The University of the State of New York

306TH HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 22, 1949 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Solve the following pair of equations, correctly group your answers and check *one* set: [7, 2, 1]

$$y = x^2 - 5x + 2$$
$$2x - y = 4$$

27 Write the equations that may be used to solve the following problems. In *each* case state what the letter or letters represent. [Solution of the equations is *not* required.]

a A man had to travel 84 miles to another town to transact some business. If he had increased his speed by 6 miles an hour he would have reached his destination $\frac{1}{3}$ of an hour earlier. What was his speed? [5]

b How much water must be added to 20 ounces of a 40% solution of a certain disinfectant to reduce it to a 25% solution? [5]

28 The side a opposite angle A of a triangle inscribed in a circle whose area is K is given by the formula $a = 2\sqrt{\frac{K}{\pi}} \sin A$.

Using logarithms, find a to the *nearest tenth of an inch* when $A = 71°$ and $K = 153$ square inches. (Use $\pi = 3.14$) [10]

29 *a* Draw the graph of $y = \cos x$ as x varies from 0 to 2π radians inclusive at intervals of $\frac{\pi}{6}$ [6]

b Using the same axes as in *a*, draw the graphs of $y = \frac{1}{2}$ and $y = -\frac{1}{2}$ [2]

c By means of the graphs drawn in answer to *a* and *b*, determine the number of values of x between 0 and 2π for which $\cos x = \pm \frac{1}{2}$ [2]

Part III

Answer two questions from part III.

30 Solve to the *nearest degree*, for all values of x between 0° and 360°:

$$\sin^2 x - 6 \sin x + 1 = 0$$ [10]

31 Prove that the following equality is an identity:

$$\frac{2 \sin^2 A}{\sin 2A} + \cot A = \sec A \csc A$$ [10]

32 A radio station R is 350 miles north of station P. Another station Q is N 65° 10′ E of R and N 25° 30′ E of P. Find the distance RQ to the *nearest mile.* [10]

*33 Derive the Law of Tangents for the oblique triangle. [10]

* This question is based upon one of the optional topics in the syllabus.

Part IV

Answer one question from part IV.

34 If the blank in each of the following statements is filled by one of the words *always, sometimes,* or *never,* the resulting statement will be true. Write the numbers (1) to (5) and opposite *each* write the word that will correctly complete the corresponding statement.

[Consider only the cases where a, b and c are real numbers.]

(1) The graph of the equation $y = ax^2 + bx + c$ .... intersects the x-axis if $b^2 - 4ac$ is less than zero. [2]

(2) The sum of the roots of $x^2 + bx + c = 0$ is ... equal to their product if $b = -c$. [2]

(3) The graph of the equation $ax^2 + by^2 = c$ is ... an hyperbola if a is not equal to b. [2]

(4) The roots of $ax^2 + c = 0$ are ... imaginary if a and c have unlike signs. [2]

(5) When drawn on the same axes, the graphs of the equations $x^2 + y^2 = a^2$ and $y = a$ are ... tangent if a is not zero. [2]

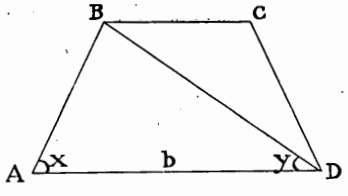
35 $ABCD$ is an isosceles trapezoid with bases AD and BC.
$\angle BAD = x$, $\angle BDA = y$ and $AD = b$

(1) Express $\angle ABD$ in terms of x and y [1]

(2) Express $\angle BDC$ in terms of x and y [2]

(3) Express BD in terms of b, x and y [3]

(4) Show that $BC = \dfrac{b \sin (x - y)}{\sin (x + y)}$ [4]



ELEVENTH YEAR MATHEMATICS

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Write an equation of the straight line whose slope is 2 and whose y-intercept is 3. 1.....

2 Combine $\sqrt{-8}$ and $\sqrt{-50}$ and express the sum in terms of i. 2.....

3 Simplify the complex fraction $\dfrac{\frac{1}{a}}{1-\frac{1}{a}}$ 3.....

4 Find the value of $8^{-\frac{1}{3}} + 8^0$ 4.....

5 If y varies inversely as x and if $y = 10$ when $x = 2$, find y when $x = 4$. 5.....

6 Solve the formula $d = \dfrac{MV}{P+M}$ for P 6.....

7 Solve the equation $\sqrt{4x} - 1 = 5$ 7.....

8 If 2 is a root of $x^2 - 5x + n = 0$, find n. 8.....

9 Find the sum of an arithmetic series of 50 terms if the first term is 5 and the last term is 45. 9.....

10 Find two geometric means between 3 and 192 10.....

11 Find the logarithm of 2.848 11.....

12 Find the logarithm of sin 36° 12′ 12.....

13 Find to the *nearest minute* the acute angle whose cosine is .7260 13.....

14 Find the value of tan 150° 14.....

15 Find the value of sin $\dfrac{\pi}{4}$ 15.....

16 If sin $A = \frac{4}{5}$ and A is in the first quadrant, find cos $2A$. 16.....

17 Find the smallest positive value of x that satisfies the equation $\cos x + 1 = 0$. 17.....

18 Express $\sec^2 A \cot^2 A$ in terms of sin A. 18.....

19 If x and y are acute angles and if sin $x = \frac{3}{5}$ and cos $y = \frac{5}{13}$, find the value of sin $(x + y)$. 19.....

20 In a circle whose radius is r, a central angle intercepts an arc whose length is $3r$. Find the number of radians in the angle. 20.....

21 At a point 150 feet from the foot of a flagpole the angle of elevation of the top of the flagpole is 21° 30′. Find to the *nearest foot* the height of the flagpole. 21.....

[OVER]

Directions (questions 22–25) — Indicate the correct answer to *each* question by writing on the line at the right the letters *a*, *b*, or *c*.

22 Using the data $a = 27$, $b = 20$, $A = 35°$, it is possible to construct (*a*) two triangles, (*b*) only one triangle, (*c*) no triangle. 22.....

23 Tan $(180° + A)$ is equal to (*a*) — tan A, (*b*) cot A, (*c*) tan A. 23.....

24 The graphs of the equations $3x + y = 2$ and $6x + 2y = 3$ (*a*) are parallel, (*b*) coincide, (*c*) intersect. 24.....

25 The graph of $x^2 - 2y^2 = 8$ is (*a*) a circle, (*b*) an ellipse, (*c*) an hyperbola. 25.....