January 26, 1983

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the answer sheet.

1. What are the factors of $3y^2 + 4y - 4$?   1_____

2. If the value of sin A is negative and the value of tan A is postive, then the terminal side of angle A lies in which quadrant?   2_____

3. Express in degrees an angle of $\dfrac{5\pi}{3}$ radians.   3_____

4. Find the value of x if $3^{x-2} = 81$.   4_____

5. If 0.00618 is written in the form 6.18×10^n, what is the value of n?   5_____

6. For what value of k will the graph of the equation $y = x^2 - 6x + k$ be tangent to the x-axis?   6_____

7. Find the value of sin 29° 34′ to four decimal places.   7_____

8. In triangle ABC, $a = 6$, $b = 8$, and sin $A = 0.3$ Find the numerical value of sin B.   8_____

9. Express $\dfrac{1}{\sqrt{3} - 1}$ as an equivalent fraction with a rational denominator.   9_____

10. If θ is a positive acute angle and tan $\theta - \sqrt{3} = 0$, what is the measure in degrees of angle θ?   10_____

11. Find the numerical value of sin (Arc cos $\dfrac{5}{13}$).   11_____

12. Express tan 185° as a function of a positive acute angle.   12_____

Directions (13-30): Write in the space provided on the answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

13. In a drama club, x pupils each contributed y dollars to buy an \$18 gift for their advisor. If three more pupils had contributed, each pupil could have contributed one dollar less to buy the same gift. Which set of equations expresses this relationship?
(1) $xy = 18$ (2) $xy = 18$
  $(x + 3)(y - 1) = 18$ $(x - 3)(y + 1) = 18$
(3) $xy = 18$ (4) $xy = 18$
  $(x + 3)(y + 1) = 18$ $(x - 3)(y - 1) = 18$ 13_____

14. The equation $x^2 + 1 = 0$ is satisfied when x is equal to
(1) 1   (2) i (3) 0   (4) -1 14_____

15. Which logarithmic equation can be used to evaluate the expression $N = 5.7 \sqrt[3]{\dfrac{36.7}{\cos 57°}}$?

(1) $\log N = \log 5.7 - \frac{1}{3}(\log 36.7 + \log \cos 57°)$

(2) $\log N = \log 5.7 + \frac{1}{3}(\log 36.7 - \log \cos 57°)$

(3) $\log N = \log 5.7 + 3(\log 36.7 - \log \cos 57°)$

(4) $\log N = \log 5.7 - 3(\log 36.7 + \log \cos 57°)$ 15____

16. The expression $\sqrt{-64} - \sqrt{-4}$ is equivalent to   (1) $16i$
(2) $10i$ (3) $6i$ (4) $4i$ 16____

17. The number $\sqrt{2}$ is   (1) an imaginary number   (2) an irrational number   (3) a repeating decimal   (4) a terminating decimal   17____

18. What is the solution set of the equation $|x + 7| = 4$?
(1) $\{3,11\}$ (2) $\{3,-11\}$ (3) $\{-3,11\}$ (4) $\{-3,-11\}$ 18____

19. The value of $\dfrac{1}{\sin \dfrac{\pi}{2}}$ is   (1) 1   (2) 2   (3) 0   (4) -1 19____

20. The fraction $\dfrac{2x - 6}{3 - x}$, where $x \neq 3$, is equivalent to
(1) -2 (2) 2   (4) -4 (4) 4   20____

21. If $f(x) = \sqrt{\dfrac{1 + x}{2}}$, what is the value of $f(\frac{1}{2})$?   (1) 1

(2) $\sqrt{2}$ (3) $\dfrac{\sqrt{3}}{2}$ (4) $2\sqrt{3}$ 21____

22. A value of x for which the expression $\dfrac{1}{1 - \cos x}$ is undefined is   (1) $0°$ (2) $30°$ (3) $45°$ (4) $90°$ 22____

23. If the sum of the roots of the equation $x^2 - 3x - 28 = 0$ is added to the product of the roots, the result is   (1) -3 (2) -25 (3) -28 (4) -31 23____

24. What is the equation of the line which passes through the point $(-1,3)$ and has a slope of $\frac{1}{3}$?   (1) $x - 3y = 10$ (2) $x - 3y = -10$ (3) $x + 3y = -10$ (4) $3x - y = 10$ 24____

