The University of the State of New York

303D HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Wednesday, June 23, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in eleventh year mathematics.

The minimum time requirement is four or five recitations a week for a school year after the completion of tenth year mathematics.

Part II

Answer two questions from part II.

26 Solve the following set of equations, group your answers and check *both* pairs: [7, 1, 2]

$$y = x^2 - 3x + 5$$
$$x + y - 8 = 0$$

27 Find to the *nearest degree* the smallest positive value of θ which satisfies the equation $\tan^2\theta + 2\tan\theta - 2 = 0$ $[\sqrt{3} = 1.732]$ [10]

28 *a* Draw the graph of the equation $y = \sin\theta$ at intervals of 30°, as θ varies from 0° to 360°. [6]

b On the set of axes used in answer to *a* draw the graphs of $y = \frac{1}{2}$ and $y = -\frac{1}{2}$ [2]

c From the graphs made in answer to *a* and *b*, find the number of values of θ between 0° and 360° which satisfy the equation $\sin^2\theta = \frac{1}{4}$ [2]

29 Points A and B, at sea level, are one mile apart and are directly east from the foot of a mountain. From A and B, the angles of elevation of the top of the mountain are x and y respectively. [angle x > angle y] If h is the height in miles of the mountain above sea level, derive the formula $h = \dfrac{1}{\cot y - \cot x}$ [10]

Part III

Answer question 30 and either question 31 or question 32.

30 Write the equations that would be used to solve the following problems: [Solution of the equations is not required.]

a How many quarts of water must be added to 8 quarts of a 7% solution of salt and water to reduce it to a 5% solution? [4]

b The balcony of an assembly hall has 960 seats. There are 50 seats in the first row and each succeeding row has two more seats than the row in front of it. How many rows are there? [6]

31 In $\triangle ABC$, $c = 30$, $b = 40$ and $A = 58°\ 20'$. Find B to the *nearest 10 minutes.* [10]

32 The bearing of a lighthouse C from a boat at point A is N 24° E. After the boat has traveled 30 miles in a direction N 80° E to point B, the bearing of the lighthouse is N 60° W. Find to the *nearest mile* the distance from B to C. [10]

Part IV

Answer one question from part IV.

33 *a* Prove the law of cosines for the case in which the triangle is acute. [7]

b Given two forces F_1 and F_2 acting on an object and θ, the angle between their lines of action. Show that the resultant R of these forces is given by the formula

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos \theta} \qquad [3]$$

34 Each of the five parts of this question is a statement that can be correctly completed by *two* and *only two* of the given choices. Write the numbers (1) to (5) on your answer paper and after *each* indicate the correct answer to the corresponding question by writing only *two* of the letters *a, b, c, d.*

(1) The graph of the equation $y = x + 4$ (*a*)passes through the point $(-4, 0)$ (*b*)is parallel to the line $x + y - 3 = 0$ (*c*)intersects the graph of $x^2 + y^2 = 4$, if both are drawn on the same axes (*d*)is inclined to the x-axis at an angle of 45° [2]

(2) If r_1 and r_2 are roots of the equation $x^2 - px + p = 0$, (*a*)$r_1 + r_2 = r_1 r_2$ (*b*)$r_1 = r_2$ if $p = 4$ (*c*)r_1 and r_2 are real and unequal if $p = 3$ (*d*)r_1 and r_2 are imaginary if $p = 5$ [2]

(3) The logarithm of $(\cos^2 \theta - \sin^2 \theta)$ (*a*)is equal to $\log \cos 2\theta$ (*b*)is equal to $2 \log \cos \theta - 2 \log \sin \theta$ (*c*)is equal to $\log (\cos \theta + \sin \theta) + \log (\cos \theta - \sin \theta)$ (*d*)is greater than zero [2]

(4) The expression $\sin (x + y)$ is equal to (*a*)$\sin x + \sin y$ (*b*)$\dfrac{1}{\cos (x + y)}$ (*c*)$\sin [180° - (x + y)]$ (*d*)$\sin x \cos y + \cos x \sin y$ [2]

(5) $\cos \theta$ is equal to (*a*)$\cos (270° + \theta)$ (*b*)$\cos (-\theta)$ (*c*)$\pm \sqrt{1 - \sin^2 \theta}$ (*d*)$\cos (180° - \theta)$ [2]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Solve for b the following formula: $A = m\ (b + c)$ ..... 1.....

2 Write the equation of the straight line whose slope is 3 and whose y-intercept is 5. ..... 2.....

3 Simplify the complex fraction $\dfrac{a - \dfrac{1}{b}}{a + \dfrac{1}{b}}$ ..... 3.....

4 If r varies inversely as s and $r = 10$ when $s = 25$, find r when $s = 40$ ..... 4.....

5 Find the value of $4^{\frac{1}{2}} \times 4^{0} \times 4^{-1}$ ..... 5.....

6 Write the expression $\dfrac{1}{3 - \sqrt{3}}$ as an equivalent fraction with a rational denominator. ..... 6.....

7 Combine $\sqrt{-16} + \sqrt{-4}$ and express the result in terms of i. ..... 7.....

8 Solve the equation $\sqrt{x^2 + x} = x + 1$ ..... 8.....

9 Find the three arithmetic means between 4 and 24. ..... 9.....

10 The first term of a geometric progression is 4 and the common ratio is 2. Find the sum of the first four terms. ..... 10.....

11 Find the logarithm of 24.78 ..... 11.....

12 Find the value of cos 41° 43′ ..... 12.....

13 Find log sin 59° 15′ ..... 13.....

14 Find the value of $\tan \dfrac{3\pi}{4}$ ..... 14.....

15 The sides of a triangle are 10 and 16 and the included angle is 13°. Find the area of the triangle. ..... 15.....

16 The base of an isosceles triangle is 20 and one of the equal angles is 11° 20′. Find to the *nearest integer* the altitude on the base. ..... 16.....

17 Sin $A = \frac{5}{13}$ and cos $A = \frac{3}{13}$. Find sin $2A$. ..... 17.....

18 Angles x and y are in the first quadrant and sin x = cos $y = \frac{3}{5}$. Find cos $(x - y)$. ..... 18.....

19 Express cot 218° as a function of a positive acute angle. ..... 19.....

20 Express $\sec^2 \theta + \tan^2 \theta$ in terms of cos θ. ..... 20.....

21 In $\triangle ABC$, $A = 45°$ and $B = 30°$. Find the ratio of side a to side b. [Answer may be left in radical form.] ..... 21.....

22 In $\triangle ABC$, $a = 4$, $b = 6$ and cos $C = \frac{9}{16}$. Find c. ..... 22.....

[OVER]

Directions (questions 23–25) — Indicate the correct answer to *each* of the following by writing the letter *a*, *b* or *c* on the dotted line at the right:

23 The graph of the equation $y = -\frac{4}{x}$ is (*a*)a straight line (*b*)a parabola (*c*)a hyperbola 23.....

24 The maximum value of 3 cos $\frac{2A}{3}$ is (*a*)1 (*b*)2 (*c*)3 24.....

25 The equation $\sin^2 x - 7 \sin x + 12 = 0$ is (*a*)true for all values of x (*b*)true for only certain values of x (*c*)not true for any value of x 25.....