The University of the State of New York

315TH HIGH SCHOOL EXAMINATION

# ELEVENTH YEAR MATHEMATICS

**Wednesday,** June 18, 1952 — 9.15 a. m. to 12.15 p. m., only

---

## Instructions

**Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.**

**Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.**

**The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.**

## Part II

**Answer two questions from part II.**

26 Find to the *nearest ten minutes* the smallest positive value of $x$ which satisfies the equation $\sin^2 x + \sin x - 1 = 0$. [10]

27 Answer *a*, *b* and *c*:

*a* What is the name of the graph of the equation $2x^2 + y^2 = 6$? [2]

*b* If the graphs of $2x^2 + y^2 = 6$ and $y = x - 1$ were drawn on the same set of axes, in how many points would they intersect? [2]

*c* Find algebraically the coordinates of the points of intersection of the two graphs. [6]

28 Answer *both a* and *b*:

*a* It takes a motor boat 50 minutes to travel upstream a distance of 5 miles and 30 minutes to travel the same distance downstream. If $r$ represents the rate of the boat in still water and $s$ the rate of the current (both $r$ and $s$ are in miles per hour), write the equations that can be used to find $r$ and $s$. [5]

*b* The tens digit of a two-digit number exceeds the units digit by 2. If the number is divided by the sum of its digits, the quotient is 6 and the remainder is 5. If $t$ represents the tens digit and $u$ the units digit, write the equations that can be used to find $t$ and $u$. [5]

29 Given the formula $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$ in which *a*, *b* and *c* are the sides of a triangle and $s = \frac{1}{2}(a + b + c)$. Using logarithms, find $A$ to the *nearest degree* if $a = 32.1$, $b = 25.6$ and $c = 37.3$. [10]

## Part III

**Answer three questions from part III.**

30 Answer both *a* and *b*:

*a* Starting with the formula for $\cos(x + y)$, derive the formula for $\cos 2x$ in terms of $\sin x$. [5]

*b* Prove that the following equality is an identity: [5]

$$\frac{\cot s + \cot r}{\cot s - \cot r} = \frac{\sin(r+s)}{\sin(r-s)}$$

31 Answer *a*, *b* and *c*:

*a* Draw the graph of $y = \sin x$ as $x$ varies from $-\pi$ to $+\pi$ radians at intervals of $\frac{\pi}{4}$. [4]

*b* On the set of axes used in *a* draw the graph of $y = \cos x$ for the same values of $x$. [4]

*c* From the graphs made in answer to *a* and *b* determine the values of $x$ between $-\pi$ and $+\pi$ radians for which $\sin x = \cos x$. [2]

32 In triangle $ABC$, $A = 42°\ 10'$, $B = 61°\ 30'$ and $c = 3.560$ ft.

*a* Find *b* to the *nearest hundredth of a foot.* [6]

*b* Using the result found in answer to *a*, find the area of triangle $ABC$ to the *nearest tenth of a square foot.* [4]

33 Two ships leave a certain point at the same time. One sails north at 10 miles per hour and the other northeast at $8\frac{1}{2}$ miles per hour. How far apart, to the *nearest tenth of a mile*, are they at the end of 2 hours? [5, 5]

34 In acute triangle $ABC$, $BD$ is perpendicular to $AC$. Segments $AD$ and $DC$ are represented by $x + y$ and $x - y$, respectively, and angles $ABD$ and $CBD$ by $r$ and $s$, respectively.

*a* Show that $(x + y) \cot r = (x - y) \cot s$. [3]

*b* Solve the equation in *a* for $x$ in terms of $y$, $\cot r$ and $\cot s$. [4]

*c* Using your answer to *b*, find $x$ to the *nearest integer* if $r = 68°$, $s = 24°$ and $y = 8$. [3]

*35 In triangle $ABC$, $AC = 83$, $AB = 56$ and $A = 40°$. Using the law of tangents, find $B$ to the *nearest minute.* [10]

* This question is based upon one of the optional topics in the syllabus.

ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil..... Name of school.....

## Part I

**Answer all questions in part I. Eact correct answer will receive 2 credits. No partial credit will be allowed.**

1 Express $\frac{3}{\sqrt{3}-1}$ as an equivalent fraction with a rational denominator. 1.....

2 If $y = 2x^0 + x^{-2}$, find the value of $y$ when $x = 3$. 2.....

3 Is 5 a root of the equation $\sqrt{x-1} = x - 7$? [Answer *yes* or *no*.] 3.....

4 If $\frac{1}{2}(A + B) = x$ and $\frac{1}{2}(A - B) = y$, express $A$ in terms of $x$ and $y$. 4.....

5 Solve for $R$: $A = \frac{R-S}{T}$ 5.....

6 Simplify the complex fraction $\frac{\frac{1}{r} + \frac{1}{s}}{\frac{1}{rs}}$ 6.....

7 Reduce the fraction $\frac{1 + \tan x}{1 - \tan^2 x}$ to lowest terms. 7.....

8 $\theta$ is an angle in the first quadrant and $\cos \theta = a$. Express $\tan \frac{\theta}{2}$ in terms of $a$. 8.....

9 Express the positive value of $\sin \theta$ in terms of $\cot \theta$. 9.....

10 Find the principal value of $\tan^{-1} \sqrt{3}$. 10.....

11 If $x$ varies inversely as $y$ and if $x = 2$ when $y = 3$, find $x$ when $y = 6$. 11.....

12 Write an equation of the straight line whose slope is 2 and which passes through the point (3,5). 12.....



[OVER]

13 The first term of an arithmetic progression is —3, the last term is 56, and the number of terms is 20. Find the sum. 13.....

14 The first term of a geometric progression is $\frac{1}{16}$ and the common ratio is 2. Find the 10th term. 14.....

15 Find log sin 18° 24′. 15.....

16 Find to the *nearest minute* the angle whose cosine is .3454 16.....

17 Find the sum of the roots of the equation $2x^2 - 5x - 4 = 0$. 17.....

18 Find the product of the roots of the equation $x^2 + 3x - 8 = 0$. 18.....

19 Find the discriminant of the equation $2x^2 - 5x - 1 = 0$. 19.....

20 Find the abscissa of the point at which the graph of the equation $y = x^2 - 6x + 9$ is tangent to the $x$-axis. 20.....

Directions (21–25): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

21 In the graph of $y = 2 \cos 3\theta$, the amplitude is (*a*) 1 (*b*) 2 (*c*) 3 21.....

22 The value of $\sqrt{\tan x}$ is always an imaginary number if $x$ is (*a*) an angle in the first or third quadrant (*b*) an angle in the second or fourth quadrant (*c*) a negative angle 22.....

23 The expression $\frac{\sin 2\theta}{2} = \sin \theta \cos \theta$ is (*a*) true for all values of $\theta$ (*b*) true for only certain values of $\theta$ (*c*) not true for any value of $\theta$ 23.....

24 The expression $\cot (\pi + \theta)$ is equal to (*a*) $\tan \theta$ (*b*) $-\cot \theta$ (*c*) $\cot \theta$ 24.....

25 A value of $\theta$ in radians which satisfies the equation $\cos^2\theta = 1$ is

(*a*) $\frac{\pi}{2}$ (*b*) $\pi$ (*c*) $\frac{3\pi}{2}$ 25.....