The University of the State of New York

324TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 22, 1955 — 9.15 a.m. to 12.15 p.m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from this part. Show all work, unless otherwise directed.

26 Find to the *nearest ten minutes* the smallest positive value of x that satisfies the equation $\sin^2 x + 2 \sin x - 1 = 0$. [10]

27 Solve the following system of equations and check: [8, 2]

$$x^2 + xy = 6$$
$$3x - y = 2$$

28 A man drove his automobile from his home to a resort hotel in exactly 4 hours. On the return trip he followed a different route which was 18 miles longer, but by increasing his average speed by 12 miles per hour, he was able to make the return trip in $\frac{3}{5}$ of an hour less time. Find the rate at which he traveled from his home to the resort hotel. [6, 4]

29 Using logarithms, find to the *nearest integer* the value of

$$\frac{38.4 \times (1.82)^2}{\sqrt[3]{0.0870}}$$

[10]

Part III

Answer at least two questions from this part. Show all work unless otherwise directed.

30 *a* Starting with the formula for $\cos(x + y)$ *derive* the formula for $\cos 2x$ in terms of $\cos x$. [5]

b Prove the identity: $\dfrac{\sin(x - y)}{\sin(x + y)} = \dfrac{\tan x - \tan y}{\tan x + \tan y}$ [5]

31 A lighthouse at A is 8000 feet west of a lighthouse at B. From a ship the bearing of A is N 72° 20′ W and the bearing of B is N 65° 30′ E. Find the distance from the ship to the lighthouse at B. [Answer may be left to the *nearest hundred feet.*] [5, 5]

32 In triangle ABC, $BC = 13''$, $AC = 16''$ and angle $C = 74°\ 20'$.

a Find AB to the *nearest inch.* [5]

b Find the area of the triangle to the *nearest square inch.* [5]

33 *a* *Sketch* the graph of $y = \sin 2x$ as x varies from 0 to 2π radians. [4]

b On the set of axes used in *a*, *sketch* the graph of $y = 2 \cos x$ as x varies from 0 to 2π radians. [4]

c From the graphs made in answer to *a* and *b*, find the value of x between π and 2π radians which satisfies the equation $\sin 2x = 2 \cos x$. [2]

*34 In triangle ABC, $BC = 72$, $AC = 56$ and angle $C = 70°$. Using the law of tangents, find A to the *nearest degree.* [10]

* This question is based on one of the optional topics in the syllabus and may be used as *one* of the questions in part III *only.*

Be sure you have answered a total of 5 questions from parts II and III.

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Express as a single term the sum of $5\sqrt{-1}$ and $-2i$. 1.....

2 Express $\frac{1}{\sqrt{7}-2}$ as an equivalent fraction with a rational denominator. 2.....

3 Find the value of $2a^0 + 3a^{-\frac{1}{2}}$ when $a = 9$. 3.....

4 If s varies inversely as t, and if $s = 6$ when $t = 2$, find s when $t = 4$. 4.....

5 Write an equation of the straight line whose slope is 3 and whose y-intercept is the same as that of the line whose equation is $y = 5x - 7$. 5.....

6 Write an equation of the circle whose center is the origin and which passes through the point $(-2, 0)$. 6.....

7 The first term of an arithmetic progression is 10 and the thirtieth term is -77. Find the common difference. 7.....

8 The first term of a geometric progression is 8 and the fourth term is 125. Find the common ratio. 8.....

9 Solve the equation $\sqrt{2 \sin x + 3} = 2$ for the smallest positive value of x. 9.....

10 Express $\tan (A + B)$ in terms of $\tan A$ and $\tan B$. 10.....

11 If $\cos x = a$, express $\sin^2 \frac{1}{2}x$ in terms of a. 11.....

12 Find $\log \tan 31° 42'$. 12.....

13 If $\log n = 0.6732$, find n. 13.....

14 In triangle ABC, $a = 5$, $b = 6$ and $\sin A = \frac{1}{3}$. Find $\sin B$. 14.....

15 In triangle ABC, $a = 5$, $b = 6$, and $\cos C = \frac{1}{3}$. Find c. 15.....

[OVER]

Directions (16–20): Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

16 The line whose equation is $2x - y + 7 = 0$ is parallel to the line whose equation is $y - 2x + 7 = 0$. 16.....

17 In the function $y = x^2 - 6x + 7$, the minimum value of y occurs when $x = 3$. 17.....

18 The product of the roots of the equation $6x^2 - 4x + 3 = 0$ is $\frac{2}{3}$. 18.....

19 In a circle whose radius is 6 inches there are $\frac{1}{2}$ radians in a central angle that intercepts an arc 4 inches long. 19.....

20 Using the data $A = 29°$, $b = 8$, $a = 5$, where a represents the side opposite angle A, two triangles can be constructed. 20.....

Directions (21–25): Indicate the correct completion for *each* of the following statements by writing the letter *a*, *b* or *c* on the line at the right.

21 If $\log 2 = 0.3010$, then $\log 2y$ is equal to (*a*) $0.3010y$ (*b*) $0.3010 + y$ (*c*) $0.3010 + \log y$ 21.....

22 In the equation $ax^2 + bx + c = 0$ (where a, b and c are real numbers) if $b^2 = 4ac$, the roots are (*a*) real and equal (*b*) real and unequal (*c*) imaginary 22.....

23 The maximum value of $3 \cos 2x$ is (*a*) 2 (*b*) 3 (*c*) 6 23.....

24 Tan $(\sin^{-1}\frac{\sqrt{3}}{2})$ may be equal to (*a*) 30° (*b*) $\sqrt{3}$ (*c*) 60° 24.....

25 The complex fraction $\dfrac{\frac{\sin x}{\cos y} + \frac{\cos y}{\sin x}}{\frac{1}{\cos y} + \frac{1}{\sin x}}$ when simplified becomes

(*a*) $\dfrac{1}{\sin x + \cos y}$ (*b*) $\sin x + \cos y$ (*c*) $\dfrac{\sin^2 x + \cos^2 y}{\sin x + \cos y}$ 25.....

FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 21–25, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $3i$ *or* $3\sqrt{-1}$

(2) $\frac{\sqrt{7}+2}{3}$

(3) 3

(4) 3

(5) $y = 3x - 7$

(6) $x^2 + y^2 = 4$

(7) -3

(8) $\frac{5}{2}$

(9) $30°$

(10) $\frac{\tan A + \tan B}{1 - \tan A \tan B}$

(11) $\frac{1-a}{2}$

(12) $9.7908 - 10$

(13) 4.712

(14) $\frac{2}{5}$

(15) 7

(16) true

(17) true

(18) false

(19) false

(20) true

(21) *c*

(22) *a*

(23) *b*

(24) *b*

(25) *c*