The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Thursday, January 25, 1973 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Jack has 11 coins which are nickels and dimes. The total value of the coins is 85¢. How many nickels does he have?

2 Solve for x: $2^{3x-1} = 32$

3 In triangle ABC, $a = 6$, $b = 8$, and $\sin A = .6$. Find the numerical value of $\sin B$.

4 The wavelength of sodium light is 0.000000589 meter. If this number is written in the form 5.89×10^n, what is the value of n?

5 Factor: $6\cot^2 x - \cot x - 15$

6 If the tangent of acute angle θ is $\frac{1}{3}$, what is the value of $\cos\theta$ in radical form?

7 Find the two binomial factors of $x + ax + ay + y$.

8 For what value of k will the roots of $kx^2 - 6x + 3 = 0$ be equal?

9 Express $\tan(-150°)$ as a function of a positive acute angle.

10 Find the logarithm of 3.895.

11 If in triangle ABC, $a = 4$, $b = 7$, and $\cos C = \frac{1}{2}$, find the value of c in radical form.

12 Write the quadratic equation whose roots are 3 and -2 in the form $x^2 + bx + c = 0$.

Directions (13–30): Write in the space provided on the separate answer sheet the *numeral* preceding the expression that best completes *each* statement or answers *each* question.

13 The expression $\dfrac{\frac{2}{x} + \frac{5}{y}}{\frac{2}{x} - \frac{5}{y}}$ is equivalent to

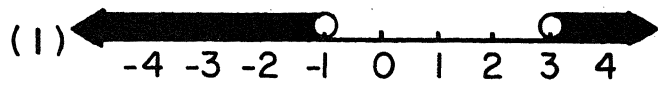
(1) $\dfrac{2y + 5x}{2y - 5x}$

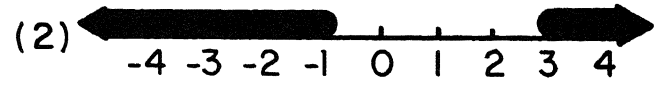
(2) $\dfrac{4}{xy}$

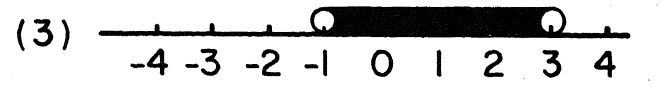
(3) $-\dfrac{7}{3xy}$

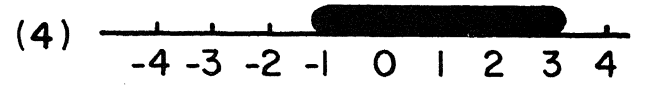
(4) $\dfrac{2y + 5x}{x^2 + y^2}$

14 Which of the following represents the graphic solution of $x^2 - 2x - 3 < 0$?

(1)


(2)


(3)


(4)


15 The roots of a quadratic equation are $\dfrac{2 \pm \sqrt{-12}}{5}$. These roots may be expressed as

(1) $\dfrac{2 \pm 12i}{5}$

(2) $\dfrac{2 \pm \sqrt{3}}{5}$

(3) $\dfrac{2 \pm 2\sqrt{3}}{5}$

(4) $\dfrac{2 \pm 2i\sqrt{3}}{5}$

16 What is the value, in degrees, of the positive acute angle θ which satisfies the equation $\tan^2\theta + \tan\theta = 2$?

(1) 27
(2) 45
(3) 63
(4) 90

17 What is the number of radians through which a wheel turns as it makes two revolutions?

(1) 2π
(2) 4π
(3) approximately 8
(4) approximately 6

18 The graphs of $y = 2$ and $y = \cos x$ are drawn on the same set of axes. In the interval $0 \leq x \leq 2\pi$, how many intersections are there?

(1) 1
(2) 2
(3) 0
(4) 4

19 The graph of the relation $ay = bx^2 + c$ in which neither a nor b is 0 is

(1) a parabola
(2) a straight line
(3) an ellipse
(4) a hyperbola

20 What is the slope of a line which has an angle of inclination of 30°?

(1) $\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$

(2) $-\sqrt{3}$ (4) $-\frac{1}{\sqrt{3}}$

21 If $i = \sqrt{-1}$, then $(1 + i)^2$ is equal to

(1) $2i$ (3) $2 - 2i$

(2) $2 + 2i$ (4) $-2 + 2i$

22 Which expression is equivalent to $\sin^2 \frac{1}{2}x$?

(1) $\frac{1 - \cos x}{2}$ (3) $\frac{1 - \sin x}{2}$

(2) $\frac{1 + \cos x}{2}$ (4) $\frac{1 + \sin x}{2}$

23 What is the value of Arc cos $\frac{1}{2}$ + Arc cos $\frac{\sqrt{3}}{2}$?

(1) 1 (3) π

(2) 0 (4) $\frac{\pi}{2}$

24 The expression $\cos 2A$ is equivalent to

(1) $2 \sin^2 A - 1$ (3) $\sin^2 A - \cos^2 A$

(2) $1 - 2 \sin^2 A$ (4) $1 - 2 \cos^2 A$

25 The numerical value of $\sec^2 30° - \tan^2 30°$ is

(1) 1 (3) $-\frac{9}{4}$

(2) $1\frac{1}{3}$ (4) $-\frac{1}{12}$

26 The expression log 6 is equivalent to

(1) log 2 + log 3 (3) log 2 · log 3

(2) log 3 + log 3 (4) 3 log 2

27 In $\triangle ABC$, if $a = 10$, $b = 20$, and $m\angle C = 45$, the area of the $\triangle$ is

(1) 50 (3) 100

(2) $50\sqrt{2}$ (4) $100\sqrt{2}$

28 The graph of all points with coordinates (x,y) such that y varies directly as x is

(1) a hyperbola, only
(2) a parabola
(3) sometimes a straight line and sometimes a hyperbola
(4) a straight line through the origin

