January 29, 1982

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided.

1. If $F = \frac{9}{5}C + 32$, find the value of C when $F = 77$. 1_____

2. If the sum of three consecutive integers is 0, what is the *smallest* integer? 2_____

3. What is the amplitude of the graph of the equation $y = 3 \cos 2x$? 3_____

4. Find the positive acute angle which satisfies the equation $\tan^2 x - 1 = 0$. 4_____

5. If x varies directly as y, and $x = 4$ when $y = 3$, what is the value of y when $x = 16$? 5_____

6. Solve for x: $\sqrt{3x} + 3 = 12$ 6_____

7. Solve for $\sin A$: $\frac{a}{\sin A} = \frac{b}{\sin B}$ 7_____

8. If $\sin A = \cos 5A$, find the number of degrees in angle A. 8_____

9. Find the slope of the straight line which passes through the points (3,5) and (6,9). 9_____

10. Express $5\sqrt{-100} + 2\sqrt{-4}$ as a monomial in terms of i. 10_____

11. Find the value of $\sin 750°$. 11_____

12. Factor: $3 \tan^2 x - 7 \tan x + 4$ 12_____

13. Express 100° in radian measure. 13_____

14. Find the numerical value of $8^{\frac{2}{3}} + 4^0$. 14_____

15. Solve the following system of equations for x in terms of r:

$$3x + y = 2r$$
$$x + y = r$$

15_____

16. Find the value of sin 65° 23′ to *four decimal places*. 16_____

Directions (17-30): Write in the space provided the ***numeral*** preceding the expression that best completes *each* statement or answers *each* question.

17. An equation whose roots are 1 and 2 is (1) $x^2 + 2x + 3 = 0$ (2) $x^2 - 2x + 3 = 0$ (3) $x^2 + 3x + 2 = 0$ (4) $x^2 - 3x + 2 = 0$ 17___

18. The expression $(0.007)^2$ is equal to (1) 4.9×10^6 (2) 4.9×10^{-2} (3) 4.9×10^{-5} (4) 4.9×10^{-6} 18___

19. If $\sin \theta$ and $\tan \theta$ have opposite signs, in which quadrants may angle θ lie? (1) I and II (2) II and III (3) I and III (4) II and IV 19___

20. What is the multiplicative inverse of $\frac{\sqrt{b}}{a}$ where $a \neq 0$ and $b \neq 0$? (1) $\frac{a\sqrt{b}}{b}$ (2) $\frac{\sqrt{a}}{b}$ (3) $\sqrt{ab}$ (4) $a\sqrt{b}$ 20___

21. For which value of c will the roots of the equation $x^2 + 4x + c = 0$ be real and equal? (1) 1 (2) 2 (3) 3 (4) 4 21___

22. The value of Arc $\sin\left(\frac{\sqrt{3}}{2}\right)$ + Arc $\tan 1$ is (1) 120° (2) 105° (3) 90° (4) 75° 22___

23. When placed in standard position, in which quadrant does an angle of $\frac{-5\pi}{6}$ radians terminate? (1) I (2) II (3) III (4) IV 23___

24. The numerical value of $\sin 240°$ is (1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) $\frac{\sqrt{3}}{2}$ (4) $-\frac{\sqrt{3}}{2}$ 24___

25. The solution set of the inequality $x^2 - 5x - 6 < 0$ is (1) $\{-1 < x < 6\}$ (2) $\{-6 < x < 1\}$ (3) $\{x > 6 \text{ or } x < -1\}$ (4) $\{x < -6 \text{ or } x > 1\}$ 25___

26. If $\log 2 = A$ and $\log 3 = B$, then $\log 6$ is equal to (1) $A + B$ (2) $A - B$ (3) AB (4) $\frac{A}{B}$ 26___

27. If $f(x) = |2x + 3|$, then $f(-5)$ equals (1) -7 (2) 7 (3) -13 (4) 13 27___

28. Which relation is a function? (1) $x = y^2$ (2) $x = 2$ (3) $y = \sin x$ (4) $x^2 + y^2 = 9$ 28___

29. In triangle ABC, $a = 3$, $b = 4$, and $c = \sqrt{13}$. What is the value of $\cos C$? (1) 1/13 (2) 1/2 (3) 1/3 (4) 1/16 29___

30. If angle x is in Quadrant II and $\sin x = 3/5$, then $\sin 2x$ is equivalent to (1) 6/5 (2) 4/5 (3) 24/25 (4) $-24/25$ 30___

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31. *a* Find to the ***nearest tenth*** the roots of the equation $2x^2 = 5x + 1$. [8]

b If $x = \csc \theta$, in which quadrant(s) does angle θ lie? [2]

32. *a* Draw the graph of the function $y = -x^2 + 2x + 1$ whose domain is $\{-2 \leq x \leq 4\}$. [6]

b From the graph drawn in part *a*, find the largest value of K for which the equation $K = -x^2 + 2x + 1$ will have real roots. [2]

c What is the range of the function over the *given domain*? [2]

33. Using logarithms, find the value of N to the ***nearest hundrdeth***:

$$N = \frac{\sqrt[3]{135 \sin 42^\circ}}{(12.3)^2} \qquad [10]$$

34. *a* For all values of x for which the expression is defined, show that the following is an identity: $\cos 2x = \dfrac{1 - \tan^2 x}{\sec^2 x}$ [6]

b Using the formula for $\cos (x - y)$, find the value of $\cos 15^\circ$ in *radical form* if $m\angle x = 45$ and $m\angle y = 30$. [4]

35. *a* On the same set of axes, sketch the graphs of $y = \sin 2x$ and $y = \frac{1}{2} \cos x$ for values of x in the interval $0 \leq x \leq 2\pi$. [4, 4]

b Find a value of x in the interval $0 \leq x \leq 2\pi$ which satisfies the equation $\sin 2x = \frac{1}{2} \cos x$. [2]

36. Given triangle ABC, with $m\angle B = 35$, $m\angle C = 41$, and $BC = 23$ meters. Find the length of the *shortest* side of the triangle to the ***nearest meter***. [10]

*37. Solve the following system of equations and check.

$$\begin{aligned} 2x - 3y + 2z &= 26 \\ x - 2y - 3z &= -5 \\ x + y - z &= -7 \end{aligned} \qquad [7, 3]$$

* This question is based on an optional topic in the syllabus.