The University of the State of New York

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REGENTS HIGH SCHOOL EXAMINATION

ELEVENTH YEAR MATHEMATICS

Thursday, January 25, 1968 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided on the separate answer sheet.

1 Factor completely: $abx^n - bcx^n$

2 Solve the following set of equations for $\sin A$:
$2 \sin A + 3 \sin B = 2$
$6 \sin A - 6 \sin B = 1$

3 The axis of symmetry of a parabola is the line $x = 6$. One of the points at which the parabola crosses the x-axis has the coordinates (4,0). What are the coordinates of the other point at which this parabola crosses the x-axis?

4 Express $\dfrac{1}{2\sqrt{3} - 1}$ as an equivalent fraction with a rational denominator.

5 Find the number whose logarithm is $9.4256 - 10$.

6 Express the sum of $2\sqrt{-50}$ and $\sqrt{-2}$ as a monomial in terms of i.

7 If $x = 8$, find the value of $3x^0 + x^{\frac{2}{3}} + 4$.

8 Find the number of degrees in the obtuse angle θ which satisfies the equation $4 \sin^2 \theta = 1$.

9 What is the slope of the line which passes through the point (4,3) if the y-intercept of the line is 2?

10 What two numbers must be inserted between 2 and 54 to form with these numbers a geometric progression of four terms?

11 If $f = \dfrac{aD}{a + D}$, find D in terms of f and a.

12 If A and B are acute angles and $\sin A = \frac{4}{5}$ and $\sin B = \frac{3}{5}$, find the numerical value of $\sin (A + B)$.

13 The tens digit of a two-digit number is 5 less than the units digit. If the units digit is represented by x, express the number in terms of x.

14 In $\triangle ABC$, if $a = 8$, $b = 7$, and $c = 5$, find the value of $\cos B$.

Directions (15–30): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

15 Which statement is true?
(1) $\sin 30° = \frac{1}{2} \sin 60°$
(2) $\sin 30° + \sin 60° = \sin 90°$
(3) $(\sin 30°)(\csc 30°) = 1$
(4) $(\sin 30°)^2 = 2$

16 What is the sum of the integers from 1 to 100, inclusive?
(1) 505
(2) 1,010
(3) 5,050
(4) 10,100

17 Which equation has a circle as its graph?
(1) $3x^2 = 5 + 3y^2$
(2) $3x^2 = 5 - 3y^2$
(3) $3x^2 = 5 - y^2$
(4) $3x^2 = 5 + y^2$

18 What is the positive value of $\tan (\text{arc } \sin \frac{1}{2})$?
(1) $\frac{1}{2}$
(2) 2
(3) $\sqrt{3}$
(4) $\dfrac{\sqrt{3}}{3}$

19 The complex fraction $\dfrac{\cos x - \dfrac{\sin^2 x}{\cos x}}{1 + \dfrac{\sin x}{\cos x}}$ is equivalent to

(1) $\cos x + \sin x$ (3) $\dfrac{1}{\cos x + \sin x}$

(2) $\cos x - \sin x$ (4) $\dfrac{1}{\cos x - \sin x}$

20 What is the maximum value of $2 + \sin x$?

(1) 1 (3) 3
(2) 2 (4) 4

21 What is the value of $-495°$ expressed in radians?

(1) $-\dfrac{\pi}{4}$ (3) $\dfrac{-11\pi}{4}$

(2) $\dfrac{3\pi}{4}$ (4) $\dfrac{11\pi}{4}$

22 The expression $\log \sqrt{\dfrac{1000}{x}}$ is equal to

(1) $\dfrac{3 - \log x}{2}$ (3) $1 - \dfrac{\log x}{2}$

(2) $\dfrac{3 + \log x}{2}$ (4) $\frac{3}{2} - \log x$

23 If x and y are real numbers, then $6(\frac{1}{2}x + \frac{1}{3}y) = 3x + 2y$ is an illustration of the

(1) associative law of addition
(2) associative law of multiplication
(3) commutative law
(4) distributive law

24 In triangle ABC, $C = 30°$ and $a = 8$. If the area of the triangle is 12, what is the length of side b?

(1) 6 (3) 3
(2) 8 (4) 4

25 The roots of the equation $x^2 + 6x - 3 = 0$ are

(1) real, unequal, and irrational
(2) real, unequal, and rational
(3) real, equal, and rational
(4) imaginary

26 As θ increases in the interval from $\dfrac{\pi}{2}$ to $\dfrac{3\pi}{2}$ radians, the value of $\cos \theta$ will

(1) always increase
(2) always decrease
(3) increase and then decrease
(4) decrease and then increase

27 The equation $\sqrt{x - 2} + 2 = x$ has

(1) 2 as its only root
(2) 2 and 3 as its roots
(3) 3 as its only root
(4) no roots

