The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Friday, June 14, 1957 — 1:15 to 4:15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 Express as a single term the sum of $\sqrt{-16}$ and $-i$. 1.....

2 Write the fraction $\frac{3}{\sqrt{6}+\sqrt{2}}$ as an equivalent fraction with a rational denominator. 2.....

3 Find the value of $(a+1)^0 + (4a)^{-\frac{1}{2}}$ when $a = 1$. 3.....

4 Perform the indicated operations and express in *simplest form:*

$$\left(\frac{1}{t}+\frac{1}{s}\right)\left(\frac{t}{s+t}\right)$$

4.....

5 In the equation $T = 80 - \frac{x}{4}$, find the value of x when $T = 60$. 5.....

6 Joe can mow a certain lawn in K minutes. What part of the lawn can he mow in 15 minutes, K being greater than 15? 6.....

7 If x varies directly as y^2 and if $x = 2$ when $y = 1$, find the value of x when $y = 3$. 7.....

8 The first term of an arithmetic progression is 2 and the 20th term is $\frac{27}{4}$. Find the common difference. 8.....

9 The first term of a geometric progression is 2 and the fourth term is $\frac{27}{4}$. Find the common ratio. 9.....

10 The graph of the equation $5x + 2y = 12$ passes through the point P, whose abscissa is 4. Find the ordinate of P. 10.....

11 Solve the following pair of equations for $\sin\theta$ in terms of a and b:

$$\sin\theta + \cos\theta = a$$
$$\sin\theta - \cos\theta = b$$

11.....

12 Find the number of radians in a central angle of a circle whose radius is 6 inches if the central angle intercepts an arc 4 inches long. 12.....

13 Find $\cos 31° 16'$. 13.....

14 If $\log \tan A = 1.0086$ and A is acute, find A. 14.....

15 If $\cos x = 0.6$, find the value of $\sin^2 \frac{1}{2}x$. 15.....

16 In triangle ABC, $a = 2$, $b = 3$, $c = 4$. Find $\cos A$. 16.....

17 In triangle ABC, $a = 9$, $b = 7$, $\sin A = 0.81$. Find $\sin B$. 17.....

Directions (18–24): Indicate the correct completion for *each* of the following by writing the letter *a*, *b*, *c* or *d* on the line at the right.

18 If 0.0000736 is written in the form 7.36×10^n, then n is equal to (*a*) —6 (*b*) —5 (*c*) —4 (*d*) 4. 18.....

19 If $\log n = K$, then $\log nt$ equals (*a*) $K + t$ (*b*) Kt (*c*) $K \log t$ (*d*) $K + \log t$ 19.....

20 The roots of the equation $6x^2 - 2x - 3 = 0$ are (*a*) real and irrational (*b*) real and rational (*c*) imaginary (*d*) equal 20.....

21 The equation $\sqrt{12 + x} = x$ has (*a*) —3 and 4 as its roots (*b*) —3 as its only root (*c*) 4 as its only root (*d*) neither —3 nor 4 as a root 21.....

22 The function $\sin\theta$ expressed in terms of $\cot\theta$ is equal to

(*a*) $\dfrac{1}{\pm\sqrt{1+\cot^2\theta}}$ (*b*) $\dfrac{\cot\theta}{\pm\sqrt{1+\cot^2\theta}}$ (*c*) $\pm\sqrt{1+\cot^2\theta}$

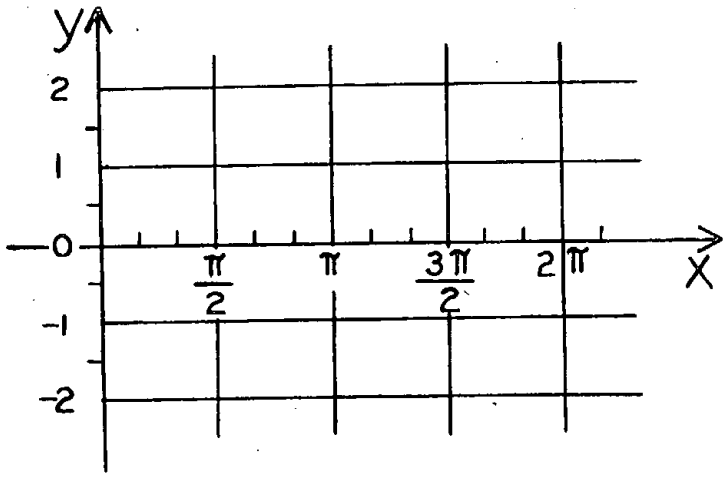
(*d*) $\dfrac{\pm\sqrt{1+\cot^2\theta}}{\cot\theta}$ 22.....

