The University of the State of New York

318TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 17, 1953 — 9.15 a. m. to 12.15 p. m., only.

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Given the equations $xy = 24$ and $x - 2y = 2$.

a If the graphs of these two equations were drawn on the same set of axes, in how many points would they intersect? [2]

b By solving algebraically the given set of equations, find the coordinates of the point (or points) of intersection of the graphs of these equations. [8]

27 Write the equations that can be used to solve the following problems. In *each* case, state what the letter or letters represent. [Solution of the equations is not required.]

a How much water must be evaporated from 32 pounds of a 4% solution of salt and water to make the result a solution that is 6% salt? [5]

b A car left a certain place and traveled at a uniform rate. One hour later a second car left the same place and traveled over the same road at a rate that was 7 miles per hour faster than the rate of the first car. The second car overtook the first at a point 210 miles from the starting point. Find the rate of the second car. [5]

28 Using logarithms, find to the *nearest integer* the value of: $\dfrac{8.3^2 \times \sqrt[3]{.947}}{\cos 42^\circ}$ [10]

29 *a* Sketch the graph of $y = \sin \theta$ as θ varies from 0 to 2π radians. [3]

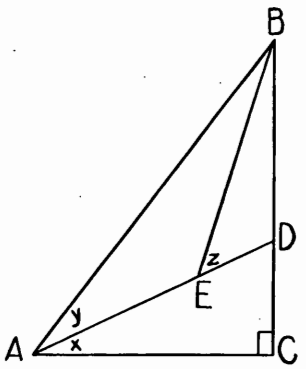
b On the same set of axes used in *a*, sketch the graph of $y = \sin 2\theta$. [6]

c How many values of θ from 0 to 2π inclusive satisfy the equation $\sin \theta = \sin 2\theta$? [1]

Part III

Answer three questions from part III.

30 In the accompanying figure ABC and ADC are right triangles, and a line through B intersects AD at E. Angles CAD, DAB and DEB are represented by x, y and z respectively.



a Show that $BD = \dfrac{BE \sin z}{\cos x}$ [5]

b Show that $BD = \dfrac{AE \sin y \sin z}{\cos x \sin (z - y)}$ [5]

[1]

[OVER]

31 The bearing of point A from point P is N 12° 40′ E and the bearing of point B from point P is N 68° 20′ E. The distance from P to A is 2.5 miles and from P to B it is 4.2 miles. Find to the *nearest tenth of a mile* the distance from A to B. [10]

32 *a* Find to the *nearest degree* the smallest positive value of x which satisfies the equation $\tan x - 2 \cot x - 2 = 0$. [8]

b How many values greater than 0° and less than 360° are there which satisfy the above equation? [2]

33 *a* Prove that the following expression is an identity: $\dfrac{\sin 2\theta + \sin \theta}{\cos 2\theta + \cos \theta + 1} = \tan \theta$ [6]

b Prove that $\dfrac{\sin (90° - \theta) - \cos (180° + \theta)}{\cot (360° + \theta)}$ is equal to $2 \sin \theta$. [4]

34 List the letters *a* through *e* on your answer paper. After each letter write the *number* of the expression that correctly completes the corresponding statement or answers the question. [10]

a The functions that are positive for angles in the fourth quadrant are (1) sine and cosine (2) cosine and tangent (3) tangent and cotangent (4) cosine and secant

b The equation $\sqrt{x^2 + 16} = 5$ (1) has no root (2) has only one real root (3) has imaginary roots (4) has two roots which are numerically equal but opposite in sign

c An imaginary number is always (1) a negative number (2) an odd root of a negative number (3) an even root of a negative number (4) none of these

d If .000062 is written in the form of 6.2×10^n, what is the value of n? (1) −6 (2) −5 (3) −4 (4) none of these

e In a circle whose radius is r, a central angle of n radians intercepts a minor arc whose length is s. Which one of the following is true?

(1) $s = \dfrac{n}{r}$ (2) $s = \dfrac{r}{n}$ (3) $n = \dfrac{s}{r}$ (4) $n = -\dfrac{r}{s}$

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the positive value of $\sin x$ which satisfies the equation $3 \sin^2 x + 5 \sin x - 2 = 0$. 1.....

2 Solve the equation $\sqrt{\frac{1 - \cos x}{2}} = \frac{1}{3}$ for $\cos x$. 2.....

3 Simplify the complex fraction: $\dfrac{\frac{x}{y} + \frac{y}{x}}{\frac{1}{xy}}$ 3.....

4 Express $\dfrac{3}{\sqrt{3} - 1}$ as an equivalent fraction with a rational denominator. 4.....

5 Find the value of $x^{\frac{3}{2}} - x^0$ when $x = 4$. 5.....

6 Find the acute angle which the line whose equation is $x - y - 4 = 0$ makes with the x-axis. 6.....

7 Write the equation of the circle whose center is the origin and which passes through the point $(0, 2)$. 7.....

8 If x varies directly as the square of y and if $x = 100$ when $y = 5$, find the value of x when $y = 4$. 8.....

9 The *sum* of the roots of a quadratic equation is 8 and their *product* is 2. Write this equation in the form of $x^2 + px + q = 0$. 9.....

10 If the discriminant of a quadratic equation with rational coefficients is 64, the roots of the equation are real, rational and unequal. Is this statement *true* or is it *false*? 10.....

11 Write the equation of the axis of symmetry of the graph of the equation $y = x^2 - 4x + 5$. 11.....

12 The first term of an arithmetic progression is 8 and the fifth term is 2. Find the common difference. 12.....

13 Express the sum of the first n terms of a geometric progression whose first term is a and whose common ratio is r. 13.....

[OVER]

14 Express cot 115° as a function of a positive acute angle. 14.....

15 Find the logarithm of .002973 15.....

16 Find log cos 39° 48′ 16.....

17 Find to the *nearest minute* the angle whose tangent is 1.1736 17.....

18 Two sides of a triangle are 20 and 24 and the included angle is 24° 50′. Find to the *nearest integer* the area of the triangle. 18.....

19 Is the following statement *true* or is it *false?* If a, b and c are the sides of a triangle and the angle opposite c is 150°, c^2 is equal to $a^2 + b^2 + 2ab \cos 30°$. 19.....

20 If $\cos \theta = a$ and θ is a positive acute angle, express $\cos \frac{\theta}{2}$ in terms of a. 20.....

21 Write an expression for the tangent of $(x - y)$ in terms of $\tan x$ and $\tan y$. 21.....

22 What is the maximum value of $\cos 2\theta$? 22.....

Directions (23–25): Indicate the correct completion for each of the following by writing the letter a, b or c on the line at the right.

23 The principal value of $\sin^{-1}(-\frac{1}{2})$ is (*a*) 30° (*b*) −30° (*c*) 210° 23.....

24 Log $(r^2 - s^2)$ equals (*a*) $2 \log r - 2 \log s$ (*b*) $\log (r + s) + \log (r - s)$ (*c*) $\log 2r - \log 2s$ 24.....

25 Sin $(45° + x)$ equals (*a*) $\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x$

(*b*) $\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x$ (*c*) $\sin 45° + \sin x$ 25.....