3 as roots
(2) 2 as its only root (3) 3 as its only root (4) neither 2 nor 3 as roots 21.....

22 If $T = 10x^2$, then $\log T$ equals (1) $1 + 2 \log x$ (2) $1 + 2x$
(3) $10 + 2 \log x$ (4) $20 \log x$ 22.....

23 If the roots of the equation $2x^2 - 3x + c = 0$ are real and irrational, then the value of c may be (1) 1 (2) 2 (3) 0 (4) -1 23.....

24 An illustration of the distributive law is (1) $(ab)\,c = a\,(bc)$ (2) $(a + b) + c = a + (b + c)$ (3) $a\,(b + c) = ab + ac$ (4) $ab + ac = ac + ab$ 24.....

25 The graphs of the equations $x^2 + y^2 = 25$ and $y = x^2$ are drawn on the same set of axes. The total number of points common to these graphs is (1) one (2) two (3) three (4) four 25.....

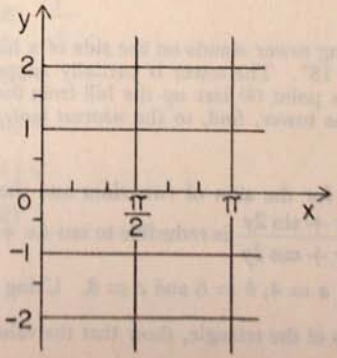
26 The period of the curve $y = 2 \sin x$ is (1) π (2) 2 (3) 2π (4) $\frac{\pi}{2}$ 26.....

27 An example of an identity is (1) $\sin^2 x - \cos^2 x = 1$ (2) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$ (3) $\tan^2 x = 1 + \sec^2 x$ (4) $\sin x + \cos x = 1$ 27.....

28 The expression $\cos(90° + \theta)$ equals (1) $\cos\theta$ (2) $-\cos\theta$ (3) $\sin\theta$ (4) $-\sin\theta$ 28.....

29 If $\tan A = \frac{2}{3}$, then the value of $\tan 2A$ is (1) $\frac{12}{5}$ (2) $\frac{12}{13}$ (3) $\frac{6}{5}$ (4) $\frac{4}{3}$ 29.....

30 On the coordinate axes at the right, *sketch* the graph of $y = \cos 2x$ from $x = 0$ to $x = \pi$.



Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 With respect to a certain rectangle and a certain square these facts are known: The sum of their areas is 68, the length of the rectangle is twice its width and a side of the square exceeds the width of the rectangle by 2. Find the dimensions of the rectangle and the length of a side of the square. [Only an algebraic solution will be accepted.] [5, 5]

32 *a* Find, in *radical form*, the roots of the equation $3x^2 - 2x = 2$. [5]

b Find *one* set of answers which satisfies the following system of equations: [5]

$$x^2 + xy = 12$$
$$y = x - 2$$

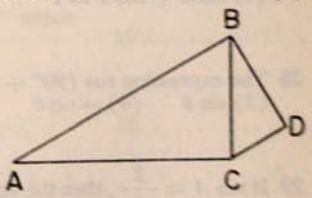
33 *a* Find all positive values of x less than $360°$ that satisfy the equation $2\sin^2 x = 1 + \sin x$. [6]

b Prove that the following equality is an identity: [4]

$$\frac{\tan x \csc^2 x}{1 + \tan^2 x} = \cot x$$

34 *a* Starting with the formula for $\cos(x + y)$, *derive* a formula for $\cos 2x$ in terms of $\cos x$. [5]

b In the figure at the right $AB \parallel CD$, $BC \perp AC$ and $BD \perp CD$. If $AB = 1$ and angle $BAC = x$, show that $CD = \sin^2 x$. [5]



35 Given the formula $V = \pi r^2 h$. By means of logarithms, find to the *nearest tenth* the value of r when $V = 5340$ and $h = 14.6$. [Use the approximation $\pi = 3.14$.] [10]

36 A vertical transmitting tower stands on the side of a hill which is uniformly inclined to the horizontal at an angle of 18°. The tower is partially supported by a cable which reaches from the top of the tower to a point 60 feet up the hill from the base of the tower. If this cable makes an angle of 38° with the tower, find, to the *nearest foot*, the height of the tower. [5, 5]

*37 *a* Use the formula for the sum of two sines and the formula for the sum of two cosines and show that $\dfrac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y}$ is reducible to $\tan(x + y)$. [5]

b In triangle ABC, $a = 4$, $b = 6$ and $c = 8$. Using a formula for a function of a half angle in terms of the sides of the triangle, show that the value of $\cos \frac{1}{2}C$ is $\dfrac{\sqrt{6}}{4}$. [5]

* This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING ELEVENTH YEAR MATHEMATICS

Friday, June 17, 1960 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $9i$

(2) $8\frac{1}{4}$

(3) -1

(4) 3

(5) $\frac{2\ (4 + \sqrt{7})}{9}$

(6) 9.6

(7) -6

(8) $y = 2x - 3$

(9) $y = 3x + 7$

(10) -204

(11) $\frac{S - 3}{3}$

(12) $75°\ 31'$

(13) 0.7

(14) 2

(15) $\frac{25}{32}$

(16) 0.20

(17) 180

(18) $\frac{1}{t}$

(19) 4

(20) 1

(21) 2

(22) 1

(23) 4

(24) 3

(25) 2

(26) 3

(27) 2

(28) 4

(29) 1

[OVER]

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

Part II

(31) Analysis [5]
8 by 4, 6 [5]

(32) *a* $\frac{1+\sqrt{7}}{3}$ and $\frac{1-\sqrt{7}}{3}$ [5]

b (3,1) *or* (—2,—4) [5]

(33) *a* 90°, 210°, 330° [6]

(35) 10.8 [10]

(36) Analysis [5]
92 [5]