23 The fraction $\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x-\sin^2 x}$ is equal to (*a*) -1 (*b*) $\cos 2x$

(*c*) $\dfrac{1}{\cos 2x}$ (*d*) $\dfrac{\cos x+\sin x}{\cos x-\sin x}$ 23.....

24 The principal value of arc cos 0 is (*a*) $-90°$ (*b*) $0°$ (*c*) $90°$ (*d*) $180°$ 24.....

25 On the axes below *sketch* the graph of $y=\cos x$. 25.....



Part II

Answer three questions from this part. Show all work unless otherwise directed.

26 Solve the following system of equations, group your answers and check them in both equations: [7, 1, 2]

$$2x^2 + y^2 = 2$$
$$2x - y + 2 = 0$$

27 *a* *Draw* the graph of the equation $y = -x^2 + 2x + 4$ from $x = -2$ to $x = 4$, inclusive. [6]

b Using the graph made in answer to part *a*, estimate to *tenths* the abscissas of the points of intersection of the graphs of $y = 2$ and $y = -x^2 + 2x + 4$. [2]

c Write the equation of the circle whose center is the origin and which passes through the y-intercept of the graph drawn in answer to part *a*. [2]

28 The members of a club were asked to contribute equally to provide a fund of \$60. When 5 members failed to contribute, each of the others had to increase his contribution by \$.40 to get the required \$60. How many members contributed to this fund? [5, 5]

29 *a* Starting with the formulas for $\sin (A + B)$ and $\cos (A + B)$, *derive* the formula for $\tan (A + B)$ in terms of $\tan A$ and $\tan B$. [5]

b Prove that the following equality is an identity: [5]

$$\tfrac{1}{2}(\cot A - \tan A) = \frac{1}{\tan 2A}$$

30 Find all values of x greater than $0°$ but less than $360°$ that satisfy the equation $2 \sin^2 x + 3 \cos x = 0$. [10]

Part III

Answer two questions from this part. Show all work.

31 Given the formula $s = 2\sqrt{\frac{A}{n}\tan\frac{180°}{n}}$.

Using logarithms, find s to the *nearest integer* when $n = 15$ and $A = 7{,}780$. [10]

32 In triangle ABC, $AB = 25$ feet, $AC = 30$ feet and angle $A = 121°$. Find BC to the *nearest foot*. [10]

33 The distance between the two points P and Q cannot be measured directly but is known to be a little more than one mile. From a point R the distance to P is 4,200 feet, and the distance from R to Q is 3,100 feet. Angle $RPQ = 33° 20'$. Find, to the *nearest hundred feet*, the distance from P to Q. [5, 5]

*34 Using the equation $\tan\frac{1}{2}C = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$, find C to the *nearest ten minutes* when the sides of a triangle are $a = 5.12$, $b = 5.41$ and $c = 7.23$. [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III *only*.

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

Friday, June 14, 1957 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. Do not allow credit if the answer to question 13 is not expressed to *four decimal places* and if the answer to question 14 is not expressed to the nearest minute. For questions 18–24, allow credit if the pupil has written the correct answer instead of the letters *a*, *b*, *c* or *d*.

(1) $3i$ *or* $3\sqrt{-1}$

(2) $\frac{3(\sqrt{6}-\sqrt{2})}{4}$

(3) $1\frac{1}{2}$

(4) $\frac{1}{s}$

(5) 80

(6) $\frac{15}{K}$

(7) 18

(8) $\frac{1}{4}$

(9) $\frac{3}{2}$

(10) -4

(11) $\frac{a+b}{2}$

(12) $\frac{2}{3}$

(13) 0.8548

(14) 84° 24′

(15) 0.2

(16) $\frac{7}{8}$

(17) 0.63

(18) *b*

(19) *d*

(20) *a*

(21) *c*

(22) *a*

(23) *c*

(24) *c*