The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

11 ELEVENTH YEAR MATHEMATICS

Tuesday, January 25, 1966 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write in the space provided on the separate answer sheet the number preceding the expression that best completes each statement or answers each question.

1 The expression $x^{-\frac{2}{3}}$ is equivalent to

(1) $\sqrt[3]{x^2}$ (3) $\dfrac{1}{\sqrt[3]{x^2}}$

(2) $\sqrt{x^3}$ (4) $\dfrac{1}{\sqrt{x^3}}$

2 If $\sin A = \dfrac{-5}{13}$ and A is a third quadrant angle, then $\tan A$ is equal to

(1) $\dfrac{12}{5}$ (3) $-\dfrac{5}{12}$

(2) $-\dfrac{12}{5}$ (4) $\dfrac{5}{12}$

3 The expression $\tan (60° + x)$ is equivalent to

(1) $\sqrt{3} + \tan x$ (3) $\dfrac{\sqrt{3} + \tan x}{1 - \sqrt{3}\tan x}$

(2) $\dfrac{\sqrt{3}}{3} + \tan x$ (4) $\dfrac{\sqrt{3} - \tan x}{1 + \sqrt{3}\tan x}$

4 If a, b, r and s are positive real numbers, which represents an *incorrect* application of the laws of exponents?

(1) $\sqrt[r]{a^s} = \left(\sqrt[r]{a}\right)^s$

(2) $\sqrt[r]{a} \cdot \sqrt[r]{b} = \sqrt[r]{ab}$

(3) $\sqrt[r]{a} + \sqrt[r]{b} = \sqrt[r]{a+b}$

(4) $\sqrt[r]{a} \div \sqrt[r]{b} = \sqrt[r]{\dfrac{a}{b}}$

5 The positive value of $\sin (\arccos x)$ equals

(1) $\sqrt{1 + x^2}$ (3) $1 + x$

(2) $\sqrt{1 - x^2}$ (4) $1 - x$

6 The complex fraction $\dfrac{s - \dfrac{s}{c}}{\dfrac{1}{c} - 1}$ is equal to

(1) s (3) $\dfrac{1}{s}$

(2) $-s$ (4) $-\dfrac{1}{s}$

7 As x increases from 90° to 270°, $\cos x$

(1) increases throughout the interval
(2) decreases throughout the interval
(3) increases and then decreases
(4) decreases and then increases

8 On a circle of radius r an arc whose length is $\dfrac{\pi r}{4}$ is laid off. The number of radians in the central angle subtended by that arc is

(1) $\dfrac{\pi}{4}$ (3) 2π

(2) $\dfrac{\pi}{2}$ (4) π

9 If $\log 2.59 = k$, then $\log 259$ is equal to

(1) $100k$ (3) $2 + k$

(2) $2k$ (4) $100 + k$

10 Which equation has a hyperbola as its graph?

(1) $x^2 = y - x + 3$ (3) $x^2 + y^2 = 36$

(2) $3y^2 = 12 - 4x^2$ (4) $3x^2 = 4y^2 + 12$

11 Which equation has both 3 and 6 as roots?

(1) $\sqrt{x-2} = \frac{3}{x}$

(2) $\sqrt{x-2} = \frac{x}{3}$

(3) $\sqrt{x-2} = -x + 4$

(4) $\sqrt{x-2} = x - 4$

12 The first two terms of a geometric progression are, respectively, $\frac{3}{2}$ and $\frac{2}{3}$. The third term of the progression is

(1) 1

(2) $\frac{3}{2}$

(3) $\frac{4}{9}$

(4) $\frac{8}{27}$

13 The first three terms of an arithmetic progression are, respectively, 6, 3, 0. The 14th term is

(1) −33

(2) −36

(3) −39

(4) −45

14 In triangle ABC, $a = 8$, $b = 9$ and $C = 135°$. The area of triangle ABC is

(1) 18

(2) 36

(3) $18\sqrt{2}$

(4) $36\sqrt{2}$

15 A certain quadratic equation has unequal, rational roots. Which is a possible value of the discriminant of the equation?

(1) 0

(2) 2

(3) −4

(4) 4

16 If one-half of a number is subtracted from three-fifths of that number, the difference is 10. If N represents the number, then an equation which could be used to find the value of N is

