The University of the State of New York

321ST HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 23, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II. All work, including computation, should be shown except in problem 29.

26 Given the equation $2\tan^2 A + 5\tan A - 8 = 0$.

a Find to the *nearest tenth* the positive value of tan A that satisfies this equation. [8]

b Using the result found in answer to *a*, find to the *nearest degree* the smallest positive value of A that satisfies this equation. [2]

27 Write the equations that would be used to solve the following problems. In each case state what the letter or letters represent. *Do not solve the equations.*

a A rectangular lot is 50 feet wide and 60 feet long. If both the width and the length are increased by the same amount, the area is increased by 1200 square feet. Find the amount by which both the width and the length are increased. [5]

b At noon a train leaves New York for Buffalo. Two hours later another train leaves New York for Buffalo on a parallel track over the same route as the first. If the second train travels 28 miles per hour faster than the first and passes the first train at 5 p. m., find the rate of each train. [5]

28 *a* Show that $\dfrac{\cos(90° - A)\cos(-A)}{\tan(180° + A)}$ is equal to $\cos^2 A$. [4]

b Prove that the following equality is an identity: [6]

$$\cot x - \tan x = 2\cot 2x$$

29 Indicate the correct completion for *each* of the following statements by listing the numbers 1–5 on your answer paper and placing after each number the letter *a*, *b* or *c*. [10]

(1) The graphs of the equations $x + 2y = 7$ and $x + 2y = 12$ when drawn on the same set of axes (*a*)intersect (*b*)are parallel (*c*)are coincident

(2) The graph of the equation $y = x^2 + 4$ is (*a*)symmetric with respect to the y-axis (*b*)symmetric with respect to the x-axis (*c*)tangent to the x-axis

(3) The graph of the equation $x^2 - y^2 = 16$ is (*a*)a circle (*b*)a hyperbola (*c*)an ellipse

(4) When drawn on the same set of axes, the graph of $y = x - 1$ will intersect the graph of $x^2 + y^2 = 25$ in (*a*)no point (*b*)one point (*c*)two points

(5) When drawn on the same set of axes, the graph of $y = x^2 + 7x - 10$ will intersect the graph of $y = 4x$ at the point (*a*) $(-5, 20)$ (*b*) $(5, -20)$ (*c*) $(2, 8)$

*30 Answer *either* *a* or *b*:

a The sides of a triangle are 5.2, 6.3 and 7.5. Find to the *nearest degree* the smallest angle of the triangle, using the following formula: [10]

$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

b Solve the following equation for all values of x greater than 0° but less than 360°: [10]

$$\frac{\sin 3x - \sin x}{\cos 3x + \cos x} + \sqrt{3} = 0$$

* This question is based on optional topics in the syllabus and may be used in place of any question in either part II or part III.

Part III

Answer three questions from part III. All work, including computation, should be shown except in problems 32 and 33.

31 *a* Two sides of a triangle are a and b, and the angle included between these sides is C. Prove that the area of the triangle is equal to $\frac{1}{2}ab \sin C$. Consider only the case in which the triangle is acute. [4]

b Using logarithms, find to the *nearest hundreth of an acre* the area of a triangular plot of ground if two sides of the plot are 9.35 rods and 14.90 rods and the included angle is 101° 20′. [1 acre = 160 square rods] [6]

32 Each of the equations in column I has *one* of the angles in column II as a value of θ which satisfies that equation. List the numbers 1–5 on your answer paper and after *each* number write one of the letters *a*–*g* that indicates the correct answer. [10]

Column I	*Column II*
(1) $\sin^2\theta + \sin\theta - 2 = 0$	(*a*) $22\frac{1}{2}°$
(2) $\tan^2\theta - 3 = 0$	(*b*) 45°
(3) $\sin 2\theta - 1 = 0$	(*c*) 90°
(4) $\cos 4\theta = 0$	(*d*) 270°
(5) $\sqrt{\tan\theta + 1} = 0$	(*e*) 300°
	(*f*) 315°
	(*g*) 330°

33 *a* Sketch the graph of $y = \tan x$ as x varies from 0° to 180°. [3]

b On the set of axes used in answer to *a*, sketch the graph of $y = \cos 2x$ as x varies from 0° to 180°. [5]

c How many values of x between 0° and 180° satisfy the equation $\tan x = \cos 2x$? [2]

34 In the *acute* triangle ABC, side $AB = 24$, side $BC = 18$ and angle $A = 40°$. Find angle C to the *nearest degree*. [10]

35 Point R is 100 miles directly east of point P. Point S is 90 miles from P, and the bearing of S from P is N 15° 40′ E. Find to the *nearest mile* the distance from S to R. [3, 7]

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the prime factors of $ax^3 - a^2x^2 + ax$. 1.....