28 In triangle ABC, $A = 60°$, $B = 45°$, and $b = 4$. What is the length of side a?

(1) $2\sqrt{6}$ (3) $\dfrac{\sqrt{6}}{2}$

(2) $2\sqrt{2}$ (4) $\dfrac{16}{3}$

29 The expression $\cot(-200°)$ is equivalent to

(1) $-\tan 20°$ (3) $-\cot 20°$
(2) $\tan 70°$ (4) $\cot 70°$

30 What is the value of $\cos \dfrac{2\pi}{3}$?

(1) $-\frac{1}{2}$ (3) $-\dfrac{\sqrt{3}}{2}$

(2) $\frac{1}{2}$ (4) $\dfrac{\sqrt{3}}{2}$

Answers to the following questions are to be written on paper provided by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find the values of θ between 0° and 360° ($0° < \theta < 360°$) which satisfy the equation of $5 \cos 2\theta - 3 \sin \theta - 4 = 0$.
[Express the approximate values of θ to the *nearest degree*.] [10]

32 *a* Express the roots of the equation $x^2 + 4x - 2 = 0$ in *radical* form. [4]

b Solve the following system of equations: [6]

$$x + y = 5$$
$$y - z = -1$$
$$x + 2z = 10$$

33 *a* *Sketch* and *label* the graph of $y = 2 \cos x$ as x varies from 0 to 2π radians. [4]

b On the same set of axes used in part *a*, *sketch* and *label* the graph of $y = \tan x$ as x varies from 0 to 2π radians. [4]

c [After the letter *c* on your answer paper write the *number* of the expression that best completes the statement.] [2]

In the equation $y = \tan x - 2 \cos x$, when x is equal to $\frac{\pi}{2}$, the value of y is

(1) 1 (2) 2 (3) undefined (4) −1

34 Using logarithms, compute the value of N to the *nearest tenth*: [10]

$$N = \frac{(\tan 15°)\sqrt[3]{435}}{0.12}$$

35 Write an equation or a system of equations that can be used to solve *each* of the following problems. In *each* case state what the variable or variables represent. [*Solution of the equations is not required.*]

a Around the outside of a rectangular picture 1 foot wide and 2 feet long, there is a border whose width is uniform and whose area is twice the area of the picture. Find the width of the border. [5]

b A two-digit number is 54 more than the number obtained by reversing its digits. If the original number is divided by the sum of its digits, the quotient is 8 and the remainder is 2. Find the original number. [5]

36 *a* Starting with the formula for $\cos (x + y)$, derive the formula for $\cos 2x$ in terms of $\sin x$. [4]

b Prove that the following equality is an identity: [6]

$$\frac{\cos 2x}{\sin x} + \frac{\sin 2x}{\cos x} = \csc x$$

37 Answer *either* *a* or *b* but *not* both:

a Two boats, A and B, observe a lighthouse at C. Boat A is 900 yards directly north of boat B. The bearing of the lighthouse from A is 119° 20′ (S 60° 40′ E). At the same time the bearing of the lighthouse from B is 46° 30′ (N 46° 30′ E). Find to the *nearest yard* the distance from the lighthouse to boat A. [4,6]

OR

b In $\triangle ABC$, $a = 19$, $b = 13$, and $c = 10$. Find angle A to the *nearest degree*. [10]

FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

Thursday, January 25, 1968 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 15–30, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) $bx^n(a - c)$

(2) $\frac{1}{2}$

(3) (8,0)

(4) $\frac{2\sqrt{3} + 1}{11}$

(5) 0.2664

(6) $11i\sqrt{2}$

(7) 11

(8) 150

(9) $\frac{1}{4}$

(10) 6, 18

(11) $\frac{fa}{a - f}$

(12) 1

(13) $11x - 50$

(14) $\frac{1}{2}$

(15) 3

(16) 3

(17) 2

(18) 4

(19) 2

(20) 3

(21) 3

(22) 1

(23) 4

(24) 1

(25) 1

(26) 4

(27) 2

(28) 1

(29) 3

(30) 1

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 12°, 168°, 210°, 330° [10]

(32) *a* $\frac{-4 \pm \sqrt{24}}{2}$ *or* $-2 \pm \sqrt{6}$ [4]

b $x = 2, y = 3, z = 4$ [6]

(33) *c* 3 [2]

(34) 16.9 [10]

(35) *a* $x =$ width of the border
$(2x + 2)(2x + 1) - 2 = 4$ [5]

b $t =$ tens digit, $u =$ units digit
$10t + u = 10u + t + 54$
$\frac{10t + u}{t + u} = 8 + \frac{2}{t + u}$ [5]

(37) *a* 683 [10]

b 111° [10]