29 The solution set of $| x + 3 | = -1$ is

(1) { } (3) {−2,−4}

(2) {−2} (4) {−4}

30 If, in triangle ABC, $m\angle A = 30$, $a = \sqrt{5}$, and $b = 4$, then angle B

1 may be either obtuse or acute
2 must be obtuse, only
3 must be acute, only
4 may be a right angle

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 *a* Find to the *nearest tenth* the values of x in the solution set of the following equation: [7]

$$x^2 + 2x - 1 = 0$$

b In part *a*, if x is replaced by $\cos \theta$, which value of $\cos \theta$ is *not* possible? [1]

c In which two quadrants does θ lie to satisfy $\{\theta \mid \cos^2 \theta + 2 \cos \theta - 1 = 0\}$? [2]

32 *a* Show that $\sin 2x = \dfrac{2 \tan x}{1 + \tan^2 x}$ is an identity for all values of x for which the expression is defined. [5]

b *Derive* a formula for the area S of acute triangle ABC in terms of two sides and a function of the included angle. [5]

33 Given the function $\{(x,y) \mid y = x^2 - 2x - 4\}$ whose domain is $-2 \leq x \leq 4$.

a When this function is graphed, what are the coordinates of the turning point? [2]

b Draw the graph of the function. [4]

c What is the range of this function over the given domain? [2]

d Using the graph made for *b*, estimate, to the *nearest tenth*, the solution set of $x^2 - 2x - 4 = 0$. [2]

34 Using logarithms, find the value of N to the *nearest integer* if $N = \dfrac{\sqrt[3]{.947}(8.3)^2}{\cos 42^\circ}$. [10]

35 A man worked a certain number of days, at a uniform daily wage, for \$450. Then he was informed that this daily wage was being diminished by \$3. Accepting the decreased wage would mean that he would have to work 5 more days than previously to earn \$450. Find his original daily wage. [*Only an algebraic solution will be accepted.*] [5,5]

36 Answer *either* *a* or *b*, but *not* both. [10]

a A plane is flying a course directly over a straight line joining two observation posts which are on level ground below the plane, and which are 1 mile (5,280 ft.) apart. The plane is observed simultaneously at each post. One post observer finds the angle of elevation of the plane to be 61°, and the other finds it to be 55°. Find to the *nearest hundred feet* the height at which the plane is flying.

OR

b Two forces of 17 lb. and 39 lb. act on a body at an angle of 142°. Find, to the *nearest pound*, the resultant of these two forces.

*37 For the two parts of this question, only solutions using the sum or difference of two sines and the sum or difference of two cosines will be accepted. Formulas used must appear in the solution.

a Express $\dfrac{\sin 50^\circ - \sin 40^\circ}{\cos 50^\circ + \cos 40^\circ}$ as a function of 5°. [7]

b Express $\cos 110^\circ - \cos 50^\circ$ as a function of 80°. [3]

* This question is based upon one of the optional topics in the syllabus.

FOR TEACHERS ONLY

11

SCORING KEY

ELEVENTH YEAR MATHEMATICS

Thursday, January 25, 1973 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 13–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 5

(2) 2

(3) 0.8

(4) -7

(5) $(3 \cot x - 5)(2 \cot x + 3)$

(6) $\frac{3}{\sqrt{10}}$ *or* $\frac{3\sqrt{10}}{10}$

(7) $(x + y)(a + 1)$

(8) 3

(9) $\tan 30°$ *or* $\cot 60°$

(10) 0.5905

(11) $\sqrt{37}$

(12) $x^2 - x - 6 = 0$

(13) 1

(14) 3

(15) 4

(16) 2

(17) 2

(18) 3

(19) 1

(20) 3

(21) 1

(22) 1

(23) 4

(24) 2

(25) 1

(26) 1

(27) 2

(28) 4

(29) 1

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) *a* 0.4, —2.4 [7]
b —2.4 [1]
c I and IV [2]

(33) *a* (1,—5) [2]
c $-5 \leq y \leq 4$ [2]
d 3.2 *or* 3.1 *or* 3.3
—1.2 *or* —1.1 *or* —1.3 [2]

(34) 91 [10]

(35) Analysis [5]
$18 [5]

(36) *a* 4,200 [10]
OR
b 28 [10]

(37) *a* tan 5° [7]
b —sin 80° [3]

DO YOU KNOW . . .

Who writes the questions used on Regents examinations?

1 the members of the Board of Regents
2 the subject supervisors in the State Education Department
3 college professors in the various disciplines
4 classroom teachers from schools throughout New York State

The correct answer is 4. Last year more than 400 classroom teachers were involved in the preparation of Regents examination questions, and many other teachers served on the committee that assembled the examinations.