(1) $\frac{N}{2} - \frac{3N}{5} = 10$

(2) $\frac{3N}{5} - \frac{N}{2} = 10$

(3) $\frac{3N}{5} = 10 - \frac{N}{2}$

(4) $\frac{3N}{5} = -10 - \frac{N}{2}$

17 If $\sqrt{-1}$ is represented by i, then $2i$ is equivalent to

(1) $\sqrt{-4}$

(2) $-\sqrt{4}$

(3) $\sqrt{-2}$

(4) $-\sqrt{2}$

18 The expression $2\sin^2\frac{A}{2}$ is equivalent to

(1) $1 - \cos A$

(2) $\frac{1 - \cos A}{2}$

(3) $1 + \cos A$

(4) $\frac{1 + \cos A}{2}$

19 The fraction $\frac{6 - \sqrt{2}}{1 + \sqrt{2}}$ is equivalent to

(1) $7\sqrt{2} - 8$

(2) $\frac{8 - 7\sqrt{2}}{3}$

(3) $4 - 7\sqrt{2}$

(4) $8 - 7\sqrt{2}$

20 Which two equations have graphs which are parallel lines?

(A) $x - 2y = 12$

(B) $y - 2x = 6$

(C) $\frac{y}{2} - \frac{x}{4} = 1$

(D) $y + \frac{x}{2} = 2$

(1) A and B

(2) A and C

(3) A and D

(4) B and D

21 The value of $\sin\frac{5\pi}{6}$ is

(1) $\frac{1}{2}$

(2) $-\frac{1}{2}$

(3) $\frac{\sqrt{3}}{2}$

(4) $\frac{-\sqrt{3}}{2}$

22 A value of x which satisfies the equation $\sin^2 x - 4\sin x + 3 = 0$ is

(1) $\frac{\pi}{2}$

(2) π

(3) $\frac{3\pi}{2}$

(4) 2π

23 If N represents an even integer, which represents an odd integer?

(1) $3N$

(2) N^3

(3) $3N + 2$

(4) $3N + 1$

24 Which equation is an identity?

(1) $\sin 4x = 4\sin x \cos x$

(2) $\cos 4x = \cos^4 x - \sin^4 x$

(3) $\sin^2 4x + \cos^2 4x = 1$

(4) $\sin^4 x + \cos^4 x = 1$

25 The logarithm of 19.87 is

(1) 0.2982 (3) 158
(2) 1.2982 (4) .1580

26 The expression $\sin(-A) + \cos(-A)$ is equivalent to

(1) $\sin A + \cos A$ (3) $\sin A - \cos A$
(2) $-\sin A - \cos A$ (4) $-\sin A + \cos A$

27 A curve whose amplitude is $\frac{1}{2}$ and whose period is π is

(1) $y = 2 \sin 2x$ (3) $y = \frac{1}{2} \sin x$
(2) $y = \frac{1}{2} \sin 2x$ (4) $y = 2 \sin \frac{1}{2}x$

28 The exact numerical value of $\sec^2 \frac{5\pi}{4} - \tan^2 \frac{5\pi}{4}$ is

(1) 1 (3) 3
(2) -1 (4) $\frac{5\pi}{4}$

29 Given quadratic equation $x^2 + px + q = 0$. What is the value of p if the roots of the equation are $2 + \sqrt{3}$ and $2 - \sqrt{3}$?

(1) 1 (3) -4
(2) -1 (4) 4

30 If $\frac{1}{t} = \frac{1}{2} - s$, then $\frac{1}{s}$ equals

(1) $2 - t$ (3) $\frac{t-2}{2t}$
(2) $t - 2$ (4) $\frac{2t}{t-2}$

Answers to the following questions are to be written on paper supplied by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Find to the *nearest tenth* the roots of $2x^2 - 3x - 1 = 0$. [10]

32 *a* On the same set of axes sketch the graphs of $y = \sin \frac{1}{2}x$ and $y = 2 \cos 2x$ as x varies from 0 to π radians. [Label each curve with its equation.] [4,4]

b From the graphs made in answer to part *a*, determine the number of values between 0 and π radians which satisfy the following equation: [2]

$$\sin \tfrac{1}{2}x = 2 \cos 2x$$

33 Solve the following system of equations. Check the solution. [8,2]

$$x + 3y = 4z - 13$$
$$2x - y + 2z = 4$$
$$4x - 6y = -z - 1$$

34 *a* Prove that the following equality is an identity: [7]

$$\csc 2x + \cot 2x = \cot x$$

b Starting with the formula for $\tan (x + y)$, derive a formula for $\tan (x - y)$. [3]

35 The square of the larger of two positive numbers exceeds twice the smaller number by 10. If the smaller number is subtracted from three times the larger number, then the difference is 9. Find the two numbers. [*Only an algebraic solution will be accepted.*] [4,6]

36 Answer *either a* or *b* but *not* both:

a The bearing of town B from town A is 47° (N 47° E) and the distance between the towns is 120 miles. The bearing of town C from town B is 75° (N 75° E). The bearing of town A from town C is 240° (S 60° W). Find to the *nearest mile* the distance from A to C. [5,5]

OR

b Two forces of 11 pounds and 14 pounds, respectively, act on a body. The angle between the two forces is 28°. Find to the *nearest pound* the magnitude of the resultant force. [10]

*37 In a unit circle draw an angle, θ, where θ is greater than 180° but less than 225°. In this figure draw the line segments whose lengths represent $\sin \theta$, $\cos \theta$ and $\tan \theta$. Label each line segment to show which function it represents. [3,2,2,3]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

ELEVENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. In problems involving logarithms, answers should be left correct to four significant digits unless directions say otherwise. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit.

(1) 3	(11) 2	(21) 1
(2) 4	(12) 4	(22) 1
(3) 3	(13) 1	(23) 4
(4) 3	(14) 3	(24) 3
(5) 2	(15) 4	(25) 2
(6) 2	(16) 2	(26) 4
(7) 4	(17) 1	(27) 2
(8) 1	(18) 1	(28) 1
(9) 3	(19) 1	(29) 3
(10) 4	(20) 2	(30) 4

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(31) 1.8 and —0.3 [10]

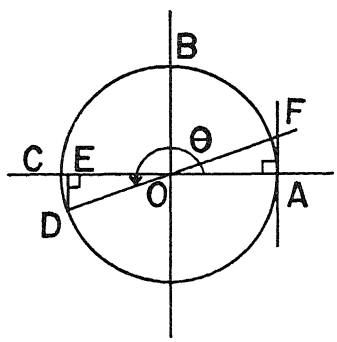
(32) *b* two [2]

(33) $x = -1$
$y = 0$ [8]
$z = 3$

(35) Analysis [4]
3, 4 [6]

(36) *a* Analysis [5]
218 [5]
b 24 [10]

(37) $OA = OD = 1$ [3]



$\sin \theta = ED$ [2]
$\cos \theta = EO$ [2]
$\tan \theta = AF$ [3]