2 Factor the expression $\sin^2\theta - 1$. 2.....

3 Find the value of: $4^0 + 4^{-\frac{1}{2}}$ 3.....

4 Write the fraction $\frac{1}{\sqrt{5} + \sqrt{2}}$ as an equivalent fraction with a rational denominator. 4.....

5 Express the complex fraction $\frac{\frac{a}{x}}{\frac{b}{x^2}}$ in simplest form. 5.....

6 Solve the following pair of equations for $\tan\theta$ in terms of a and b:

$$\tan\theta + \sin\theta = a$$
$$\tan\theta - \sin\theta = b$$

6.....

7 If r varies directly as s and if $r = 13$ when $s = 52$, find s when $r = 100$. 7.....

8 Write an equation of the straight line that passes through the origin and has a slope of 2. 8.....

9 The parabola whose equation is $y = ax^2$ passes through the point $(2, 3)$. Find the value of a. 9.....

10 Solve the equation $\sqrt[3]{x-2} = \frac{1}{2}$ for the value of x. 10.....

11 Express as a single term the sum of $\sqrt{-50}$ and $2i\sqrt{2}$. 11.....

12 The first term of a geometric progression is 1 and the common ratio is 2. Find the sum of the first ten terms. 12.....

13 Find the logarithm of cos 57° 28′ 13.....

[OVER]

14 Find the antilogarithm of 3.4060 14.....

15 Express the logarithm of $\frac{\sqrt[3]{a}}{b}$ in terms of log a and log b. 15.....

Directions (16–18): For *each* of the following, tell whether the statement is always true, sometimes true or never true, by writing one of the words *always, sometimes* or *never* on the line at the right.

16 If the first term of an arithmetic progression is a and the nth term is b, then the common difference is $\frac{b-a}{n-1}$. 16.....

17 The sum of the roots of the equation $x^2 - px + p = 0$ is equal to their product. 17.....

18 If the coefficients of a quadratic equation are integers and if the discriminant of the equation is positive, the roots of the equation are real, unequal and rational. 18.....

Directions (19–25): Indicate the correct completion for *each* of the following by writing the letter *a, b* or *c* on the line at the right.

19 Cos $(A + B)$ is equal to (*a*) cos A cos B + sin A sin B (*b*) cos A cos B — sin A sin B (*c*) sin A sin B — cos A cos B 19.....

20 Cos θ expressed in terms of tan θ is (*a*) $\pm \frac{1}{\sqrt{1 + \tan^2 \theta}}$

(*b*) $\pm \frac{\tan \theta}{\sqrt{1 + \tan^2 \theta}}$ (*c*) $\pm\sqrt{1 + \tan^2 \theta}$ 20.....

21 $\text{Tan}^2 \frac{x}{2}$ is equal to (*a*) $\frac{1 + \cos x}{2}$ (*b*) $\frac{1 - \cos x}{2}$ (*c*) $\frac{1 - \cos x}{1 + \cos x}$ 21.....

22 The expression $\cos 2x = 1 - 2 \sin^2 x$ is (*a*) true for all values of x (*b*) true for only certain values of x (*c*) not true for any value of x 22.....

23 If $y = \frac{1}{(1 + \sin A)^2}$ and if A increases from 0 to $\frac{\pi}{2}$ radians, the value of y (*a*) increases (*b*) decreases (*c*) increases and then decreases 23.....

24 The principal value of $\cos^{-1} \frac{1}{2}$ is (*a*) 30° (*b*) 60° (*c*) —60° 24.....

25 When drawn on the same set of axes the graph of $y = 2 \sin A$ will intersect the graph of $y = \sin 2A$ when A is equal to (*a*) 90° (*b*) 180° (*c*) 270° 25.....

• FOR TEACHERS ONLY

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INSTRUCTIONS FOR RATING
ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–25, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $ax(x^2 - ax + 1)$
(2) $(\sin\theta + 1)(\sin\theta - 1)$
(3) $1\frac{1}{2}$
(4) $\frac{\sqrt{5} - \sqrt{2}}{3}$
(5) $\frac{ax}{b}$
(6) $\tan\theta = \frac{a+b}{2}$
(7) 400
(8) $y = 2x$
(9) $\frac{3}{4}$
(10) $2\frac{1}{8}$
(11) $7i\sqrt{2}$
(12) 1023
(13) 9.7306 — 10
(14) 2547
(15) $\frac{1}{3}\log a - \log b$
(16) always
(17) always
(18) sometimes
(19) *b*
(20) *a*
(21) *c*
(22) *a*
(23) *b*
(24) *b*
(25) *b*