The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Friday, June 13, 1958 — 1:15 to 4:15 p.m., only

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Find the factors of $\tan^2 x - 4$. 1.....

2 Combine into a single fraction: $\frac{3}{x-2} - \frac{2}{x}$. 2.....

3 Find the value of $(x+2)^0 + (9x)^{-\frac{1}{2}}$ when $x = 4$. 3.....

4 Write the fraction $\frac{5}{3-\sqrt{2}}$ as an equivalent fraction with a rational denominator. 4.....

5 Find the slope of the line whose equation is $3x - y = 4$. 5.....

6 Simplify completely: $\dfrac{\frac{x}{x+1}}{1-\frac{x}{x+1}}$ 6.....

7 The first term of an arithmetic progression is 3 and the sixteenth term is $\frac{27}{4}$. Find the common difference. 7.....

8 If the number 0.0000086 is written in the form 8.6×10^n, find the value of n. 8.....

9 Write an equation that expresses the relationship between x and y shown in the following table:

x	-3	-1	3	6
y	-6	0	12	21

9.....

10 Solve for x: $7 - \sqrt{2x+1} = 4$ 10.....

11 Find the antilogarithm of $9.7353 - 10$. 11.....

12 Find $\cos 27° 14'$. 12.....

13 Express in degrees an angle of $\frac{3}{5}\pi$ radians. 13.....

14 In triangle ABC, $\sin A = \frac{1}{4}$, $\sin B = \frac{2}{5}$ and side $a = 30$. Find side b. 14.....

15 In triangle ABC, side $b = 5$, side $c = 6$ and $\cos A = -\frac{1}{3}$. Find side a. 15.....

16 Find the positive value of $\tan(\arccos \frac{3}{5})$. 16.....

17 Express $\sin 220°$ as a function of a positive acute angle. 17.....

18 A central angle of 2.5 radians intercepts an arc of 10 inches. Find the number of inches in the radius of the circle. 18.....

Directions (19–25): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

19 The graph of $x^2 + y^2 = 6$ is (*a*) a circle (*b*) an ellipse (*c*) a parabola (*d*) a hyperbola 19.....

20 If the roots of an equation are real, rational and unequal, the discriminant of the equation may be (*a*) -9 (*b*) 0 (*c*) 9 (*d*) 13 20.....

21 If θ is a positive fourth quadrant angle less than 360°, then

(*a*) both $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$ are positive

(*b*) $\sin \frac{\theta}{2}$ is positive and $\cos \frac{\theta}{2}$ is negative

(*c*) $\sin \frac{\theta}{2}$ is negative and $\cos \frac{\theta}{2}$ is positive

(*d*) both $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$ are negative 21.....

22 Tan 70° equals (*a*) $2 \tan 35°$ (*b*) $\frac{\tan 50° + \tan 20°}{1 + \tan 50° \tan 20°}$

(*c*) $\frac{\tan 50° + \tan 20°}{1 - \tan 50° \tan 20°}$ (*d*) $\frac{\tan 50° - \tan 20°}{1 - \tan 50° \tan 20°}$ 22.....

23 $\sin A + \frac{\cos^2 A}{\sin A}$ equals (*a*) 1 (*b*) $\csc A$ (*c*) $\cos A$ (*d*) $\sec A$ 23.....

24 The amplitude of the function $4 \sin 3\theta$ is (*a*) 1 (*b*) 12 (*c*) 3 (*d*) 4 24.....

25 Two different triangles can be constructed if, in triangle ABC, side $b = 10$, side $a = 6$ and angle A equals (*a*) 35° (*b*) 40° (*c*) 45° (*d*) 50° 25.....

Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 Given the equation $\tan^2 x - 4 \tan x + 1 = 0$.

a Solve for $\tan x$ correct to *hundredths*. [7]

b Using the result(s) obtained in *a*, find, to the *nearest degree*, all values of x between 0° and 90° which satisfy the given equation. [3]

27 Solve the following system of equations and check both sets of answers. [8, 2]

$$x^2 - xy + y = 5$$
$$2x + y = 3$$

28 *a* *Sketch* the graph of $y = \cos x$ for values of x from $-\pi$ to π radians. [5]

b On the same set of axes used in *a*, *sketch* the graph of $y = 2 \sin x$ for values of x from $-\pi$ to π radians. [4]

c From the graphs made in answer to *a* and *b*, determine the number of values of x between $-\pi$ and π radians which satisfy the equation $\cos x = 2 \sin x$. [1]

29 At a certain school the total receipts for a junior dance were $150. If ten more couples had attended the dance, the price per couple could have been reduced a half dollar without causing any change in the total receipts. What was the price per couple? [5, 5]

30 Write the equation or equations that would be used in solving the following problems. In *each* case state what the letter or letters represent. [*Solution of the equations is not required.*]

a The rate of a motorboat in still water is 10 miles per hour. The boat traveled 12 miles upstream and returned. The round trip required $2\frac{1}{2}$ hours. What was the rate of the current? [5]

b The units digit of a two-digit number exceeds the tens digit by 3. The number with the digits reversed is 20 less than twice the original number. Find the original number. [5]

31 Using logarithms, find to the *nearest hundredth* the value of

$$\sqrt[3]{\frac{94.7 \tan^2 59°}{3.14}}. \quad [10]$$

Part III

Answer two questions from this part. Show all work.

32 *a* Starting with the formula for $\cos(x + y)$, *derive* the formula for $\cos 2x$ in terms of $\cos x$. [5]

b Prove the identity: $\dfrac{\sin 2\theta}{1 + \cos^2\theta - \sin^2\theta} = \tan\theta$. [5]

33 In triangle ABC, $BC = 30$ feet, angle $A = 30°\ 10'$ and angle $C = 103°\ 20'$.

a Find AC to the *nearest foot*. [6]

b Find the area of triangle ABC to the *nearest square foot*. [4]

34 A lighthouse A is 15 miles from ship B and 24 miles from ship C. The bearing of B from A is N 22° 50′ E and the bearing of C from A is N 74° 30′ E. Find to the *nearest mile* the distance from B to C. [6, 4]

*35 Two forces of 33 pounds and 47 pounds, respectively, act on a body at an angle of 45° 40′ with each other. Using the law of tangents, find to the *nearest ten minutes* the angle formed by the resultant and the greater force. [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III only.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 11 is not expressed to *four significant figures* and if the answer to question 12 is not expressed to *four decimal places*. For questions 19–25, allow credit if the pupil has written the correct answer instead of the letters *a, b, c* or *d*.

(1) $(\tan x + 2)(\tan x - 2)$

(2) $\frac{x+4}{x(x-2)}$

(3) $1\frac{1}{6}$

(4) $\frac{5(3+\sqrt{2})}{7}$

(5) 3

(6) x

(7) $\frac{1}{4}$

(8) -6

(9) $y = 3x + 3$

(10) 4

(11) 0.5436

(12) 0.8892

(13) 108

(14) 48

(15) 9

(16) $\frac{4}{3}$

(17) $-\sin 40°$ *or* $-\cos 50°$

(18) 4

(19) *a*

(20) *c*

(21) *b*

(22) *c*

(23) *b*

(24) *d*

(25) *a*