25. The accompanying diagram represents the graph in the interval $-\pi \leq x \leq \pi$ of which function?
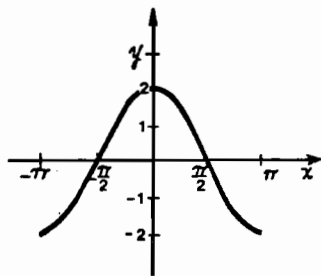
(1) $y = \sin 2x$ (2) $y = \cos 2x$ (3) $y = 2 \sin x$ (4) $y = 2 \cos x$ 25_____

26. The expression $\sin \theta \cot \theta$ is equivalent to   (1) 1   (2) $\csc \theta$
(3) $\dfrac{1}{\sin \theta}$ (4) $\cos \theta$ 26_____

27. The equation $\sqrt{x + 2} + x = 4$ has   (1) 7 as its only root
(2) 2 as its only root   (3) both 2 and 7 as roots   (4) no roots   27_____

28. When $x = y$, the expression $\cos x \cos y - \sin x \sin y$ is equivalent to   (1) $2 \sin x$ (2) $2 \cos x$ (3) $\cos 2x$ (4) $\sin 2x$ 28_____

29. Which is the inverse relation of $\{(1,3),(2,5),(3,6)\}$?
(1) $\{(3,1),(5,2),(6,3)\}$ (2) $\{(3,6),(2,5),(1,3)\}$
(3) $\{(-1,-3),(-2,-5),(-3,-6)\}$ (4) $\{1,\tfrac{1}{3}),(2,\tfrac{2}{5}),(3,\tfrac{3}{6})\}$ 29_____

30. Using the data m$\angle A = 30$, $a = 16$, and $b = 8$, which statement about triangle ABC is true?   (1) It cannot be constructed.
(2) It must be acute.   (3) It must be obtuse.   (4) It may be either obtuse or acute.   30_____

Part II

Answer four questions from this part.
Show all work unless otherwise directed.

31. *a* Find, to the *nearest tenth*, all values of $\sin \theta$ in the solution set of the equation $5 \sin^2 \theta + 1 = 5 \sin \theta$.   [8]

   b Using the results found in part *a*, determine the quadrant(s) in which angle θ may lie.   [2]

32.   *a* Sketch and label the graph of $y = 2 \sin \frac{1}{2}x$ as x varies from 0 to 2π radians inclusive.   [5]

   b On the same set of axes, sketch and label the graph of $y = \cos x$ as x varies from 0 to 2π radians inclusive.   [3]

   c From the graphs sketched in parts *a* and *b*, what is the value of $2 \sin \frac{1}{2}x - \cos x$ when $x = \pi$?   (1) 1   (2) 2   (3) 3   (4) -1 [2]

33.   Solve the following system of equations and check your solution(s) in both equations.
$$x^2 - 4x - y - 5 = 0$$
$$y = 3x - 11 \qquad\qquad [8, 2]$$

34.   *a* If angle x is a positive acute angle and $\sin x = 8/17$, find the value of $\cos 2x$.   [3]

   b If angle y is a positive acute angle and $\tan y = \dfrac{\sqrt{15}}{7}$, find the value of $\sin \frac{1}{2}y$.   [4]

   c If θ is a positive acute angle, express $\tan \theta$ in terms of $\cos \theta$.   [3]

35.   A valley is crossed by a bridge, $\overline{PQ}$, whose length is 100 meters. Point R is in the valley, directly below the bridge. The angle of depression of R from Q is 60° 20′ and $QR = 85$ meters. Find the distance from P to R, to the *nearest ten meters*.   [5, 5]

36.   *a* Using logarithms, find N to the *nearest tenth*:
$$N = \sqrt{676 \cos^3 16° 20'} \qquad [8]$$
   b If $\log x = \log 3 + \log 2$, find the value of x.   [2]

*37.   *a* On the same set of axes, graph each of the following relations.
$$\{(x,y) \mid xy \leq 6\}$$
$$\{(x,y) \mid y - x > 2\} \qquad [8] \qquad\qquad *$$

   b Give the coordinates of one point in the solution set of $\{(x,y) \mid xy \leq 6$ and $y - x > 2\}$.   [2]

* This question is based on an optional topic in the syllabus.